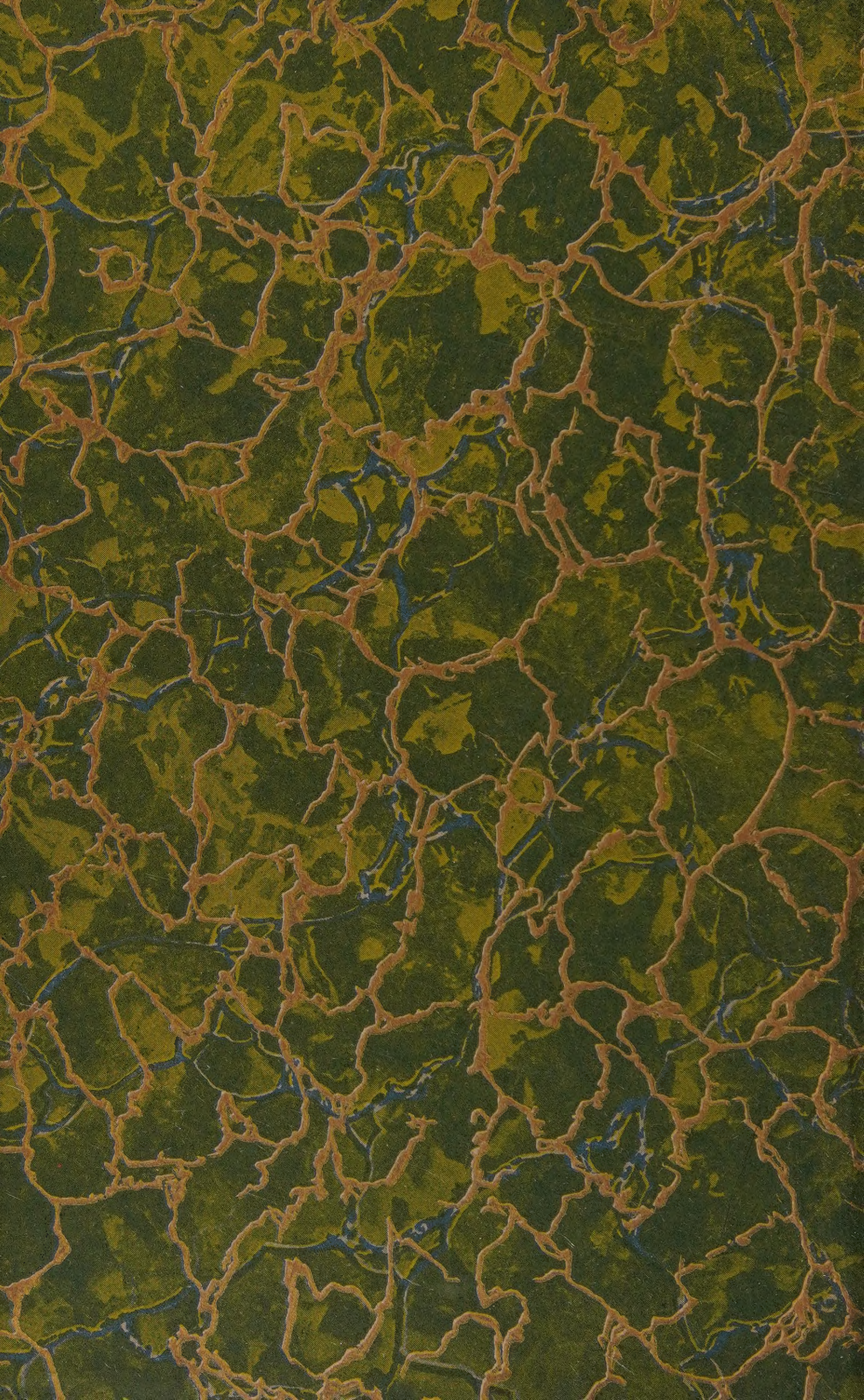

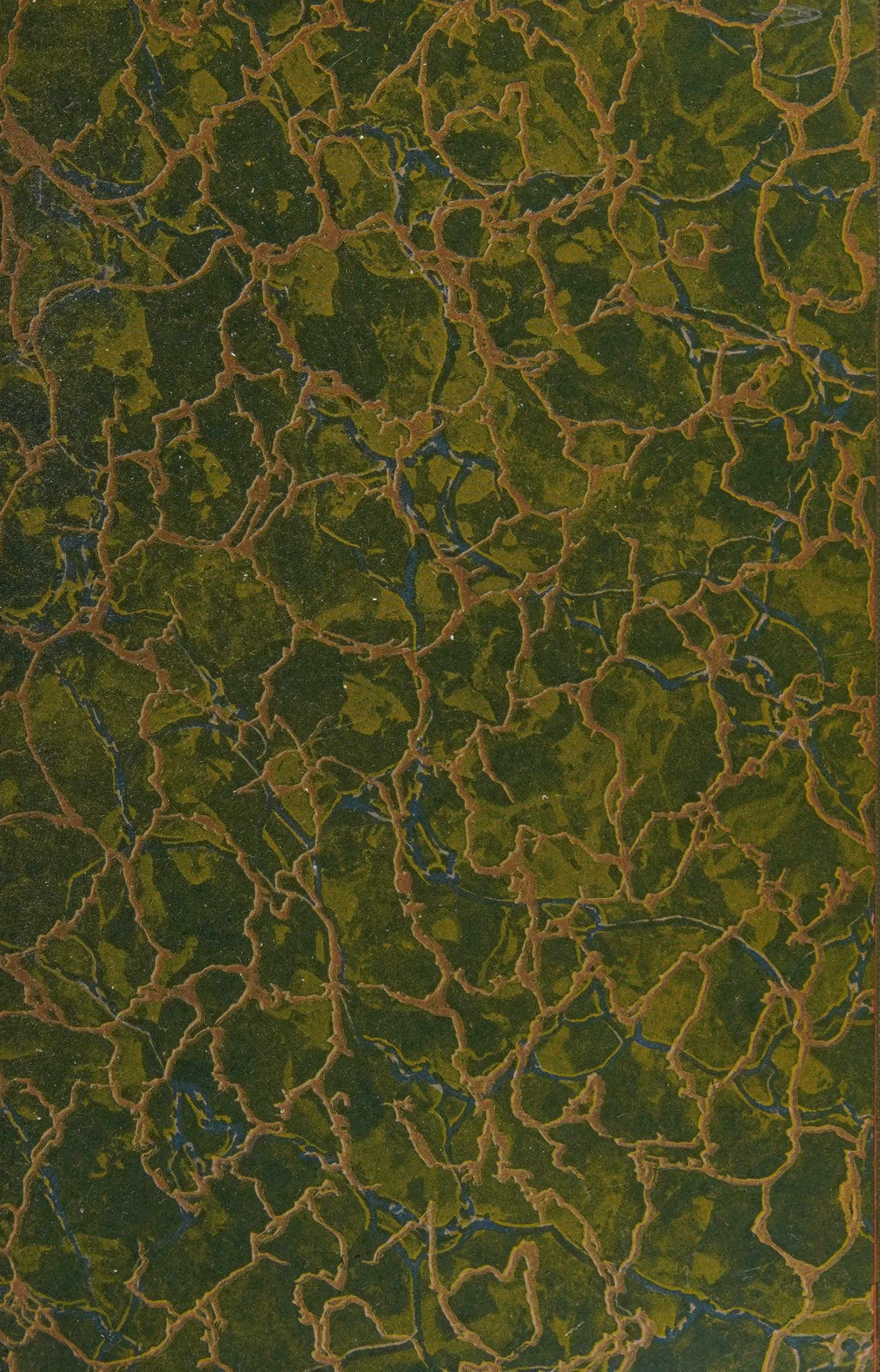

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Applied Mechanics and Strength of Materials

198 ILLUSTRATIONS

Prepared Under Supervision of

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LINK MECHANISMS
GEARING
GEAR TRAINS AND CAMS
PULLEYS AND BELTING
MATERIALS OF CONSTRUCTION
STRENGTH OF MATERIALS
THE TESTING OF MATERIALS

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the nonessentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

International Textbook Company

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NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents, the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

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LINK MECHANISMS

RELATIVE MOTIONS OF LINKS

INTRODUCTION

1. Mechanics.—The science that treats of the motions of bodies and of the forces that produce or tend to produce such motions is called **mechanics.**

Applied mechanics comprises the principles of pure mechanics as applied to the design and construction of machinery or to works of engineering. The part of applied mechanics that relates to machinery is called the **mechanics of machinery.**

The mechanics of machinery may be divided into two chief branches: **kinematics of machinery,** treating of the motions of machine parts without regard to the forces acting; **dynamics of machinery,** treating of the forces acting on machine parts and of the transmission of force from one part to another.

2. Free and Constrained Motion.—A body is said to be **free** when it may move in any direction in obedience to the forces acting on it. A body is **constrained** when the nature or direction of its motion is determined by its connection with other bodies.

Examples of free bodies are the moon, the sun, and other heavenly bodies. Examples of constrained bodies are seen in every machine. Thus, the crosshead of an engine is constrained by the guides, and the shaft by the bearings in which it turns.

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In constrained motion, every point of the constrained body is forced to move in a definite path, no matter what may be the direction of the force that causes the motion. Any force that tends to give the body some other motion is at once neutralized by an equal and opposite force developed in the constraining members. For example, the block *a*, Fig. **1**, is enclosed by stationary guides *b* and *c* and its only possible motion therefore is a sliding motion along the line *EF*. Suppose that the acting force *P* has the direction of the line *MN*. Then, if *a* were free, the force *P* would cause it to move along the line *M N*. Let *P* be resolved into the components *H* and *V*, respectively, acting parallel and perpendicular to *EF*. The force *H* causes *a* to move along *EF*; the force *V*, on the other hand, would cause it to move vertically downwards, but the downward push of *a* against the guide *c* develops a reaction *R* in the guide *c* that is equal and opposite to *V*. Hence, *V* and *R* are balanced, the net vertical force is zero, and as a result there can be no motion vertically. In this way, any force that tends to cause *a* to move in any direction except along the line *EF* creates a reaction in one of the restraining bodies *b*, *c* that exactly neutralizes that tendency.

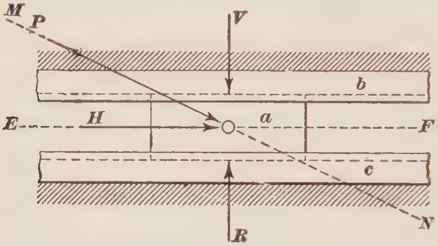

FIG. 1

3. Definition of a Machine.—A **machine** is an assemblage of fixed and moving parts so arranged as to utilize energy derived from some external source for the purpose of doing work.

In the operation of machinery, motion and force are communicated to one of the movable parts and transmitted to the part that does the work. During the transmission, both the motion and the force are modified in direction and amount, so as to be rendered suitable for the purpose to which they are to be applied.

The moving parts are so arranged as to have certain definite motions relative to each other, the effect of which is to compel the part doing the work to move in the required way. The nature of these movements is independent of the amount of force transmitted; in other words, in a model of a machine operated by hand, the relative motions of the parts will be precisely the same as in the machine itself, although in the latter case a great amount of power may be transmitted and much work done.

GENERAL KINEMATIC PRINCIPLES

KINDS OF CONSTRAINED MOTION

4. Plane Motion.—All constrained motions of rigid bodies, however complicated, may be divided into three classes; viz., *plane motion*, *spherical motion*, and *screw motion*.

A body is said to have **plane motion** when all its points move in parallel planes. Nearly all the motions of machine parts belong to this class. For example, the motions of all parts of the steam engine, except the governor, are plane; all points of the piston, piston rod, and crosshead move in equal parallel straight lines; all points of the crank, shaft, and flywheel move in circles of various radii that lie in parallel planes; and points of the connecting-rod describe oval curves, which likewise lie in parallel planes. There are two special cases of plane motion of much importance, namely, *rotation* and *translation*. A body is said to **rotate** about an axis when all points of it move in parallel circles whose centers all lie on the axis; this motion is very common. Thus, a flywheel rotates about the axis of the shaft, a pulley about the axis of the line shaft, etc. A body has a motion of **translation** when the direction of a straight line in that body is always parallel to or coincides with its original direction. The paths of any points in the body may be either straight or curved. The motions of the piston of a steam engine and the parallel rod of a locomotive are examples of translation in straight and curved paths, respectively.

5. Spherical Motion.—A body has **spherical motion** when each point of it remains always at a definite distance from some fixed point so as to move in the surface of an imaginary sphere with the fixed point as a center. In machinery, there are few examples of spherical motion; the universal joint and the balls of the steam-engine flyball governor are two familiar examples of this motion.

6. Screw motion consists of a rotation about a fixed axis combined with a translation along the axis. An example is the motion of a nut on a bolt.

Of the three forms of motion, plane motion occurs most frequently in machinery. Unless the contrary is stated, it is assumed in the following pages that the motion of a machine part is plane.

PLANE MOTION OF A RIGID BODY

7. Point Paths.—During the motion of a rigid body, such as a machine part, each point of the body traces a line, which is called the **path** of the point. In plane motion, the path lies in a plane, and in the case of a machine part the path is usually a closed curve; that is, a path that, if followed continuously, will bring the body to its original position. The path of any point of a body rotating about a fixed axis is a circle, which is a closed curve. The direction in which a point is moving at any instant is the direction of the tangent to the path of motion at the given point.

Also, in plane motion, the motions of any two points of a body determine the motion of the body as a whole. Thus, in the case of a connecting-rod, if the motions of any two points of the rod, at any instant, are known, the motion of the entire rod for that instant is determined.

8. The Instantaneous Center.—In Fig. 2, A and B are two points of a rigid figure, which may be in any plane section of the rigid body parallel to the plane of motion, and m and n, respectively, are their paths. The direction of motion of A at a certain instant is the tangent a, at the point A, to the path *m*. Similarly, the tangent *b* to the path *n* shows the direction of motion of *B* at the same instant. Let the lines *e* and *f* be drawn through *A* and *B* perpendicular, respectively, to the tangents *a* and *b*, and let *O* be their intersection. Suppose that some point be chosen, say *E*, on the line *e*, and that the figure be rotated about this point. For the sake of clearness, imagine the figure to be a disk of paper with a pin stuck through it at the point *E*. Evidently, when the disk is rotated, the direction of motion of *A* will be perpendicular to *AE*, that is, in the direction of the tangent *a*. Hence, a rotation about any point on the line *e* will cause *A* to move, for the instant, in the direction of the tangent *a*.

In the same way, a rotation about any point in the line *f* will cause *B* to move in the direction of the tangent *b*. Therefore, by choosing as the center of rotation the point *O*, which lies on both *e* and *f*, both *A* and *B* will be caused to move for the instant in their proper directions. But, in plane motion, the motions of two points of a body determine the motion of the whole body. Hence, the actual motion for the instant is a rotation about the point *O*, which for this reason is called the **instantaneous center** of the figure. Whatever may be the motion of a figure in a plane, it is possible to find a point about which a rotation will, for an instant, give the figure the same motion.

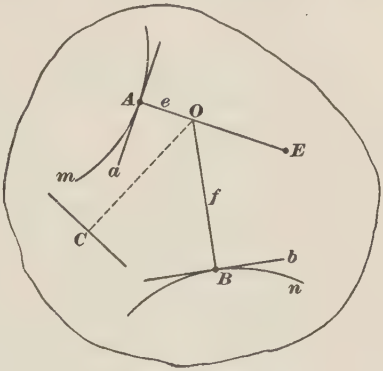

FIG. 2

Having the center *O*, it is easy to find the direction of motion of any third point, as *C*. Since, for the instant, the whole figure is rotating about *O*, the point *C* is moving in a direction perpendicular to *CO*. In general, at a given instant, any point of the figure is moving in a direction perpendicular to the line joining it to the center *O*.

9. The instantaneous center can always be found if the motions of two points are known. Through the two points lines are drawn perpendicular to the directions in which the points are moving, and the intersection of these two lines is the instantaneous center. In Fig. 3, for example, there are two links *a*, *b* that turn about the fixed points *A* and *B* and are pinned at *C* and *D* to a body *c*. The center of the pin *D* is a point of *c* as well as of *a*, and it is moving in a direction at right angles to the link *a*; hence, a line drawn from *D* at right angles to this direction is the center line of the link. Likewise, the link *b* stands at right angles to the direction of motion of *C* and therefore the intersection *P* of the center lines of the links, extended, is the instantaneous center for the motion of the body *c*. To find the direction of motion of any other point of *c*, as *E*, join *E* to *P*; then the direction of motion of *E* is perpendicular to *PE*.

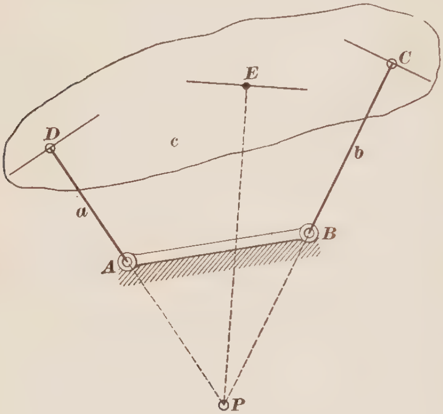

FIG. 3

In Fig. 4, the motion of the connecting-rod *B C* is considered. All points in the crank *A B* describe circles about the fixed point *A*. The end *B* of the connecting-rod *B C* rotates about the fixed point *A*, while the end *C* moves to and fro in the straight line *A X*. Evidently, the perpendicular to the direction of motion of *B* is the crank *A B*, and the perpendicular to the motion of *C* is a line perpendicular to *A X* at *C*; therefore the instantaneous

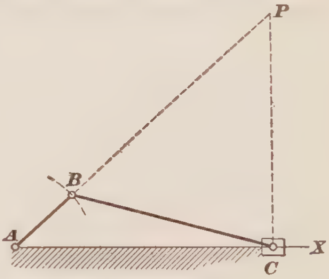

FIG. 4

center P lies at the intersection of the center line of the crank $A\,B$, extended, and a perpendicular to $A\,X$ at the point C.

10. If two points A, B, of a body, Fig. 5, are moving in the same direction, the perpendiculars m and n are parallel. In this case, the body has a motion of translation, and all points of it are moving in the same direction with the same speed.

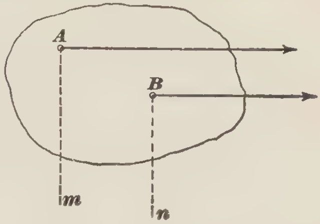

FIG. 5

11. Angular Velocity. A body rotating about a fixed axis is said to have unit angular velocity when the distance moved through in 1 second by any point is equal to the distance between the point and the axis. Hence, if the point chosen is 1 foot from the axis, the angular velocity of the body is unity when this point travels 1 foot in 1 second, that is, when its speed is 1 foot per second; if the point has a speed of 5 feet per second, the angular velocity is 5; and so on.

Suppose that a point at a distance of r feet from the axis has a speed of v feet per second. Then, as the speeds are proportional to the radii, a point at a distance of 1 foot from the axis has a speed of $\frac{1}{r} \times v = \frac{v}{r}$ feet per second, and the angular velocity is therefore $\frac{v}{r}$ units. Let w denote the angular velocity; then,

$$w = \frac{v}{r} \qquad (1)$$

and

$$v = r\,w \qquad (2)$$

Formula **1** may be used to determine the angular velocity of a body rotating about an instantaneous, instead of a fixed, axis, or, what is the same thing, the angular velocity of a point of a plane figure rotating about an instantaneous center. Thus, in Fig. 3, suppose that the velocity of the point D is 20 feet per second and the distance from D to the instantaneous center P is 4 feet; then the angular velocity of the body c about the center of rotation P is, by formula **1**.

$$w = \frac{v}{r} = \frac{20}{4} = 5$$

Having the angular velocity of the body, the linear velocity of any other point, as C or E, is easily found by formula **2**; thus,

linear velocity of C = angular velocity × PC
linear velocity of E = angular velocity × PE

In the foregoing consideration of angular velocity, the unit is based on the angle subtended by an arc equal to the radius of the circle forming the path of the point in motion; this angle is called a *radian*. In other words, a **radian** is the angle subtended by an arc equal in length to its radius.

The length of the circumference of a circle, that is, the arc subtending the angle of one complete revolution, or 360°, is $2\pi r$. This angle, measured in radians, is therefore equal to $\frac{2\pi r}{r} = 2\pi$ radians. It follows, therefore, that one radian is equal to $\frac{360}{2\pi} = 57.296°$.

RELATIVE MOTION

12. Two bodies, each of which is moving relative to some fixed body, have, in general, relative motion. For example, the crank and the connecting-rod of an engine have each a certain motion relative to the frame; each has also a motion relative to the other, which motion is a turning about the axis of a crankpin.

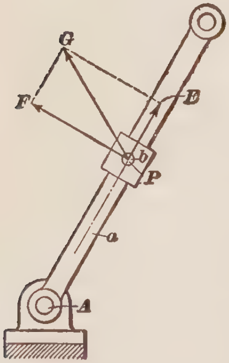

FIG. 6

An illustration of the principle of relative motion is shown in Fig. 6. A block b is constrained to slide on a rod a, and this rod is pinned at A to a fixed body, so that the only possible motion of a is a rotation about A. Now, if a does not move, the block b simply slides along a; if, however, *a* rotates while *b* slides along it, the actual motion of *b* will have the direction PG, but its motion relative to *a* is in the direction PE, just as though *a* were at rest. That is, the motion of *b* relative to *a* is not in the least affected by the motion of *a*. In general, the relative motion of two bodies is not affected by any motion they may have in common. This is a principle of great importance, and may be further illustrated by the following familiar examples: The relative motions of the parts of a marine engine are not influenced by the rolling of the ship. The relative motions of the moving parts of a locomotive are not affected by the motion of the locomotive on the track.

From this principle, it follows that in studying the relative motions of two bodies, any motion common to both may be neglected; also, if desirable, a common motion may be given them without affecting their relative motions.

LEVERS

13. Use of Levers.—Levers are used in mechanisms to guide a moving point, as the end of a moving rod, or to transfer motion from one line to another. There are three kinds of levers: (1) Levers whose lines of motion are parallel; (2) levers whose lines of motion intersect; and (3) levers having arms whose center lines do not lie in the same plane.

In proportioning levers, the following points should in general be observed; they apply to all three cases just mentioned:

1. When in mid-position, the center lines of the arms should be perpendicular to the lines along which they give or take their motions, so that the lever will vibrate equally each way.
2. If a vibrating link is connected to the lever, its point of attachment should be so located as to move equally on each side of the center line of motion of the link.
3. The lengths of the lever arms must be proportional to the distances through which they are to vibrate.

14. Reversing Levers.—An example of a lever that illustrates the foregoing principles is shown in Fig. 7. The crank SR is the driver, and through the connecting-rod RH gives a motion to the point H approximately along the center line AB, which is transferred by the lever EH to the line CD. The lever vibrates equally each way about its fulcrum or center O, as indicated by the lines cb and da. When in mid-position, its center line EH is perpendicular to the lines of motion CD and AB. The horizontal distances traversed by the points E and H, respectively, are proportional to the arms EO and HO, or $y : EO = x : HO$. The vibrating link EK connects the point E with the rod KD, which is constrained to move in a straight line by the guide g, and, in accordance with principle 2, the lever is so proportioned that the point E will be as far above the center line of motion CD, when in mid-position, as it will be below it in the two extreme positions; that is, the points c and d are as far below the line as the point E is above it.

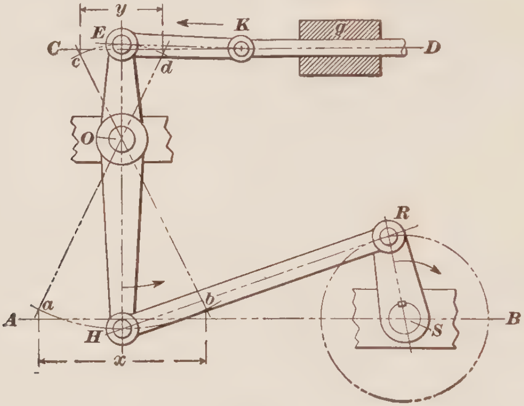

FIG. 7

At the bottom of the lever, where the rod HR connects with the crank, the same principle holds, the point H being as far below the line AB as the points a and b are above it. Frequently, the distance between the center lines CD and AB is given, and the extent of the motion along these lines, from which to proportion the lever. A correct solution to this problem is troublesome by calculation, because it is not known at the start how far above and below their respective lines of motion the points E and H should be.

A graphic solution is shown in Fig. 8. Draw the center lines of motion CD and AB, and a center line ST perpendicular to them. Draw ME parallel to ST at a distance from it equal to $\frac{1}{2}y$, or half the stroke along CD; also, the parallel line HN, on the other side of ST, and at a distance from it equal to $\frac{1}{2}x$, or half the stroke along AB. Connect points M and N by a straight line; where this line intersects ST, as at O, will be the center or fulcrum of the lever. With O as a center, find by trial the radius of an arc that will cut ST as far below the line AB as it does HN above this line, or so that the distance n will be equal to the distance m. As an aid in determining the correct radius, describe an arc cutting ST, with O as a center and a radius ON. The distance n will be a little more than $\frac{1}{2}l$. Now, draw a straight line through points H and O. The part included between HN and ME determines the length of the lever. In this case, the length of the shorter arm is equal to OE, and that of the longer arm to OH.

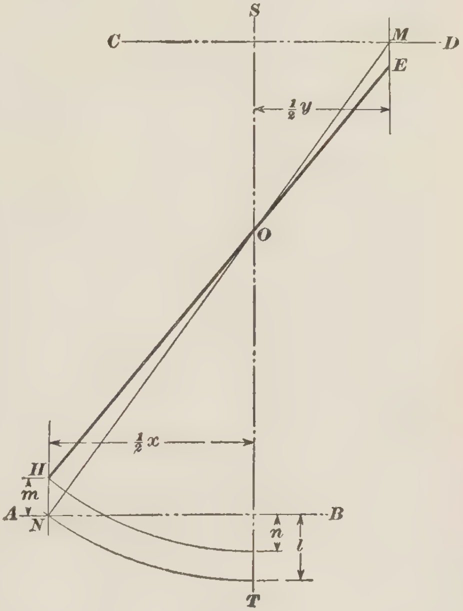

FIG. 8

15. Non-Reversing Levers.—Fig. 9 shows the same construction applied to a lever in which the center O is at one end of the lever. This lever does not reverse the motion like the previous one, since, when the motion along $A\,B$ is to the right or left, the motion along $C\,D$ will be in the same direction. The figure is lettered like the preceding one, so that the construction will be easily understood.

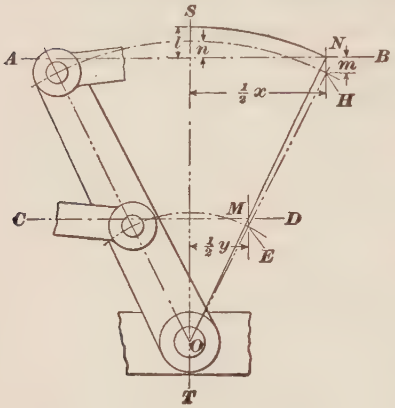

FIG. 9

16. Reducing Motions.—It is often desirable that a lever mechanism shall reproduce on a greater or smaller scale, along one line, the exact motion that occurs along another line; that is, for every change in the rate of the motion along one line a corresponding change shall be produced along the other line. Figs. 10 and 11 illustrate three indicator reducing motions that accomplish this.

In Fig. 10, the lower end of the lever attaches to the crosshead of the engine through the swinging link $H\,R$. The indicator string is fastened to the bar $C\,D$, which receives its motion from the lever through the link $E\,K$, and slides through the guides g, g in a direction parallel to the line of motion $A\,B$ of the crosshead. In order that the bar $C\,D$ shall have the same kind of motion as the crosshead, it is necessary that the lengths of the links $E\,K$ and $H\,R$ shall be proportional to their respective lever arms; thus $O\,H : H\,R = O\,E : E\,K$. The pins must be so placed

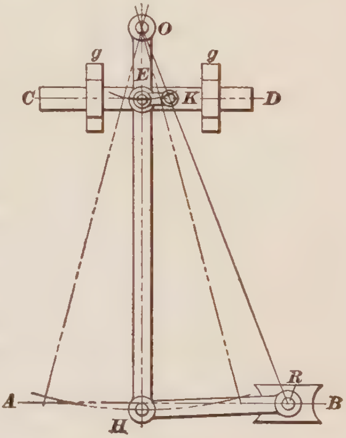

FIG. 10

that the connecting links will be parallel; if parallel at one point of the stroke, they will be so at all points. When the links EK and HR are parallel, $OK : OR = OE : OH$, and the length of the indicator diagram will bear the same ratio to the length of the stroke as OE bears to OH.

It is to be observed that the pins O, K, and R are in one straight line, and, in general, it may be said that any arrangement of the lever that will keep these three pins in a straight line for all points of the stroke will be a correct one.

In Fig. 11, two such arrangements are shown. In the first, the pins K and R are fast to the slide and crosshead, respectively, and slide in slots in the lever. In the second, they are fast to the lever, the slots being in the slide and crosshead. In both, the pins K and R are in a straight line with the pin O during the whole stroke.

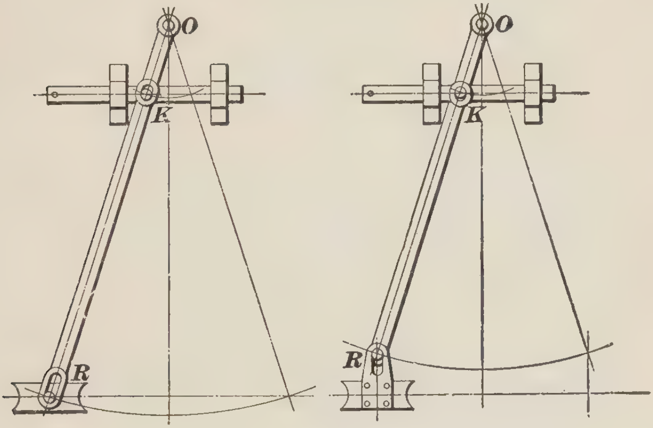

FIG. 11

17. Bell-Crank Levers.—Levers whose lines of motion intersect are termed **bell-crank levers**; they are used very extensively in machine construction for changing the direction and the amount of motion. The method of laying out a lever of this kind to suit a given condition is as follows:

In Fig. 12, suppose the angle CAB, made by the lines of motion, to be given, and that the motion along AB is to be twice that along *A C*. Draw *c d* parallel to *A C* at any convenient distance from *A C*. Draw *a b* parallel to *A B* and at a distance from *A B* equal to twice the distance of *c d* from *A C*.

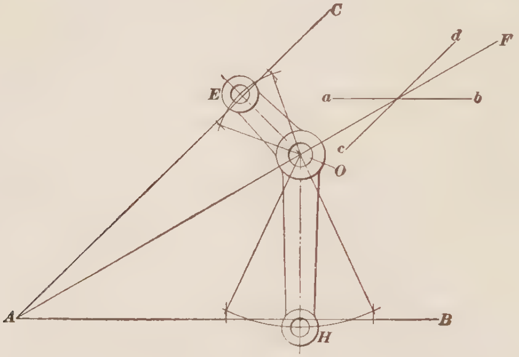

FIG. 12

Through the intersection of these two lines and the apex *A* of the angle, draw the line *A F*. The center *O* of the bell-crank may be taken at any point on *A F* suited to the design

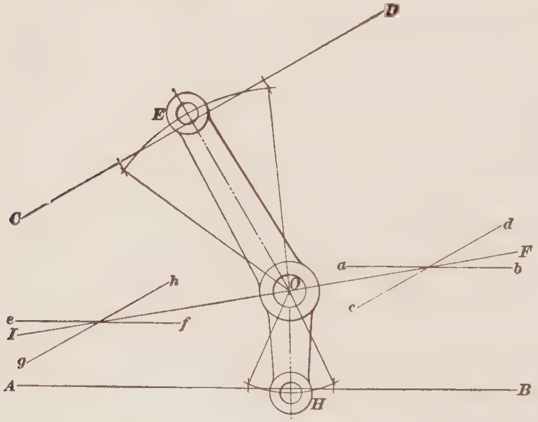

FIG. 13

of the machine. Having chosen point *O*, draw the perpendiculars *O E* and *O H*, which will be the center lines of the lever arms.

In Fig. 13, a construction is shown that may be employed when the two lines *CD* and *AB* do not intersect within the limits of the drawing. In Fig. 14, the same construction is applied to a non-reversing lever, in which the center *O* falls outside of the lines *AB* and *CD*. The figures are lettered alike, and the following explanation applies to both. Draw *cd* parallel to *CD* and *ab* parallel to *AB*, as before, so that the distance of *cd* from *CD* : distance of *ab* from *AB* = amount of motion along *CD* : amount of motion along *AB*. Again, draw lines *gh* and *ef* in exactly the same way, but taking care to get their distances from *CD* and *AB* different from those of the lines just drawn. Thus, if *cd* should be 6 inches from *CD*, make *gh* some other distance, as 4 inches or 8 inches, and then draw *ef* at a proportionate distance from *AB*. Through the intersections of *ab*

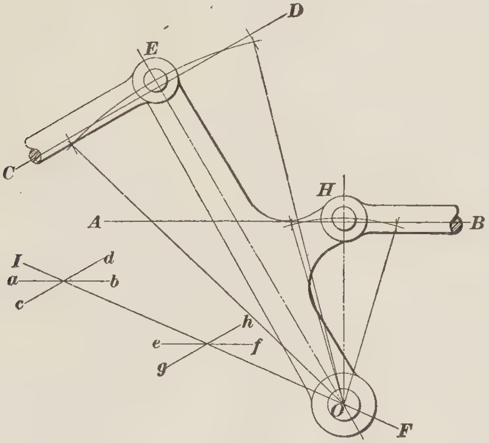

FIG. 14

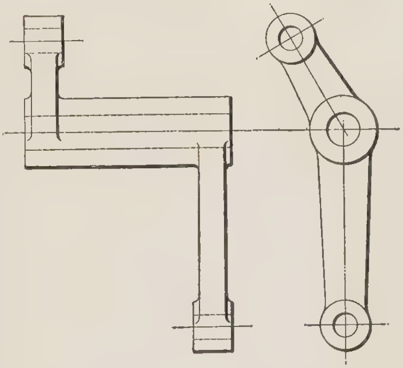

FIG. 15

with *c d* and *e f* with *g h*, draw the line *I F*, which will be the line of centers for the fulcrum *O*.

Levers falling under the third classification are usually bell-crank levers, with their arms separated by a long hub, so as to lie in different planes. They introduce no new principle. Fig. 15 shows the general construction of a lever of this kind.

18. Slow-Motion Mechanism.—A mechanism consisting of two connected levers, or of a crank and lever, is sometimes used to secure slow motion in one of the levers. Such an arrangement is shown in Fig. 16 where two levers, *a* and *b*, are arranged to turn on fixed centers and are connected by the rod *r*. The lever *a* is actuated by the handle *h* secured to the same shaft. If *h* and the lever *a* be turned counter-clockwise, the lever *b* will turn clockwise, but with decreasing velocity, which will become zero when the lever *a* and the rod *r* lie in the same line. Any further motion of *a* will cause *b* to return toward its first position, its motion being slow at first and then faster.

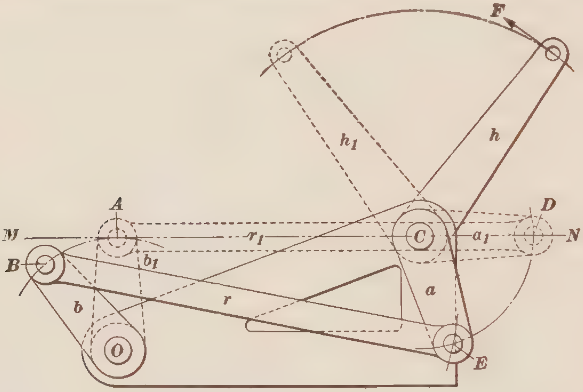

FIG. 16

To obtain the greatest advantage, the lever *b* should be so placed that it will occupy a position perpendicular to the link *r* at the instant when *a* and *r* are in line. To lay out the motion, therefore, supposing the positions of the centers and lengths of the levers to be known, describe the arc *B A* about the center *O*, with a radius equal to the length of the lever *b*. Through *C*, the center of the lever *a*, draw the line *MN* tangent to the arc just drawn; *r* and *a* must then be in line along the line *MN*, and the lever *b* must be perpendicular to this line when in position b_1. Generally, there will be a certain required amount of movement for the lever *b*. To secure this, draw *b* in its extreme left-hand position; then with *C* as a center and a radius equal to the length of *a*, draw the arc *ED*. Set the compasses to the length *AD*, and from *B* as a center draw an arc cutting the arc *ED* at *E*. *CE* will be the second position of the lever *a*. In this combination, if the parts were proportioned to allow *a* to rotate like a crank, *r* and *a* would come into line twice during each revolution.

This mechanism has been used to operate platen printing presses, in which oscillation of the handle *h* moves the platen to and from the type through the lever *b*. It is also used to operate the exhaust valves in Corliss engines.

THE STEAM-ENGINE MECHANISM

19. Relative Positions of Crank and Crosshead. The frame, connecting-rod, crosshead, and crank of a steam engine form what is called a **four-link slider-crank** mechanism. This mechanism is shown in diagrammatic form in Fig. 17, with the frame at *a*, the crank at *b*, the connecting-rod at *c*, and the crosshead at *D*. It is customary to speak of these parts as links, the frame *a* being termed the fixed link, since it is stationary, and the motions of the other links are considered with respect to the fixed link. The crank rotates about the point *O* with a uniform motion, and the crankpin *E* describes the crankpin circle. The crosshead moves to and fro in a straight line, and the connecting-rod has a rather complex motion, since one end moves in a circle, while the other reciprocates in a straight line. The object of this mechanism is to transform the to-and-fro motion of the piston and crosshead into continuous rotary motion of the crank and shaft, and thereby enable the energy of the steam to be utilized in doing useful work.

In the study of the steam-engine mechanism, there are two points to be investigated: (1) the position of the crosshead, or piston, in the case of the actual engine, for any crank position; and (2) the velocity of the crosshead or piston at any position of the crank. In Fig. 17, let *m* be the crankpin circle. From the dead centers *F* and *G*, with a radius equal to the length *DE* of the connecting-rod, draw arcs at *H* and *K*. Then *H* and *K* are the extreme positions of the crosshead pin. To find the position of the crosshead for any crank position, as, for example, *OL*, take *L* as a center, and with a radius equal to *DE* draw an arc cutting *HK* at *N*. Then *N* is the position of the crosshead when the crank is at *OL*. To find the position of the crank for a given crosshead position, as *R*, take *R* as a center, and with *DE* as a radius draw an arc cutting the crankpin circle at *S* and *T*. Then, when the crosshead is at *R* the crank may be at *OS* or at *OT*.

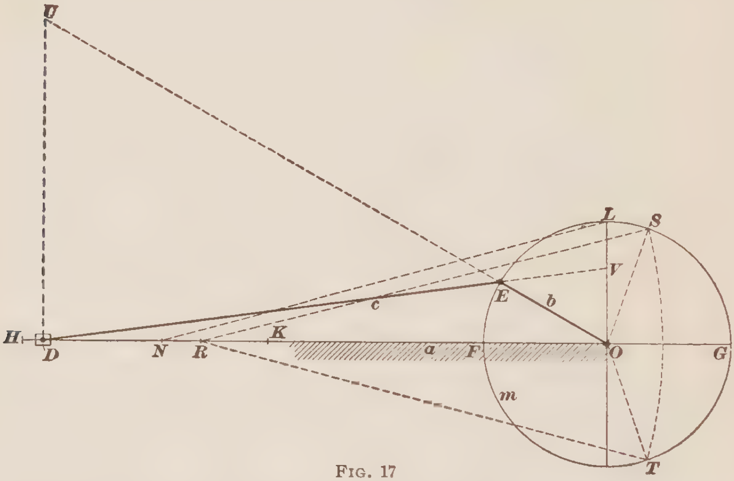

FIG. 17

20. Relative Velocities of Crosshead and Crankpin.—The crankpin moves with a practically uniform velocity, while the crosshead has a variable velocity. Starting from rest, at the beginning of the stroke, the velocity of the crosshead increases to a maximum, and then decreases to zero at the end of the stroke before starting on the return. When the velocity of the crankpin is known, the velocity of the crosshead can be easily determined for any crank position. In Fig. 17, produce the center line OE of the crank indefinitely, and at D erect a line perpendicular to HO. This perpendicular will intersect the center line of the crank at a point U, which, according to Art. **9,** is the instantaneous center for the connecting-rod DE, since the points D and E are rotating, for the instant, about U. For the instant, then, the linear velocities of D and E are proportional to their distances from U, since the linear velocities of points in a rotating body are proportional to their radii. Hence,

$$\frac{\text{the linear velocity of } E}{\text{the linear velocity of } D} = \frac{UE}{UD}$$

Extend the center line of the connecting-rod until it intersects the vertical line OL at V. By geometry, it may be proved that the triangle VOE is similar to the triangle DUE, and that their corresponding sides are proportional. That is,

$$\frac{OE}{OV} = \frac{UE}{UD} = \frac{\text{the linear velocity of } E}{\text{the linear velocity of } D}$$

Therefore, *the velocity of the crankpin is to the velocity of the crosshead as the length of the crank is to the intercept that the connecting-rod, or the connecting-rod produced, cuts off on the perpendicular through the center of the main bearing.*

21. Velocity Diagrams for Crosshead.—The conclusion reached in the last article may be used in constructing a diagram that will represent the velocities of the crosshead at different points of its travel. Such a diagram is shown by the curve *1–2′–3′*, etc., Fig. 18.

It is assumed that the crankpin of an engine moves with a constant velocity, and the length AB of the crank may be assumed to represent, to some scale, this velocity. Then, by placing the center of the crosshead pin in each of the positions *1*, *2*, *3*, etc. and extending the center lines of the corresponding connecting-rod positions until they intersect the vertical line $A\,C$, the points $1''$, $2''$, $3''$, etc. are located. At the position *1*, the velocity of the crosshead is zero, and at the position *2* its velocity is represented by the distance $A\,2''$. This distance is laid off vertically above *2*, making $2-2' = A\,2''$. Similarly, the distances $3-3'$, $4-4'$, etc. are equal, respectively, to $A\,3''$, $A\,4''$, etc., and represent the velocities of the crosshead at the points *3*, *4*, etc. By locating from eight to twelve crosshead positions, enough points will be obtained to permit a curve to be drawn through them, and this curve $1-2'-3'-4'$, etc. is the velocity diagram of the crosshead. The height of any point on this curve above the horizontal line *1–10* gives the velocity of the crosshead, and hence of the piston also, for the corresponding position of the crank, to the same scale that the length of the crank represents the linear velocity of the crankpin.

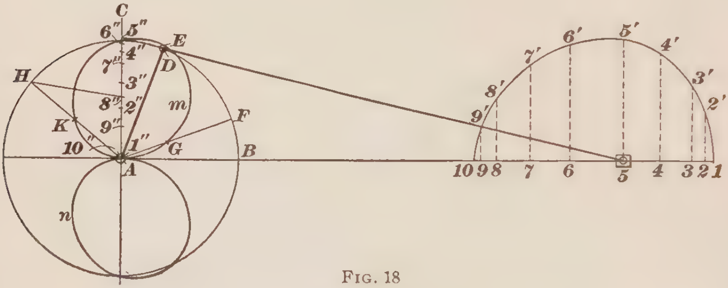

FIG. 18

Instead of laying off the velocities vertically above *1–10*, they may be laid off on the corresponding crank positions. That is, in the position $A\,D$ of the crank, the velocity $A\,E$ of the crosshead is laid off equal to $A-5''$; on $A\,F$, the velocity $A\,G$ is laid off equal to $A-2''$; and so on for each crank position. By taking several crank positions, a number of points, A, G, E, etc., are obtained, through which a smooth curve *m* may be drawn. This curve is then the diagram of crosshead velocities for the forward stroke.

To determine the crosshead velocity at any position of the crank, simply draw the crank in the desired position and note the distance from the center A to the point where the crank-line intersects the curve m. For example, if the crank is at AH, AK represents the corresponding crosshead velocity. If, now, the crosshead velocities be determined for the positions 10 to 1 on the return stroke, they will be found to be equal to the corresponding velocities on the forward stroke. Hence, if the curve n is plotted in the same manner as m, but below the center line, as shown, it will represent the velocity diagram for the return stroke. Moreover, the two curves m and n will be exactly alike, and will be symmetrical with respect to the center line. The curve $1-4'-7'-10$ may also be used to find the crosshead velocity at any point on either stroke.

22. Crank and Slotted Crosshead.—If the connecting-rod in Fig. 17 be increased to a very great length, an arc drawn through L will be nearly a straight line coinciding with LO, and the horizontal movement of the crankpin will, therefore, be practically the same as that of the crosshead. If the connecting-rod were increased to an infinite length, the two movements would be exactly the same. Fig. 19 shows the crank and slotted crosshead mechanism by which this is accomplished. Consider the crank ob as the driver. The crankpin b is a working fit in the block r, which is arranged to slide in the slotted link l. The rods f and h are rigidly attached to the link, and are compelled to move in a straight line by the guides g, g'. As the crank rotates, the rods f and h are given a horizontal motion exactly equal to the horizontal motion of the pin b. If the crank rotates uniformly, the motion of the sliding rods is said to be **harmonic**, and the mechanism itself is often called the **harmonic-motion** mechanism.

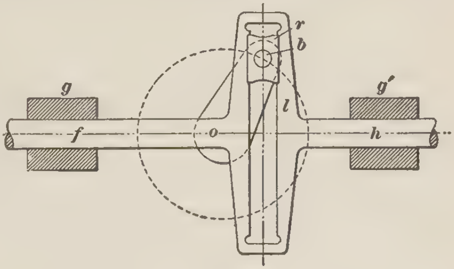

FIG. 19

Harmonic motion may be defined as the motion of the foot of a perpendicular let fall on the diameter of a circle from a point moving with uniform velocity along the circumference.

23. The Togglejoint.—The **togglejoint,** shown in Fig. 20, is a mechanism for producing a heavy pressure by the application of a small effort. It will be seen that it resembles the steam-engine mechanism with crank and connecting-rod of about the same length. The effort F is applied at the joint B and the resistance P at the slide. The mechanical advantage, that is, the ratio of resistance to effort, depends on the angle between the links. It is evident that the point Q is the instantaneous center of motion of the link AB. Now, as the angle ABQ becomes smaller, that is, as the links approach the position in which they would form a straight horizontal line, the center Q moves toward the joint A, and the velocity of A relative to that of B grows less.

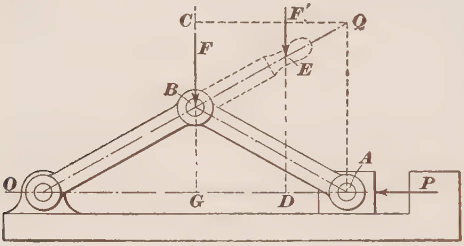

Fig. 20

Since Q is the instantaneous center and QA is perpendicular to the direction of the force P, the product $P \times QA$ is the moment of P about Q. The moment of F about Q is $F \times QC$. Considering the mechanism to be, for the instant, in equilibrium, and neglecting friction, $P \times QA = F \times QC$, or

$$P = F \times \frac{QC}{QA} \qquad (1)$$

This formula may be used to calculate the pressure P when the force acts vertically at B.

If the force acts at some other point than B, as, for example, F' at E, it becomes necessary to find the equivalent pressure at the joint B. Assuming that F' and F are parallel, and taking moments about O, $F' \times OD = F \times OG$, or $F = F' \times \frac{OD}{OG}$. But, $\frac{OD}{OG} = \frac{OE}{OB}$, since the triangles OGB and ODE are similar. Hence, $F = F' \times \frac{OE}{OB}$. Substituting this value of F in formula **1**,

$$P = F' \times \frac{OE \times QC}{OB \times QA} \qquad (2)$$

In case the force acts other than vertically, it is necessary to find its vertical component, which, when substituted for F or F' in the proper formula, will give the value of P. The distances QC and QA will vary according to the position of the toggle. However, by laying out the mechanism accurately to a fairly large scale, these distances may be measured with sufficient accuracy, so that the pressure P may be calculated for any position.

EXAMPLE 1.—In Fig. 20, let F equal 80 pounds; QC, 24 inches; and QA, 5 inches; find the pressure P.

SOLUTION.—Applying formula **1**,

$$P = 80 \times \frac{24}{5} = 384 \text{ lb. Ans.}$$

EXAMPLE 2.—In a togglejoint like that shown in Fig. 20, the force F' is 32 pounds; OB, 12 inches; OE, 30 inches; QC, 30 inches; and QA, 6 inches; what is the pressure P?

SOLUTION.—Applying formula **2** and substituting the values given,

$$P = 32 \times \frac{30 \times 30}{12 \times 6} = 400 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. In Fig. 20, if QA is 8 inches and QC is 36 inches, what force must be applied vertically at B to produce a pressure P of 180 pounds?
Ans. 40 lb.

2. In a togglejoint similar to that in Fig. 20, the arms OB, AB, and BQ are equal, each being 14 inches in length; the height BG is 4 inches; and the force F is 2,800 pounds. Find the pressure P, remembering that CQ equals GA, and that the triangles OBG and OQA are similar.
Ans. 4,695.74 lb.

3. If, in Fig. 20, OE is 21 inches; QC, 24 inches; OB, 16 inches; and QA, 6 inches; find the pressure P when the vertical force F' is 88 pounds.
Ans. 462 lb.

4. In case a force F of 200 pounds, Fig. 20, should act at right angles to $O B$ at B, find the pressure P when $Q C$ equals 32 inches; $Q A$, 8 inches; and $O Q$, 60 inches, remembering that the angle that F makes with the vertical is equal to the angle $Q O A$.
Ans. 793 lb., nearly

5. A point revolves about an axis at a speed of 4,200 feet per minute; if the point is 5 feet from the axis, what is its angular velocity in radians per second? Ans. 14 radians per sec.

6. The angular velocity of a point is 25 radians per second and its distance from the axis of revolution is 8 feet; what is its linear velocity?
Ans. 200 ft. per sec.

7. A flywheel having an outside diameter of 24 feet makes 56 revolutions per minute; find the angular velocity of the wheel, in radians per second. Ans. 5.86 radians per sec.

QUICK-RETURN MOTIONS

24. Vibrating-Link Motion.—**Quick-return motions** are used in shapers, slotters, and other machines, where all the useful work is done during the stroke of a reciprocating piece in one direction. During the working stroke, the tool must move at a suitable cutting speed, while on the return stroke, when no work is performed, it is desirable that it should travel more rapidly. The mechanism shown in Fig. 21, known as a **vibrating-link motion,** is applied to shaping machines operating on metal. Motion is received from the pinion p, which drives the gear g. The pin b is fast to the gear, and

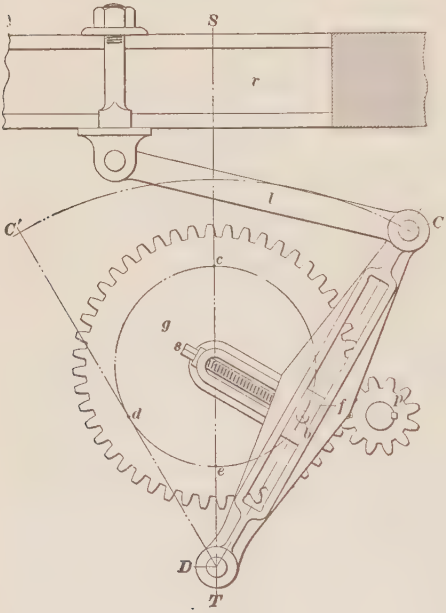

FIG. 21

pivoted to it is the block *f*, which is fitted to the slot of the oscillating link *CD*. As the gear rotates, the pin describes the circle *bedc*, the block slides in the slot of the link *CD*, and causes *CD* to oscillate about the point *D*, as indicated by the arc *CC′*, the path of the joint *C*. The rod *l* connects the upper end of the link with the tool slide, or ram, *r*, which is constrained by guides (not shown) to reciprocate in a straight horizontal line.

During the cutting stroke, the pin *b* travels over the arc *dcb*, or around the greater arc included between the points of tangency of the center lines *C′D* and *CD*. During the return stroke, the pin passes over the shorter arc *bed* and as the gear *g* rotates with a uniform velocity, the times of the forward and return strokes will be to each other as the length of the arc *dcb* is to the length of the arc *bed*. The throw of the slotted link and the travel of the tool can be varied by the screw *s*, which moves the block *f* to and from the center of the gear. The rod *l*, instead of vibrating equally above and below a center line of motion, is so arranged that the force moving the ram during the cutting stroke will always be downwards, causing it to rest firmly on the guides.

25. To lay out the motion, proceed as follows: Draw the center line *ST*, Fig. 22, and parallel to it the line *mn*, the distance between the two being equal to one-half the longest stroke of the tool. About *O*, which is assumed to be the center of the gear, describe the circle *bdc* with a radius equal to the distance from the center of the pin *b* to the center of the gear *g*, Fig. 21, when set for the longest stroke. Divide the circumference of the circle at *b* and *d* into upper and lower arcs extending equally on each side of the center line *ST*, and such that their ratio is equal to that of the times of the forward and return strokes. In this case the time of the forward stroke is double that of the return stroke, and the circle is divided into three equal parts, as shown at *b*, *d*, and *c*, thus making the arc *db* equal to one-half the arc *dcb*. Draw the radial lines *Ob* and *Od*. Through *b* draw *C D* perpendicular to *O b*; the point *D* where it intersects *S T* will be the fulcrum, and the point *C* where it intersects *m n* the upper end of the slotted lever. Through *C* draw the horizontal line *C C′*, making *C′ E* equal to *C E*. Draw *C′ D*, which will be tangent to the circle at *d*, thus giving the other extreme position of the lever.

To plot the motion, draw the center line of motion *R L* through the point at which the connecting-rod attaches to the tool slide. Divide the circle *d c b* into a number of equal parts, as *1*, *2*, *3*, etc., and from *D* draw lines through these points, extending to the arc *C′ C*. Number the points of division on the circle, and give corresponding numbers to the points of intersection on the arc *C′ C*. With these last points as centers, and with a radius equal to the required length of the connecting-rod *l*, in Fig. 21, strike arcs cutting the line *R L* and number these intersections so that they will correspond to the other points. In Fig. 22, assuming the gear to turn with a uniform motion, the tool slide will move along *R L* from point *1* to *2* during the first one-twelfth of a revolution; during the next one-twelfth revolution, from

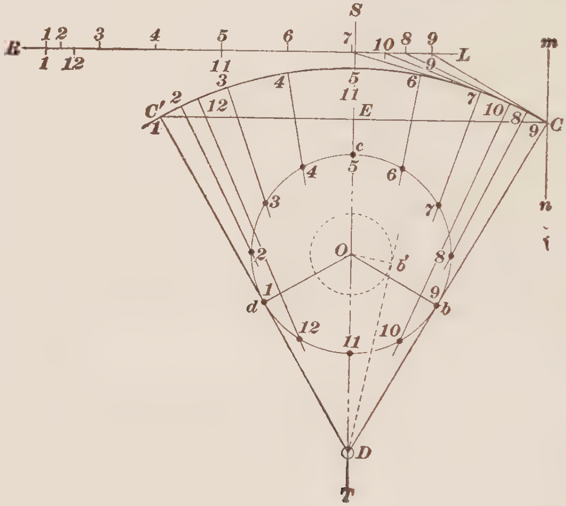

FIG. 22

point *2* to *3*, etc., on the forward stroke. On the return stroke, from point *9* to *10*, *11*, *12*, and *1*, the motion is much less uniform.

A property of this motion is that, as the radius Ob is diminished to shorten the stroke, the return becomes less rapid, as can be seen from the figure by comparing the motion when Ob is the radius with the motion when Ob' is the radius.

26. Whitworth Quick-Return Motion.—This mechanism is shown in principle in Fig. 23. The pin b, inserted in the side of the gear f, gives motion to the slotted link CD, as in the vibrating link motion. This motion closely resembles the previous one, the difference being that the center D of the slotted link lies within the circle described by the pin b, while in the previous case it lies without it. To accomplish this result, a pin p is provided for the gear to turn on, and is made large enough to include another pin D placed eccentrically within it, which acts as the center for the link CD. With this arrangement, the slotted link, instead of oscillating, follows the crankpin during the complete revolution, and thus becomes a crank. The stroke line RL passes through the center D, which is below the center O of the pin p. The forward or working stroke occurs

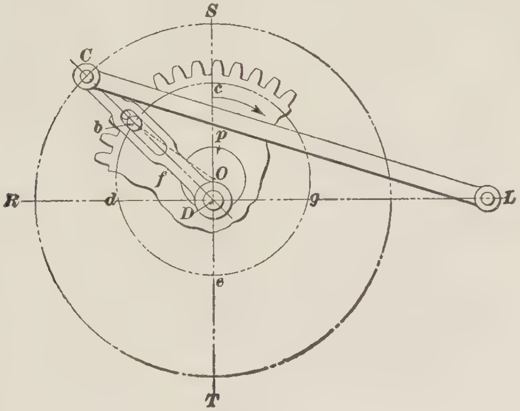

FIG. 23

while the crankpin *b* passes over the arc *dcg*, and the quick return occurs while it travels over the arc *ged*. During each of these intervals, the link *CD* completes a half revolution, and, consequently, must move more rapidly, while the crankpin describes the shorter arc *ged*.

27. To proportion the motion, it is only necessary to so locate the stroke line *RL* that it will divide the crankpin circle *dcge* into two arcs, *dcg* and *ged*, in the ratio of the times of the forward and return strokes. The point *D* where this line cuts the center line *ST* is the position for the center of the slotted crank. The motion is plotted as in Fig. 24. Divide the crankpin circle *dcge* into a number of equal parts. From the center of the slotted crank, draw radial lines through these points to the outer circle, which represents the path of pin *C*, Fig. 23, using the latter points of intersection as centers, and, with a radius equal to the length of the connecting-rod, strike off points on the stroke line, which will show the movement of the tool for equal amounts of rotation of the driving gear.

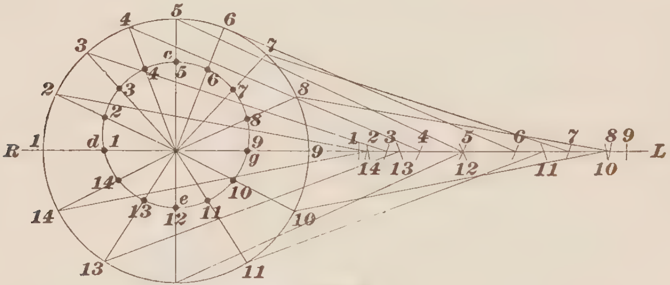

FIG. 24

Fig. 25 shows the mechanism as practically constructed. The gear *g* is driven with a uniform velocity in the direction of the arrow by the pinion *h*. It rotates on the large pin *p*, which is a part of the frame of the machine, and carries the pin *b* which turns in the block *k*. This block is capable of sliding in a radial slot in the piece *f*, as is shown in the sectional view. This piece *f* is supported by the shaft *d*, which turns in a bearing extending through the lower part of the large pin. The line *R L* drawn through the center of *d* is the line of motion of the tool slide. The connecting-rod that actuates the tool slide is pinned to the stud *c*, which in turn is clamped to the piece *f*.

28. The radial slot *t*, in Fig. 25, holds the stud *c*, to which the connecting-rod is attached; the slot provides for the adjustment of the *length* of the stroke, and if the point of attachment of the rod to the tool slide is also made adjustable, the *position* of the stroke, as well as its length, can be changed. Thus, in the case of a shaping machine, it is not only desirable to regulate the *distance* passed over by the tool, but to have the stroke extend *exactly to a certain point*. These two adjustments are often required in mechanisms where reciprocating pieces are employed.

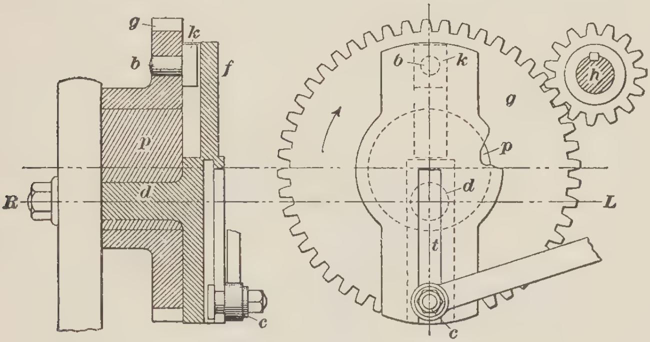

Fig. 25

STRAIGHT-LINE MOTIONS

29. Parallel Motion.—A **parallel motion,** more properly called a **straight-line motion,** is a link mechanism designed to guide a reciprocating part, as a piston rod, in a straight line. In the early days of the steam engine, parallel motions were extensively used to guide the pump and piston rods; but they are now seldom met with, except on steam-engine indicators, where they are employed to give a straight-line motion to the pencil. Very few parallel motions produce an absolutely straight line, and it is customary to design them so that the middle and two extreme positions of the guided point will be in line.

30. The Watt Parallel Motion.—The best-known motion is the one shown in Fig. 26, which was invented by James Watt in 1784. The links *A B* and *C D* turn on fixed centers *A* and *D*. The other ends *B*, *C* are connected by the link *C B*, which has the point *O* so chosen that it will pass through three points O_1, O, O_2 in the straight line *S S* perpendicular to the links *C D* and *A B* when in their middle positions. When the point *O* is at the upper extremity of its motion at O_1, the linkage assumes the position $A\,B_1\,C_1\,D$; at the lower extremity it assumes the position $A\,B_2\,C_2\,D$.

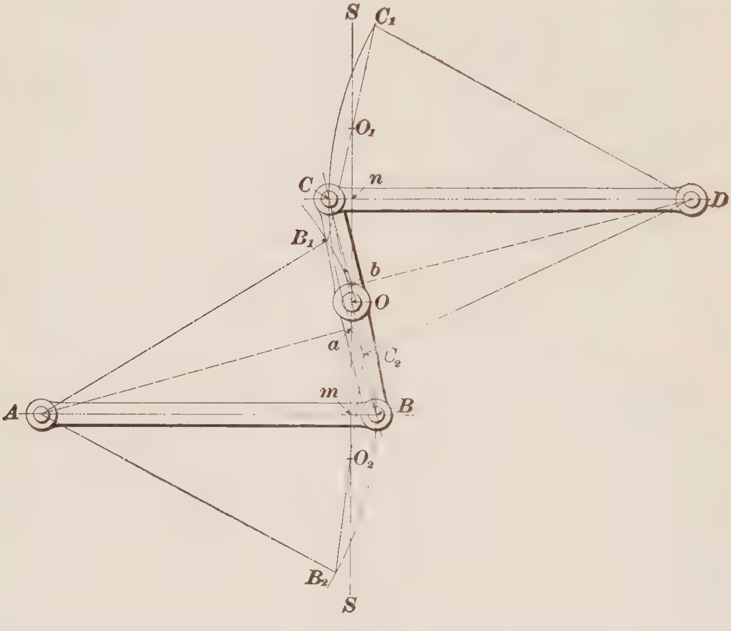

FIG. 26

Having given the length $O_1\,O_2$ of the stroke, the middle position *O* of the guided point, the center of one lever *A*, and the perpendicular distance between the levers when in mid-position, the motion may be laid out as follows: Let *S S* be the path of the guided point, *O* its middle position, *A* the given center, and *A B* and *C D* indefinite parallel lines, representing the middle positions of the levers. From *m*, where *A B* intersects *S S*, lay off on *S S* the distance *m a*, equal to one-fourth of the stroke. Join *A* with *a* and draw an indefinite line *a B*, perpendicular to *A a*. The point *B*, where *a B* intersects line *A B*, is the right-hand extremity of lever *A B*, and the lower extremity of the link *C B*. The point *C* is obtained by drawing an indefinite line through *B* and *O*; this intersects the line *C D* in the required point. To find the center *D*, lay off *n b* equal to one-fourth stroke; connect *C* and *b*, and from *b* draw an indefinite line perpendicular to *C b*; the center will be at its intersection with *C D*.

If the positions of both centers should be known, mark points *a* and *b* as before. Draw *A a* and *b D*, and through *b* and *a* draw perpendiculars to these lines; the points *B* and *C*, where they intersect the center lines of the levers, are the extremities of these levers. Join *B* and *C* by the link *B C*, and the point *O*, where the center line of this link cuts the line of motion *S S*, is the position of the guided point *O* on the link *B C*.

UNIVERSAL JOINT

31. Hooke's coupling, or the **universal joint,** shown in Fig. 27, is used to connect two shafts, the center lines of

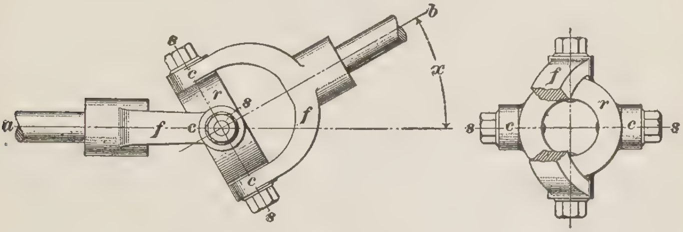

Fig. 27

which are in the same plane, but which make an angle with each other. It is generally constructed in the following manner: Forks *f*, *f* are fastened to the ends of the shafts *a* and b, and have bosses c, c tapped out to receive the screws s, s. The ends of these screws are cylindrical, and are a working fit in corresponding bearings in the ring r. The details of construction may be seen in the right-hand part of this figure. In heavy machinery, the forks are forged and welded to the shafts. The ring r furnishes an example of spherical motion, and the point of intersection of the two axes is the center. The motion transmitted by the universal joint is not uniform; that is, if the driving shaft turns uniformly, the driven shaft will make the same number of revolutions, but it will have alternately greater and less angular velocities at different points of a single revolution, and there are only four positions of the joint in one revolution in which the two shafts have the same speed.

32. The maximum and minimum speeds may be found as follows: In Figs. 28 and 29, let m be the driving shaft and n the driven shaft, the center lines of both shafts being in the plane of the drawing; also, let the center lines of the two shafts intersect at O, and denote by e the acute angle between them. In Fig. 28, the fork of the shaft n is in the plane of the two shafts, and in this position it is evident that the plane of the ring connecting the two forks is perpendicular to the shaft n, for in this position both arms SS' and RR' are perpendicular to n.

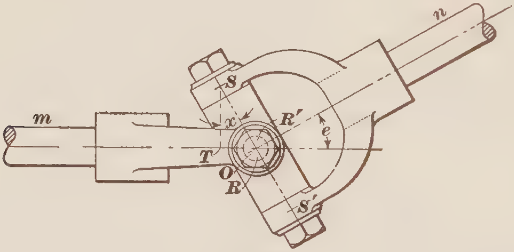

FIG. 28

From formula **2,** Art. **11,** $v = rw$; that is, the linear velocity of a rotating point equals the radius times its angular velocity. Let v denote the linear velocity of the point S, and w_n the angular velocity of the shaft n; then, since S

is at a distance OS from the shaft n, the linear velocity equals OS times the angular velocity of n, or $v = OS \times w_n$. Now, when the ring is in this position, it is moving, for the instant, as if it were rigidly attached to shaft m, and hence S may be considered for this instant as a part of m. From S, therefore, let the perpendicular ST be dropped on the shaft m, and since the velocity of S is v, the relation of the linear velocity to the angular velocity w_m of the shaft m is $v = ST \times w_m$. But, as the linear velocity v of the point S is the same in both cases,

$$OS \times w_n = ST \times w_m, \text{ or } w_n = w_m \times \frac{ST}{OS}$$

But in the triangle OST, $\frac{ST}{OS} = \cos OST$, and the angle OST keeps changing as the shafts revolve. Let x denote this variable angle OST. Then, $\frac{ST}{OS} = \cos x$, and

$$w_n = w_m \times \cos x \qquad (1)$$

When the fork on m is in the plane of the shafts, as in Fig. 29, the same course of reasoning will apply. Hence, in this position, $v = OR \times w_m$ and $v = RT \times w_n$. Therefore, $RT \times w_n = OR \times w_m$, or

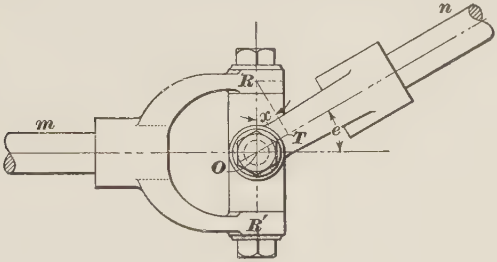

FIG. 29

$$w_n = w_m \times \frac{OR}{RT} = w_m \times \frac{1}{\cos x} \qquad (2)$$

These two formulas give the extremes of angular velocity of the driven shaft, formula 1 being for the minimum, and formula 2 for the maximum, angular velocity of n.

33. From the foregoing formulas, it is apparent that when the shaft m rotates at a uniform speed, the shaft n is given a variable speed, which is alternately less and greater than the speed of the driving shaft. However, there are four positions in each revolution at which the angular velocities of the driving and driven shafts are equal, and these are the positions in which the distances of the points S or R from the two axes are exactly equal. In the positions shown in Figs. 28 and 29, the angles e and x are equal, and the driven shaft n has its least and greatest angular velocities, respectively. If the driving shaft rotates uniformly, the least speed of the driven shaft is equal to the speed of the driving shaft multiplied by the cosine of the angle between the two axes, and the greatest speed is equal to the speed of the driver multiplied by $\frac{1}{\cos x}$. Thus, if the shafts revolve at the rate of 100 revolutions per minute, and the angle e between them is 30°, the least speed of the driven shaft will momentarily be $100 \times \cos 30° = 100 \times .866 = 86.6$ revolutions per minute, and the greatest speed, $\frac{100}{\cos 30°} = \frac{100}{.866} = 115.47$ revolutions per minute.

34. To obviate the variation in angular velocity, the **double universal joint** is used, as shown in Fig. 30. Let a and b be the two shafts to be connected. Draw their center lines, intersecting at O and bisect the angle $A O B$ by

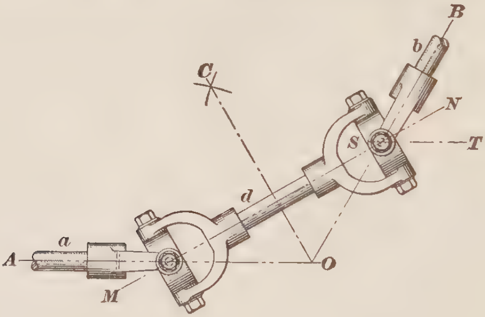

FIG. 30

the line *O C*. The center line *M N* of the connecting shaft *d* must now be drawn perpendicular to *O C*. Care must be taken that the forks on the intermediate shaft lie in the same plane. Thus constructed, a uniform motion of *a* will result in a uniform motion of the driven shaft *b*, though the intermediate shaft will have a variable motion. Let x = angle between *a* and *d* = angle between *d* and *b*, and denote the three angular velocities by w_a, w_d, and w_b. Then, with the forks as shown in the figure, $\frac{w_d}{w_a} = \cos x$, and $\frac{w_b}{w_d} = \frac{1}{\cos x}$. Multiplying,

$$\frac{w_d}{w_a} \times \frac{w_b}{w_d} = \cos x \times \frac{1}{\cos x}, \text{ or } \frac{w_b}{w_a} = 1, \text{ whence } w_b = w_a$$

It can be proved that this law holds good for all positions.

If, therefore, the forks on the intermediate shaft are in the same plane, and the intermediate shaft makes the same angle with the driving and driven shafts, the angular velocity of the driven shaft is equal to that of the driving shaft.

This arrangement is often employed to connect parallel shafts, as would be the case in Fig. 30 if *O B* took the direction *S T*. In such a case it makes no difference what the angle x is, except that, if the joints are expected to wear well, it should not be too great. If the forks of the intermediate link *d* are at right angles, the variation of motion of the shaft *b* is greater than in the single coupling. In this case, the greatest and least angular velocities of *B* would be, respectively, $w_a \times \frac{1}{\cos^2 x}$ and $w_a \times \cos^2 x$.

GEARING

TOOTHED GEARING

ROLLING CURVES AND SURFACES

1. Direct-Contact Transmission.—Of the various ways of transmitting motion from one machine part to another, that by direct contact is the most common. That is, a surface of one of the parts is in contact with a similar surface of another part, and the motion of the first causes the motion of the second. In Fig. 1, for example, the parts *a* and *b* are in contact at *P* and the rotation of *a* in the direction shown by the arrow will cause a rotation of *b* in the same direction.

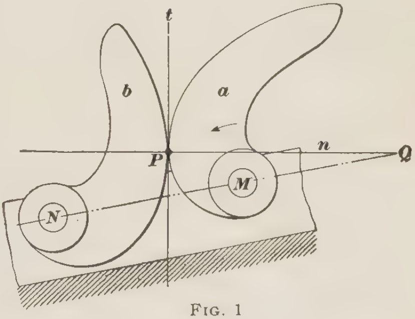

FIG. 1

2. Condition of Rolling.—The relative motion of the two bodies in contact may be: (*a*) sliding motion; (*b*) rolling motion; (*c*) combined sliding and rolling motion. When the point of contact *P* is not situated on the line joining the centers of rotation *M* and *N*, there will be sliding; but when, as in Fig. 2, *P* lies in the same line with *M* and *N*, the bodies will roll on each other without sliding. The condition for rolling, therefore, is that the point of contact shall lie in the line of centers.

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3. Nature of Rolling Curves.—In order that two curves may rotate about fixed points and roll together, the curves, known as **rolling curves,** must be specially constructed to give the required motion. A pair of logarithmic spirals, the curves CPE and DPF of Fig. 2, will roll together about axes through their foci. But these are not closed curves; that is, it is impossible to start from any point on the curve and, by following its outline in one direction, return to the starting point. Hence, the curve CPE, Fig. 2, cannot be used to transmit motion continuously to the curve DPF when revolving in one direction, but it may reciprocate through a partial revolution, and the two curves will remain in contact. On the other hand, a pair of closed curves, as the circles shown in Fig. 3, will roll in contact and may turn continuously in one direction. Two equal ellipses, Fig. 4, will roll together if the distance between the axes of rotation is equal to the length of the major axis.

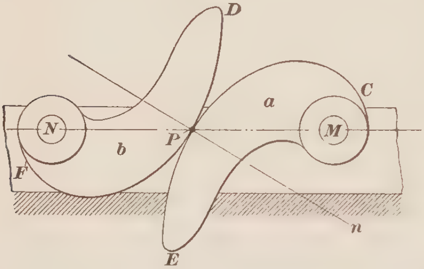

FIG. 2

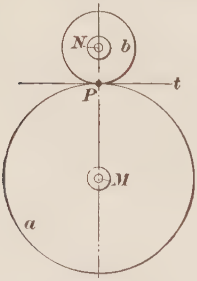

FIG. 3

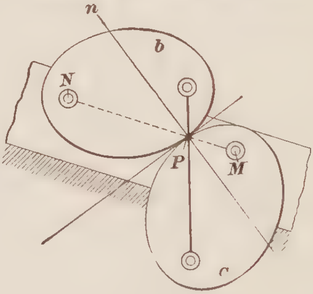

FIG. 4

4. Angular Velocity Ratio.—In Fig. 1, let a line n be drawn through P at right angles to the common tangent t to the two curves. A line like this, perpendicular to the tangent, is called the **normal** to the curve at the given point.

Let Q denote the point in which this normal n cuts the line through the centers M and N; then it can be proved, by a rather difficult process, that the segments MQ and NQ are to each other inversely as the angular velocities of a and b; that is,

$$\frac{\text{angular velocity of } b}{\text{angular velocity of } a} = \frac{MQ}{NQ} \qquad (1)$$

This is a general rule that applies to all cases of transmission by direct contact.

In Figs. 2, 3, and 4, the normal must cut the line MN in the point P itself; hence, for cases of pure rolling,

$$\frac{\text{angular velocity of } b}{\text{angular velocity of } a} = \frac{MP}{NP} \qquad (2)$$

When the radii MP and NP remain constant throughout the entire revolution, as in the case of circular gears, and the motion of the gears is uniform, the angular velocities may be measured in revolutions per minute.

Let N_a = angular velocity of a, in revolutions per minute; and

N_b = angular velocity of b, in revolutions per minute.

Then formula 2 becomes

$$\frac{N_b}{N_a} = \frac{MP}{NP} \qquad (3)$$

When P lies between M and N, as in Figs. 2, 3, and 4, the curves a and b rotate in opposite directions. If Q or P lies outside of MN, as in Figs. 1 and 5, they rotate in the same direction.

When the rolling curves are circles, as in Figs. 3 and 5, the point of contact P will always be at the same distance from the centers M and N; but for any other forms of curves, the position of P will change. It follows that the velocity ratio, that is, the fraction $\frac{N_b}{N_a}$, is constant for rolling circles, but is variable in other cases. Thus, in Fig. 4, suppose a

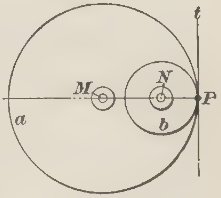

FIG. 5

to rotate clockwise about M with a constant number of revolutions per minute. As the rotation proceeds, the point P moves toward N, the ratio $\frac{MP}{NP}$ increases, and in consequence the angular velocity of b increases and attains its maximum value when P is nearest to N.

5. Rolling Cones.—While the bodies represented in Figs. 1 to 5 are shown as plane figures rotating about fixed points, they are, in reality, representations of solids with definite thicknesses rotating about parallel axes. Cases of parallel axes of rotation are the most common, but frequently non-parallel axes must also be used. Let OM and ON, Fig. 6, be two axes of rotation intersecting in the point O; if on these axes are constructed two frustums of cones, a and b, respectively, these cones will roll together and one will transmit motion to the other.

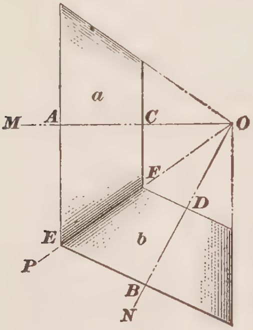

FIG. 6

Assuming that there is no slipping, the velocity ratio may be found in the same manner as for rolling cylinders. At E, for example, the circles forming the bases of the frustums are in contact, and the velocity ratio is $\frac{N_b}{N_a} = \frac{EA}{EB}$, since the angular velocities are inversely proportional to the radii. Similarly, at any other point, as F, the velocity ratio is $\frac{N_b}{N_a} = \frac{FC}{FD}$. But since triangle OCF is similar to triangle OAE, and triangle ODF is similar to triangle OBE, $EA : FC = OE : OF$, and $EB : FD = OE : OF$, so that $EA : FC = EB : FD$; from which, by transposing, $EA : EB = FC : FD$. That is, $\frac{EA}{EB} = \frac{FC}{FD}$, which means that, for a pair of cones running in contact without slipping, the velocity ratio is constant and is equal to the inverse ratio of the corresponding radii at any point.

SPUR GEARING

GENERAL PRINCIPLES

6. Positive Driving.—When motion is transmitted by friction gearing, that is, by cylinders or cones that roll in contact, there is more or less slipping of the contact surfaces; hence, when it is necessary to transmit motion with a definite and unvarying velocity ratio, friction wheels cannot be used, and some means must be employed to insure positive driving. In the case of rolling wheels or cylinders, the circumferences may be provided with projections *a*, *a*, Fig. 7, and with corresponding grooves *b*, *b*, so that the projections of one wheel fit into the grooves of the other. Evidently the same number of these projections pass the line of centers in a given time on wheel *B* as on wheel *A*, so that a given number of revolutions of the driver *A* causes a definite number of revolutions of the follower *B*.

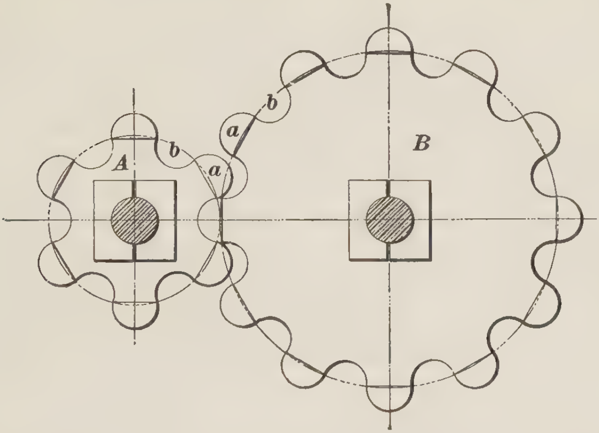

Fig. 7

7. Condition of Constant Velocity Ratio.—The projections on the rolling surfaces insure that the number of revolutions of the follower per minute or per hour is the same that would be obtained by rolling without slipping, but the angular velocity from projection to projection will not be constant unless the bounding curves of the projections, or **teeth**, as they are called in gear-work, have certain forms. The shape of the tooth depends on the principle stated in Art. **4**, namely, that when the driver and follower have direct contact, the angular velocities are inversely as the segments into which the line of centers is cut by the common normal at the point of contact.

In Fig. 8, a pair of teeth m and n are attached to the parts a and b and are in contact at C. The common normal NN at C cuts the line of centers AB in the point P; then if V_a and V_b denote the angular velocities of a and b, respectively,

$$\frac{V_a}{V_b} = \frac{BP}{AP}$$

In order, therefore, that this ratio of the velocities may be strictly constant, the point P must always lie in the same position on the line of centers. Hence the following law:

Law.—*In order that toothed wheels may have a constant velocity ratio, the common normal to the tooth curves must always pass through a fixed point on the line of centers.*

8. Pitch Surfaces and Pitch Lines.—In Fig. 8 let the circles e and f, with centers A and B, be the outlines of two rolling cylinders in contact at P. Let the tooth curves m and n be so constructed that their common normal shall always pass through P. The velocity ratio produced by the rolling of these cylinders is precisely the same as that caused by the teeth m and n; for in the case of rolling cylinders the velocities are inversely proportional to the radii, that is,

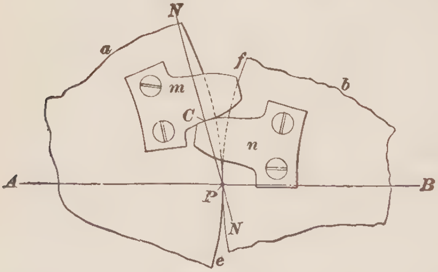

FIG. 8

$$\frac{V_a}{V_b} = \frac{N_a}{N_b} = \frac{BP}{AP}$$

The bounding surfaces *e* and *f* are called **pitch surfaces,** and the lines *e* and *f* are called **pitch lines** or **pitch curves,** or, in the case of circular gears, **pitch circles.** The point of tangency *P* of the pitch lines is called the **pitch point,** and it must lie on the line of centers. This is the meaning of the term as generally used by designers, and is the meaning intended wherever the term *pitch point* is mentioned in this treatise on gearing. In the machine shop, however, this term is frequently used in a different sense, being there considered as any point in which the tooth outline intersects the pitch circle, as indicated in Fig. 9.

In the case of circular pitch lines, the pitch point *P* lies in a fixed position; but if the pitch lines are non-circular, as in the case of rolling ellipses, Fig. 4, the pitch point moves along the line of centers. In order to include non-circular wheels, the law of Art. **7** may be made general, as follows:

Law.—*In order that the motion produced by tooth driving shall be equivalent to the rolling of two pitch surfaces, the common normal to the tooth curves must at all times pass through the pitch point.*

The object, then, in designing the teeth of gear-wheels is to so shape them that the motion transmitted will be exactly the same as with a corresponding pair of wheels or cylinders without teeth, which run in contact without slipping. In actual work, two general systems of gear-teeth are used. The one is known as the *involute system* and the other as the *cycloidal system*, both of which will be discussed in succeeding paragraphs.

9. Definitions.—Referring to Fig. 9, which shows part of a circular gear-wheel, the following definitions apply to the lines and parts of the tooth.

The circle drawn through the outer ends of the teeth is called the **addendum circle;** that drawn at the bottoms of the spaces is called the **root circle.** In the case of non-circular gears, these would be called, respectively, the **addendum line** and **the root line.**

The **addendum** is the distance between the pitch circle and the addendum circle, measured along a radial line. The **root** is the distance between the pitch circle and the root circle, measured along a radial line. The term addendum is also frequently applied to that portion of a tooth lying between the pitch and addendum circles, and the term root to that portion of the tooth lying between the pitch and root circles.

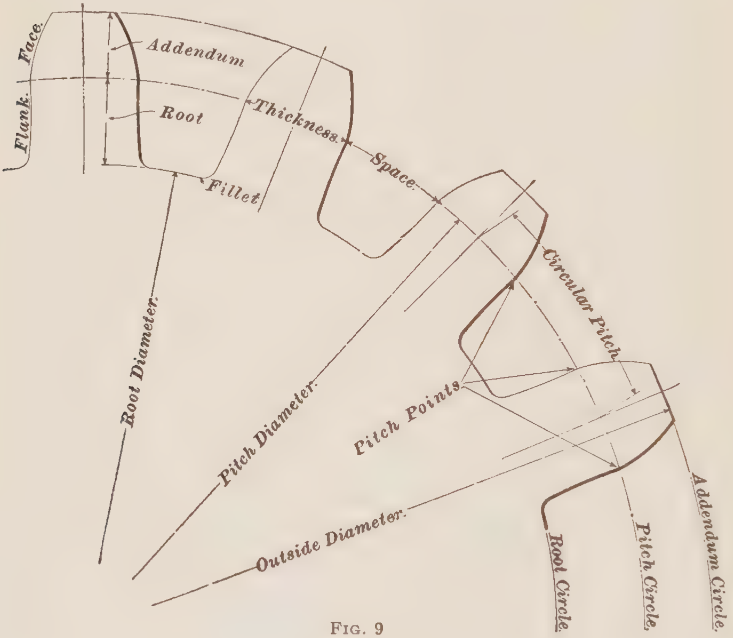

FIG. 9

The working surface of the addendum, that is, the part of the working surface outside of the pitch circle, is called the **face** of the tooth. The working surface of the root is called the **flank** of the tooth.

The diameter of the pitch circle is called the **pitch diameter.** When the word *diameter* is applied to gears, it is always understood to mean the pitch diameter unless otherwise specially stated—as **outside diameter,** or **diameter at the root.**

The distance from a point on one tooth to a corresponding point on the next tooth, measured along the pitch circle, is the **circular pitch.**

The **fillet** is a curve of small radius joining the flank of the tooth with the root circle, thus avoiding the weakening effect of a sharp corner.

THE INVOLUTE SYSTEM

10. Production of the Involute Curve.—In general, the **involute** of any curve may be defined as the curve that is described by a point in a cord as it is unwound from the original curve, keeping the unwound portion of the cord straight. Thus, suppose that a cord is wound around the curve *a*, Fig. 10, and let *P* be any point on the cord. Then, as the cord is unwound, the point *P* will describe a curve *m* that is an involute of the curve *a*, $P_1 A$ being the last position of the cord shown.

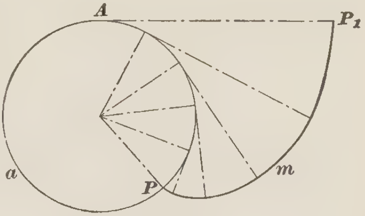

FIG. 10

In the case of an involute to a circle, it is convenient to conceive the curve described as follows: Suppose *a*, Fig. 11, to be a circular pulley having a cord wound around it, and let the pulley be pinned at a fixed point *O*. At the end *P* of the cord a pencil is attached, and there is a fixed groove

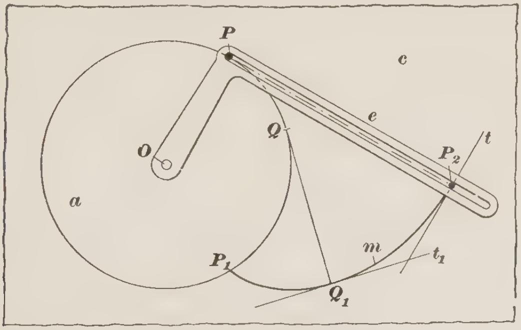

FIG. 11

or guide *e* tangent to the pulley at P. Suppose also that the pulley has attached to it a sheet of paper or cardboard *c*. Now take hold of the pencil and pull it along the groove *e*. The pulley and paper will thus be caused to rotate about O, and the pencil will trace on the moving paper the involute $P_1 P_2$, or *m*. If now the pulley is turned backwards, so that the string is wound up, the pencil will move from P_2 to P and will retrace the curve $P_2 P_1$ on the moving paper, the point P_1 moving to P, its original position.

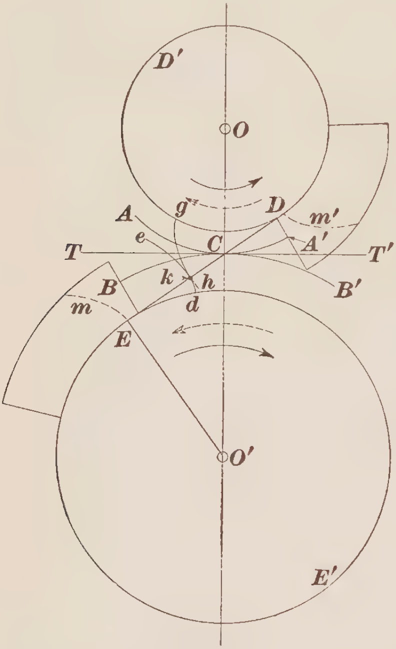

FIG. 12

Let *t* be a tangent to the curve *m* at the point P_2. From the manner in which the curve is produced, it is evident that the tangent is perpendicular to $P P_2$. Similarly, the tangent t_1 at any other point Q_1 is perpendicular to $Q_1 Q$, and in general any tangent to the circle *a* is a normal to the involute *m*.

11. Involute Tooth Curves.—Let O and O', Fig. 12, be the centers of two cylinders that are a short distance apart, and $D E$ a cord that has been wound several times around them in opposite directions. If the cylinder $D D'$ be turned in the direction of the full arrow, the cord $D E$ will cause the cylinder $E E'$ to turn in the opposite direction, as shown by the full arrow, and points on the cord $D E$ will describe portions of the involute curve. In order to better comprehend this, imagine a piece of paper to be attached to the bottom of each cylinder, as shown, and that the width of each piece is the same as the distance between the cylinders. Now, suppose that a pencil is attached to the cord at E, the point of tangency of the line DE with the cylinder EE', in such a manner that it can trace a line on the piece of paper attached to the cylinder EE' if a proper motion be given to the cord DE. Turn the cylinder DD' in the direction of the full arrow, that is, rotate it counter-clockwise. The point E will travel toward the cylinder DD' in the straight line ED, and will gradually move away from the cylinder EE'. During this movement, the pencil attached at E will trace on the piece of paper the involute curve m. In the same manner, if the pencil be attached at D and the cylinder EE' be rotated in the direction of the dotted arrow, the involute m' will be traced on the piece of paper attached to the cylinder DD'. Suppose that those parts of the pieces of paper to the right of the curve m and to the left of the curve m' be removed, and that EE' be rotated until curve m takes the position ed; also, that DD' be rotated until m' takes the position gh, the two curves being in contact at k on the line DE. The cord DE, being tangent to the circle DD', is normal to the involute gh, and being also tangent to the circle EE', it is normal to the involute de. This would be true for any positions in which the curves de and gh could be moved into contact by rotating the cylinders. Moreover, this line DE always passes through a fixed point C on the line of centers. Therefore, de and gh are two curves whose common normal at the point of contact always passes through one fixed point C so long as the curves remain in contact. If, now, the pitch circles AA' and BB' be drawn through this point C, the motion transmitted by the involutes de and gh used as the outlines of teeth will be precisely equivalent to that produced by the rolling of these circles without slipping.

12. Properties of Involute Teeth.—The circles DD' and EE', Fig. 12, to which DE is tangent, are called **base circles.** It will be noted as a characteristic of involute teeth that the whole of the face and the part of the flank between the pitch and base circles is a continuous curve.

For this reason, involute gears are sometimes called **single-curve** gears.

If the centers O and O', Fig. 12, be moved apart so that the pitch circles do not touch, the relative velocities of DD' and EE' evidently remain unchanged, since the cylinders are connected by the cord DE. The curves described by the point k are also the same, because they are still involutes of the same circles. From this, it follows that the distance between the centers of involute gears may be varied without disturbing the velocity ratio or the action of the teeth—a property peculiar to the involute system.

In Fig. 12 let TT' be drawn through C at right angles to the line of centers OO'; then TT' is the common tangent to the pitch circles AA' and BB'. As already stated, the line ED is the common normal of the tooth curves, and is therefore the line of action of the pressure between the teeth. The angle between the common normal to the tooth surfaces and the common tangent to the pitch circles is called the **angle of obliquity.** In the case of involute teeth, this angle is constant.

13. Angle and Arc of Action.—The angle through which a wheel turns from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of contact has reached the line of centers is the **angle of approach;** the angle through which it turns from the instant the point of contact leaves the line of centers until the teeth are no longer in contact is the **angle of recess.** The sum of these two angles forms the **angle of action.** The arcs of the pitch circles that measure these angles are called the arcs of **approach, recess,** and **action,** respectively.

In order that one pair of teeth shall be in contact until the next pair begin to act, the arc of action must be at least equal to the circular pitch.

The **path of contact** is the line described by the point of contact of two engaging teeth. In the case of involute gears the path of contact is part of the common tangent to the base circles. The arc of action depends on the addendum, or what amounts to the same thing, on the length of the tooth. With short teeth, the arc of action must necessarily be small; and if a long arc is desired, the teeth must be made long.

14. Standard Interchangeable Gears.—In order that two gears may run properly together, two conditions must be satisfied: (1) They must have the same circular pitch, and (2) they must have the same obliquity. If, therefore, all involute gears were made of the same obliquity, any pair of wheels having teeth of the same pitch would work properly together, and such gears would be said to be interchangeable.

The tooth selected for the standard is one having an angle of obliquity of 15°; that is, in Fig. 12, angle TCE = angle $CO'E = 15°$. With this obliquity, then, in the triangle $O'EC$, $O'E = O'C \cos CO'E = O'C \cos 15° = .966\ O'C$; that is, the radius of the base circle equals .966 times the radius of the pitch circle. The distance between the base circle and pitch circle is thus about one-sixtieth of the pitch diameter.

In the interchangeable series of standard gears, the smallest number of teeth that a gear may have is twelve, for with a smaller number the arc of contact will be smaller than the circular pitch, in which case one pair of teeth will separate before the next pair comes into contact, and the gears will not run.

15. The Involute Rack.—A **rack** is a series of gear-teeth described on a straight pitch line. It is usually a metal bar, in which the teeth are cut, although they may be cast. A rack, therefore, may be considered as a portion of the circumference of a gear-wheel whose radius is infinitely long, and whose pitch line may consequently be regarded as straight.

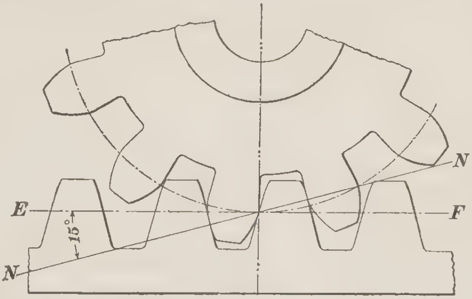

FIG. 13

In the involute rack, Fig. 13, the sides of the teeth are straight lines making an angle of $90^\circ - 15^\circ = 75^\circ$ with the pitch line EF. Thus, on the contact side, the tooth outlines are perpendicular to the line of action NN. To avoid interference, the ends of the teeth should be rounded to run with the 12-tooth pinion. A **pinion** is a small gear meshing with a rack or with a larger gear.

16. Involute Internal Gears.—An **annular,** or **internal, gear** is one having teeth cut on the inside of the rim. The pitch circles of an annular gear and its pinion have internal contact, as shown in Fig. 5.

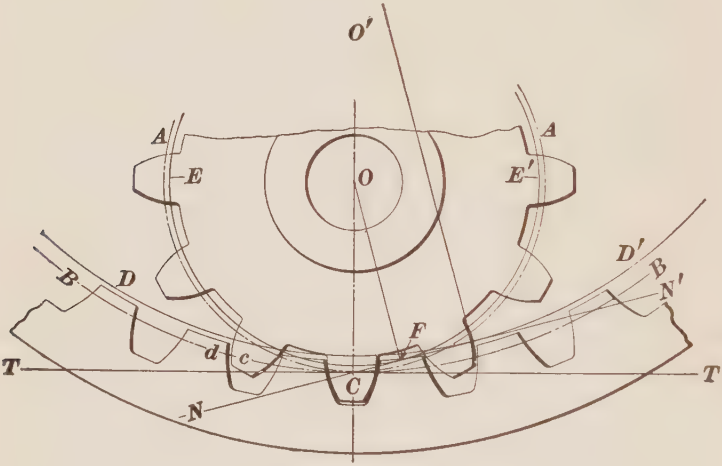

FIG. 14

The construction of an annular gear with involute teeth is shown in Fig. 14. The obliquity of 15° is shown by the angle TCN, and the base circles EE' and DD' are drawn tangent to the line of action NN', with O and O', respectively, as centers. The addendum circle for the internal gear should be drawn through F, the intersection of the path of contact NN' with the perpendicular OF drawn from the center of the pinion. The teeth will then be nearly or quite without faces, and the teeth of the pinion, to correspond, may be without flanks. If the two wheels are nearly of the same size, points *c* and *d* will interfere; this interference may be avoided by rounding the corners of the teeth.

THE CYCLOIDAL OR ROLLED-CURVE SYSTEM

17. The Cycloid.—The name **cycloid** is given to the curve traced by a point on the circumference of a circle as it rolls on a straight line. Thus, in Fig. 15 the circle *m* rolls on the line *A B*, and the point *P* on the circumference traces the path *A C B*, which is a cycloid. The rolling circle is called the **generating circle,** the point *P* the **tracing point,** and the line *A B* the **base line.**

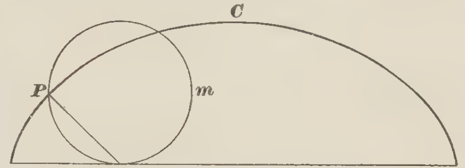

FIG. 15

If the base line *A B* is an arc of a circle, the curve traced by the point *P* is called an **epicycloid** when the generating circle rolls on the convex side, and a **hypocycloid** when it rolls on the concave side. Fig. 16 shows an epicycloid, and Fig. 17 a hypocycloid.

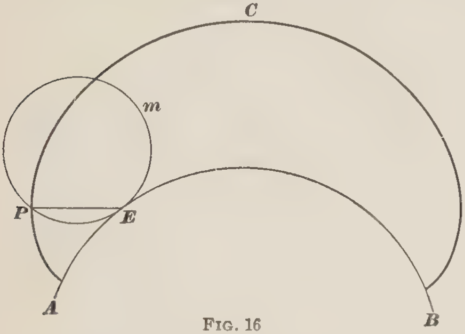

FIG. 16

The one property of cycloidal curves that makes them peculiarly suitable for tooth outlines is that the common normal through the point of contact will always pass through the pitch point. In Figs. 15, 16, and 17, *P* is the tracing point in each case and *E* is the point of contact of the generating circle and the line on which it rolls. Now, it is clear that the generating circle *m* is for the instant turning about *E* as a center, and the point *P* is therefore moving in a direction at right angles to *PE*. In other words, the tangent to the curve at *P* is perpendicular to *PE*, or *PE* is normal to the curve.

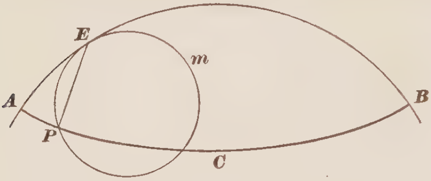

FIG. 17

A particular form of the hypocycloid is shown in Fig. 18. The generating circle *m* has a diameter equal to the radius of the circle *n*. In this case, a point *P* of *m* describes a straight line *ACB*, which passes through the center *C* of *n* and is therefore a diameter.

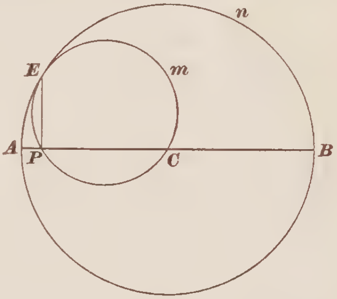

FIG. 18

18. Generation of Tooth Outlines.—In Fig. 19 let *a* and a_1 be two pitch circles in contact at *E* and suppose a third circle *m* to be in contact with them at *E*. If the circle *m* rolls on the outside of the circle *a*, the tracing point *P* describes the epicycloid BPB_1. If, however, it rolls on the inside of circle a_1, *P* describes the hypocycloid CPC_1. Now let an arc of the curve BPB_1 be taken as the outline of the face of a tooth on *a*, and let an arc of the hypocycloid CPC_1 be taken as the outline of the flank of a tooth on a_1. These curves are in contact at *P*, and, moreover, *PE* is normal to each of them. Hence, the common normal through the point of contact passes through the pitch point, and the curves satisfy the condition required of tooth outlines.

The one rolling circle *m* generates the faces of the teeth of *a* and the flanks of the teeth of a_1. A second circle m_1, rolling on the inside of *a* and on the outside of a_1, may be used to generate the faces of gear-teeth for a_1 and the flanks for *a*. While not necessary, it is customary that the generating circles shall have the same diameter. If a series of gears of the same pitch have their tooth outlines all generated by rolling circles of the same diameter, any gear will run with any other gear of the series; that is, the gears are interchangeable.

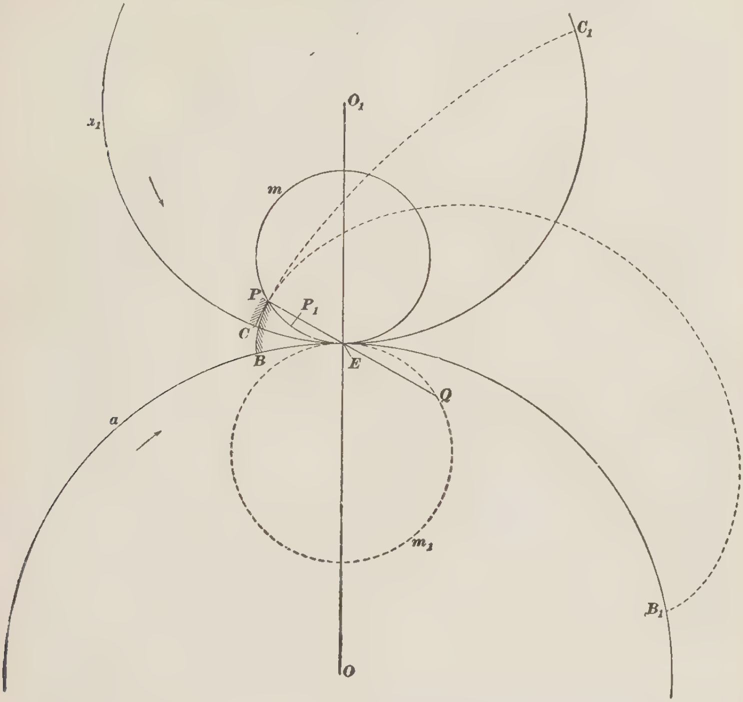

FIG. 19

19. Size of Generating Circle.—In Figs. 20 to 22 is shown the effect of different sizes of generating circles on the flanks of the teeth. In the first, the generating circle has a diameter

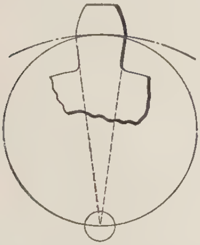

FIG. 20

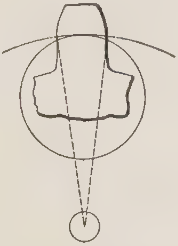

FIG. 21

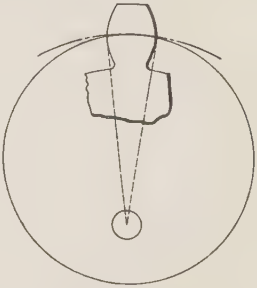

FIG. 22

equal to the radius of the pitch circle, the hypocycloid is a straight line, and the flanks described are radial. In the second, with a smaller circle, the flanks curve away from the radius, giving a strong tooth, and in the third, with a larger circle, the flanks curve inwards, giving a weak tooth, and one difficult to cut. It would seem, therefore, that a suitable diameter for the generating circle would be one-half the pitch diameter of the smallest wheel of the set, or one-half the diameter of a 12-tooth pinion, which, by common consent, is taken as the smallest wheel of any set.

It has been found, however, that a circle of five-eighths the diameter of the pitch circle will give flanks nearly parallel, so that teeth described with this circle can be cut with a milling cutter. For this reason, some gear-cutters are made to cut teeth based on a generating circle of five-eighths the diameter of a 12-tooth pinion, or one-half the diameter of a 15-tooth pinion.

It is more common practice to take the diameter of the generating circle equal to one-half the diameter of a 12-tooth pinion rather than one-half the diameter of a 15-tooth pinion; this size, therefore, is taken in this discussion of the subject of gearing.

20. Obliquity of Action.—Neglecting friction, the pressure between two gear-teeth always has the direction of the common normal. In the case of involute teeth, this direction is always the same, being the line of action ED, Fig. 12. With cycloidal teeth, however, the direction of the normal is constantly changing, and hence the direction of pressure between the teeth is likewise variable. In Fig. 19, let the tooth curves be just coming into contact at the point P. The direction of the pressure between them at that instant is PE, since PE is the common normal at P. As the gears rotate, however, the point of contact P approaches E, moving along the arc PP_1E, and the common normal PE, therefore, makes a greater angle with O_1O, until, when P reaches E, the common normal is at right angles to O_1O.

Beyond the point *E*, the point of contact follows the arc *E Q* until the teeth leave contact, and the direction of the common tangent gradually changes from a position at right angles to $O_1 O$ to the position *E Q*. Hence, with cycloidal teeth, the obliquity of the tooth pressure is greatest when the teeth first come in contact, decreases to zero at the pitch point, and increases again to a maximum at the final point of contact. As a rule, the greatest obliquity should not exceed 30°.

21. Rack and Wheel.—In Fig. 23 is shown a rack and 12-tooth pinion. Tooth outlines for the rack in the cycloidal system are obtained by rolling the generating circles *o o′* on the straight pitch line *B B*; the curves are therefore cycloids. The generating circle *o* passes through the center *O* of the pinion and rolls on the pitch circle *A A*, and therefore describes radial flanks.

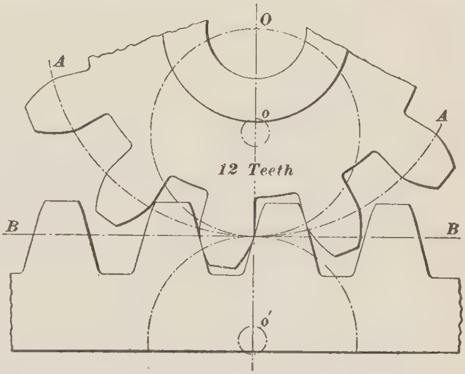

FIG. 23

22. Epicycloidal Annular Gears.—Annular gears, or internal gears, as already explained, are those having teeth cut on the inside of the rim. The width of space of an internal gear is the same as the width of tooth of a spur gear. Two generating circles are used, as before, and if they are of equal diameter, the gear will interchange with spur wheels for which the same generating circles are used.

In Fig. 24 is shown an internal gear with pitch circle *A A*, inside of which is the pinion with pitch circle *B B*. The generating circle *o*, rolling inside of *B B*, will describe the flanks of the teeth for the pinion, and rolling inside of *A A*, the faces of the teeth, for the annular wheel. Similarly, the corresponding faces and flanks will be described by *o′*. The only special rule to be observed in regard to epicycloidal internal gears is that the difference between the diameters

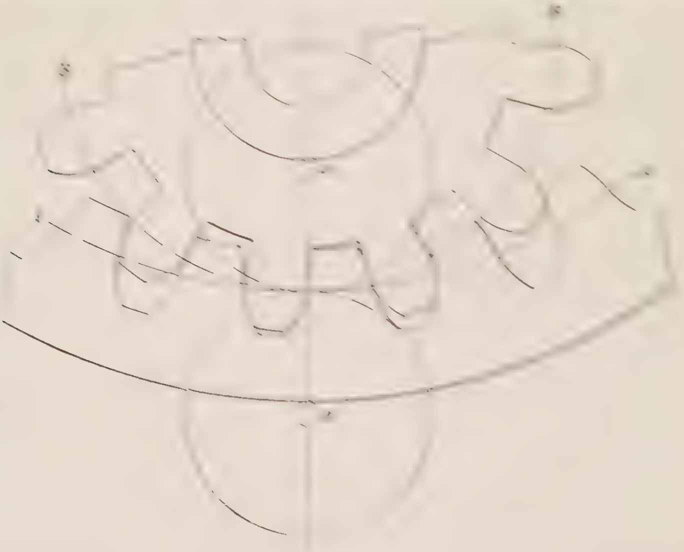

FIG. 24

of the pitch circles must be at least as great as the sum of the diameters of the generating circles.

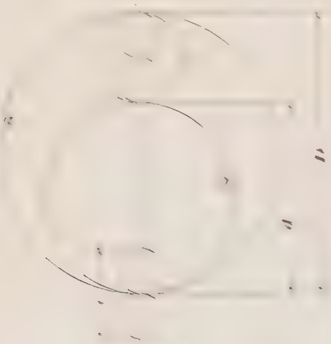

FIG. 25

This is illustrated by Fig. 25. a is the pitch circle of an internal gear and b that of the pinion. Then, for correct action, the difference D − d of the diameters must be at least as great as c, the sum of the diameters of the generating circles. To take a limiting case, suppose a to have 36 teeth and b 24 teeth. A wheel with a diameter equal to D − d, as shown dotted at c, would, therefore, have 36 − 24 = 12 teeth. In the 12-tooth interchangeable system this latter would be the smallest wheel of the series and the generating circles would be half its diameter. From this, it follows that, if $D - d$, the diameter of e, is not to be exceeded by the sum c of the diameters of the generating circles, b is the largest wheel that can be used with a. Hence, when the interchangeable system is used, the number of teeth in the two wheels must differ by at least the number in the smallest wheel of the set. If it is desired, for example, to have 18 and 24 teeth, and the gears are to interchange, generating circles of half the diameter of a 6-tooth pinion will be used, this being taken as the smallest wheel.

23. Formerly, the cycloidal system was used almost exclusively, but in later years the involute system has largely taken its place. The distinctive features of the involute system lie in the great strength of the tooth and in the fact that the distance between the centers of two meshing gears may be varied somewhat without affecting the uniformity of the velocity ratio. The chief objection that has been raised against involute teeth is the obliquity of action, which causes increased pressure on the bearings. If the obliquity does not exceed 15°, however, this objection is not serious.

PROPORTIONS OF GEAR-TEETH

24. Circular Pitch.—The circular pitch of a gear has been defined as the distance between corresponding points on adjacent teeth measured along the pitch circle. It is obtained by dividing the circumference of the pitch circle by the number of teeth in the gear. If the circular pitch is taken in even dimensions, as 1 inch, $\frac{3}{4}$ inch, etc., the circumference of the pitch circle will also be expressed in even dimensions; but the diameter will be a dimension that can only be expressed by a fraction or a decimal. Thus, if a gear has 40 teeth and a 1-inch circular pitch, the pitch circle has a circumference of 40 inches and a diameter of 12.732 inches.

25. Diametral Pitch.—Ordinarily it is more convenient to have the pitch diameter a dimension easily measured, and for this reason a new pitch, called the **diametral pitch,** has been devised. This pitch is not a measurement like the circular pitch, but a ratio. It is the ratio of the number of teeth in the gear to the number of inches in the diameter; or it is the number of teeth on the circumference of the gear for 1 inch of the pitch-circle diameter. It is obtained by dividing the number of teeth by the pitch diameter, in inches.

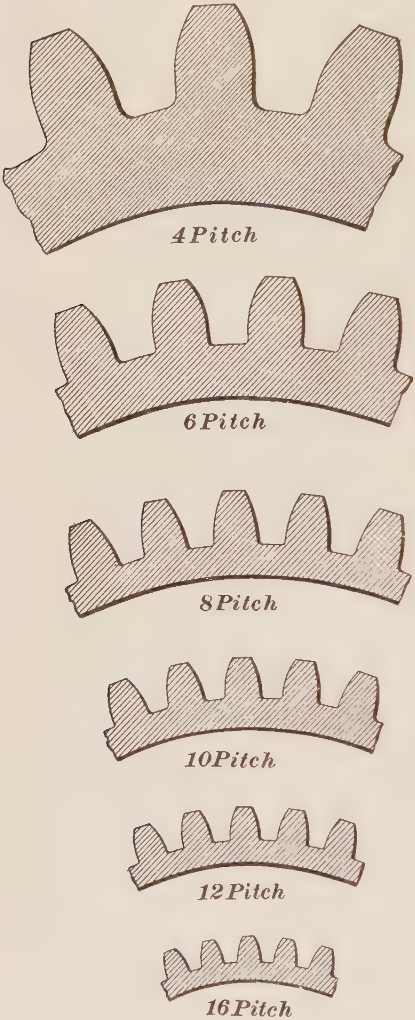

FIG. 26

For example, take a gear that has 60 teeth and is 10 inches in diameter. The diametral pitch is the ratio of 60 to 10 $= \frac{60}{10} = 6$, and the gear is called a 6-pitch gear. From the definition, it follows that teeth of any particular diametral pitch are of the same size, and have the same width on the pitch line, whatever the diameter of the gear. Thus if a 12-inch gear had 48 teeth, it would be a 4-pitch gear. A 24-inch gear, to have teeth of the same size, would have twice 48 or 96 teeth, and $96 \div 24 = 4$, the same diametral pitch as before.

Fig. 26 shows the sizes of teeth of various diametral pitches.

26. Relation of Diametral to Circular Pitch.—The relation between the diametral and circular pitches of a gear may be readily obtained.

Let p = diametral pitch;
p_1 = circular pitch, in inches;
n = number of teeth;
d = diameter of pitch circle, in inches.

TABLE I

CIRCULAR AND DIAMETRAL PITCHES

Diametral Pitch	Circular Pitch Inches	Circular Pitch Inches	Diametral Pitch
2	1.571	2	1.571
$2\frac{1}{4}$	1.396	$1\frac{7}{8}$	1.676
$2\frac{1}{2}$	1.257	$1\frac{3}{4}$	1.795
$2\frac{3}{4}$	1.142	$1\frac{5}{8}$	1.933
3	1.047	$1\frac{1}{2}$	2.094
$3\frac{1}{2}$	.898	$1\frac{7}{16}$	2.185
4	.785	$1\frac{3}{8}$	2.285
5	.628	$1\frac{5}{16}$	2.394
6	.524	$1\frac{1}{4}$	2.513
7	.449	$1\frac{3}{16}$	2.646
8	.393	$1\frac{1}{8}$	2.793
9	.349	$1\frac{1}{16}$	2.957
10	.314	1	3.142
11	.286	$\frac{15}{16}$	3.351
12	.262	$\frac{7}{8}$	3.590
14	.224	$\frac{13}{16}$	3.867
16	.196	$\frac{3}{4}$	4.189
18	.175	$\frac{11}{16}$	4.570
20	.157	$\frac{5}{8}$	5.027

The length of the circumference of the pitch circle is πd inches; hence, from the definition of circular pitch,

$$p_1 = \frac{\pi d}{n} \qquad (1)$$

From the definition of diametral pitch,

$$p = \frac{n}{d} \qquad (2)$$

Multiplying these equations together, member by member,

$$p_1 \times p = \frac{\pi d}{n} \times \frac{n}{d} = \pi = 3.1416 \qquad (3)$$

Then,
$$p_1 = \frac{\pi}{p}, \text{ and } p = \frac{\pi}{p_1} \qquad (4)$$

that is, the product of the numbers expressing the two pitches is π, and when one pitch is known, the other may be found by dividing π by the known pitch.

Example.—If the circular pitch is 2 inches, the diametral pitch is 3.1416 ÷ 2 = 1.571, nearly. If the diametral pitch is 4, the circular pitch is 3.1416 ÷ 4 = .7854 inch.

Table I gives in the first two columns values of the circular pitch corresponding to common values of the diametral pitch, and in the last two columns values of the diametral pitch corresponding to common circular-pitch values.

27. Backlash and Clearance.—Referring to Fig. 27, t and s denote, respectively, the thickness of tooth and width of space measured on the pitch line. Evidently, $t + s = p_1$, the circular pitch, and s must be at least equal to t, in order that the tooth on the other gear may enter the space without binding. In practice, s is made somewhat greater than t, to provide for irregularities in the form or spacing of the teeth, and the difference between s and t is called the **backlash.**

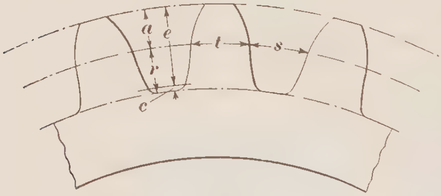

Fig. 27

The root r is also made greater than the addendum a, so that there is a clearance between the point of the tooth of one wheel and the bottom of the space of the engaging wheel. This space is shown at c, Fig. 27 and is called

bottom clearance, or simply **clearance.** When the wheels have equal addenda and equal roots, as in the interchangeable system, then $c = r - a$.

The sum $r + a$ is the whole depth of the tooth and the working depth is $e = 2a$, double the addendum.

28. Tooth Proportions for Cut Gears, Using Diametral Pitch.—Cut gears of small and medium size, such as are used on machine tools, are invariably based on the diametral pitch system. For gears of this character the following proportions are used:

$$p = \text{diametral pitch} = \frac{n}{d}$$

$$a = \text{addendum} = \frac{1}{p} \text{ inches}$$

$$r = \text{root} = \frac{9}{8p} \text{ inches}$$

$$a + r = \text{whole depth} = \frac{17}{8p} \text{ inches}$$

$$t = \text{thickness of tooth} = \frac{1.57}{p} \text{ inches}$$

$$c = \text{clearance} = \frac{1}{8p} \text{ inches}$$

Example.—The diametral pitch of a cut gear is 4; what are the tooth dimensions?

Solution.—Addendum $= \frac{1}{p} = \frac{1}{4}$ in.; root $= \frac{9}{8p} = \frac{9}{32}$ in.; clearance $= \frac{1}{8p} = \frac{1}{32}$ in.; whole depth of tooth $= \frac{17}{8p} = \frac{17}{32}$ in.; thickness of tooth $= \frac{1.57}{p} = .39+$ in. Ans.

29. Tooth Proportions for Gears Using Circular Pitch.—With gears of large size, and usually with cast gears of all sizes, the circular-pitch system is used. In these cases, it is usual to make the addendum, whole depth, and thickness of tooth conform to arbitrary rules based on the circular pitch. However, none of these rules can be considered absolute. Machine-molded gears require less clearance and backlash than hand-molded, and very large gears should have less, proportionately, than smaller ones. Table II gives the proportions that have been used successfully and will serve as an aid in deciding on suitable dimensions. Column 1 is for ordinary cast gears and column 2 is for very large gears having cut teeth. Circular pitch = p_1.

TABLE II

PROPORTIONS FOR GEAR-TEETH

	1	2
Addendum	$.30 p_1$	$.30 p_1$
Root	$.40 p_1$	$.35 p_1$
Whole depth	$.70 p_1$	$.65 p_1$
Thickness of tooth	$.48 p_1$	$.495 p_1$
Width of space	$.52 p_1$	$.505 p_1$

30. Gear-Blanks.—A **gear-blank** is a disk of metal along the circumference of which gear-teeth are to be cut. The blank is turned in a lathe, and its diameter is made equal to the outside diameter of the desired gear. In making a gear-blank, therefore, the outside diameter rather than the pitch diameter is required.

Let D = outside diameter;
d = pitch diameter;
a = addendum.

Then for any gear,

$$D = d + 2a \qquad (1)$$

But, in the case of diametral pitch, $d = \frac{n}{p}$ and $a = \frac{1}{p}$. Substituting these values,

$$D = \frac{n+2}{p} \qquad (2)$$

which may be used to find the outside diameter when the number of teeth and the diametral pitch are known.

EXAMPLE 1.—A wheel is to have 48 teeth, 6 pitch; to what diameter must the blank be turned?

SOLUTION.—By formula 2, $D = \frac{n+2}{p} = \frac{48+2}{6} = 8.33$ in. Ans.

EXAMPLE 2.—A gear-blank measures $10\frac{1}{2}$ inches in diameter and is to be cut 4-pitch; how many teeth should the gear-cutter be set to space?

SOLUTION.—From formula **2**, $D = \frac{n+2}{p}$, or $n = D \times p - 2 = 10\frac{1}{2} \times 4 - 2 = 42 - 2 = 40$ teeth. Ans.

EXAMPLES FOR PRACTICE

1. (*a*) How many teeth has a $2\frac{1}{2}$-pitch gear, 4 feet in diameter? (*b*) What is the circular pitch of this gear?

Ans. $\begin{cases} (a) \text{ 120 teeth} \\ (b) \text{ 1.257 in.} \end{cases}$

2. What is the outside diameter of a gear-blank from which a wheel is to be cut having 50 teeth 4-pitch? Ans. 13 in.

3. The pitch diameter of a gear is 25 inches; what is its outside diameter, supposing it to be 6-pitch? Ans. 25.333 in.

4. A gear-blank measures 10.2 inches in diameter and is to be cut 10-pitch; how many teeth should the gear-cutter be set to space?

Ans. 100 teeth

CONSTRUCTION OF TOOTH PROFILES

31. Approximate Methods.—The exact construction of the outlines of gear-teeth involves a great amount of work, and hence is seldom employed except where great accuracy is required. In laying out the patterns of cast gears, especially if the pitch is small, it is not necessary to employ exact constructions, since the cast teeth will depart somewhat from the true form, however carefully the pattern may have been made. The part of the curve, whether involute, epicycloid, or hypocycloid, used as the tooth outline is so short that for practical purposes one or more circular arcs closely approximating the true curve may be used for the profile.

Many systems of drawing tooth profiles by means of arcs of circles have been proposed, among which may be mentioned especially the *odontograph* and the *odontograph table*. The **odontograph** is a templet for drawing tooth outlines. Its form is generally determined by a set of radii that give a curve that closely approximates the correct tooth form. Tables of radii that give tooth outlines that approximate very closely to the correct forms are also used, and these are called **odontograph tables.**

TABLE III

INVOLUTE ODONTOGRAPH TABLE FOR STANDARD INTERCHANGEABLE GEARS

Number of Teeth	Divide by the Diametral Pitch		Multiply by the Circular Pitch	
	Face Radius	Flank Radius	Face Radius	Flank Radius
10	2.28	.69	.73	.22
11	2.40	.83	.76	.27
12	2.51	.96	.80	.31
13	2.62	1.09	.83	.34
14	2.72	1.22	.87	.39
15	2.82	1.34	.90	.43
16	2.92	1.46	.93	.47
17	3.02	1.58	.96	.50
18	3.12	1.69	.99	.54
19	3.22	1.79	1.03	.57
20	3.32	1.89	1.06	.60
21	3.41	1.98	1.09	.63
22	3.49	2.06	1.11	.66
23	3.57	2.15	1.13	.69
24	3.64	2.24	1.16	.71
25	3.71	2.33	1.18	.74
26	3.78	2.42	1.20	.77
27	3.85	2.50	1.23	.80
28	3.92	2.59	1.25	.82
29	3.99	2.67	1.27	.85
30	4.06	2.76	1.29	.88
31	4.13	2.85	1.31	.91
32	4.20	2.93	1.34	.93
33	4.27	3.01	1.36	.96
34	4.33	3.09	1.38	.99
35	4.39	3.16	1.39	1.01
36	4.45	3.23	1.41	1.03
37–40	4.20		1.34	
41–45	4.63		1.48	
46–51	5.06		1.61	
52–60	5.74		1.83	
61–70	6.52		2.07	
71–90	7.72		2.46	
91–120	9.78		3.11	
121–180	13.38		4.26	
181–360	21.62		6.88	

32. The Involute Odontograph Table.—Table III is an involute odontograph table that gives a series of radii for constructing arcs of circles for approximate tooth outlines. The centers of these arcs all lie on the base circle. For gears with from 10 to 36 teeth, the tooth profile consists of two circular arcs of different radii, one for the face of the tooth and one for the part of the profile between the pitch circle and base circle. For gears with more than 36 teeth, a single arc is used for the curve between base and addendum circles. The flanks from the base line to the bottoms of the spaces are straight radial lines, but at the bottom fillets should be put in to avoid sharp corners.

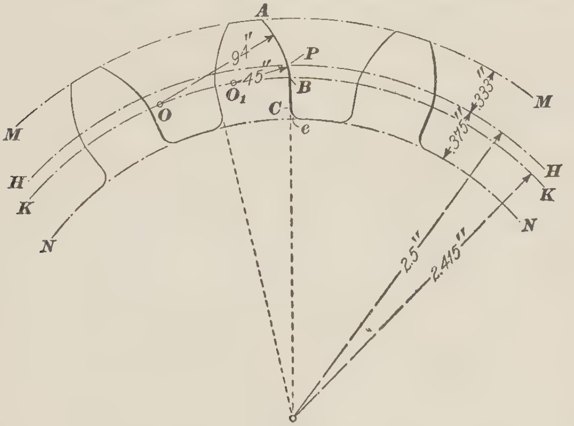

FIG. 28

The construction of tooth outlines by the odontograph table is illustrated in Fig. 28. Suppose that it is required to draw a gear with 15 teeth, 3 diametral pitch. Then, as previously explained, pitch diameter $= \frac{n}{p} = \frac{15}{3} = 5$ inches; addendum $= \frac{1}{p} = \frac{1}{3} = .333$ inch; root $= \frac{9}{8p} = \frac{9}{8 \times 3} = .375$ inch; by Art. **14**, radius of base circle $= .966 \times 2.5 = 2.415$ inches; circular pitch $= \frac{3.1416}{3} = 1.047$ inches; and thickness of tooth $= \frac{1.57}{3} = .523$ inch. With these data, the pitch, base, addendum, and root circles are readily drawn, as shown at *H*, *K*, *M*, and *N*, Fig. 28.

Let *P* be some pitch point. Referring to the odontograph table, the numbers in the columns for the face and flank radii are, respectively, 2.82 inches and 1.34 inches; but these must be divided by the diametral pitch 3, giving .94 inch and .45 inch, respectively, as the radii. Take points O and O_1 on the base circle, so that $OP = .94$ inch and $O_1P = .45$ inch. With O as a center, strike the arc PA, and with O_1 as a center, strike the arc PB. From B draw the radial line BC and finish the outline with the fillet *e*. The radius of this fillet differs in practice. Some authorities recommend a radius equal to one-seventh the width of the space between two teeth, measured along the addendum circle, while others recommend a smaller radius. When the form of the tooth is such that the root is liable to be weak, the radius should be made as large as possible. Knowing the circular pitch and thickness of tooth, the pitch points for other teeth may be laid off and the profiles constructed in a similar manner.

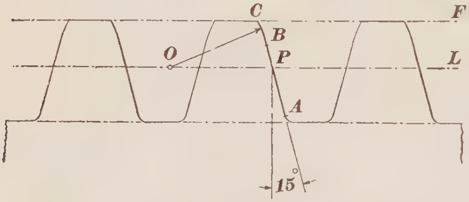

FIG. 29

The construction of the involute rack is shown in Fig. 29. The side of the tooth is a straight line AB inclined at an angle of 15° with a line at right angles to the pitch line L. To correct for interference, the points of the teeth are cut away as follows: From a point B midway between the pitch line L and the addendum line F, strike a circular arc BC from a center O on the pitch line with a radius $OB = \frac{2.10}{p}$ inches, or $.67p_1$ inch.

33. The Cycloidal Odontograph Table.—For the cycloidal system of interchangeable gears Table IV gives the radii of circular arcs for faces and flanks, and also the

TABLE IV

CYCLOIDAL ODONTOGRAPH TABLE FOR STANDARD INTERCHANGEABLE GEARS

Number of Teeth	Divide These Numbers by the Diametral Pitch				Multiply These Numbers by the Circular Pitch			
	Face		Flank		Face		Flank	
	Radius	Distance	Radius	Distance	Radius	Distance	Radius	Distance
10	1.99	.02	−8.00	4.00	.62	.01	−2.55	1.27
11	2.00	.04	−11.05	6.50	.63	.01	−3.34	2.07
12	2.01	.06	∞	∞	.64	.02	∞	∞
13–14	2.04	.07	15.10	9.43	.65	.02	4.80	3.00
15–16	2.10	.09	7.86	3.46	.67	.03	2.50	1.10
17–18	2.14	.11	6.13	2.20	.68	.04	1.95	.70
19–21	2.20	.13	5.12	1.57	.70	.04	1.63	.50
22–24	2.26	.15	4.50	1.13	.72	.05	1.43	.36
25–29	2.33	.16	4.10	.96	.74	.05	1.30	.29
30–36	2.40	.19	3.80	.72	.76	.06	1.20	.23
37–48	2.48	.22	3.52	.63	.79	.07	1.12	.20
49–72	2.60	.25	3.33	.54	.83	.08	1.06	.17
73–144	2.83	.28	3.14	.44	.90	.09	1.00	.14
145–300	2.92	.31	3.00	.38	.93	.10	.95	.12
301 to rack	2.96	.34	2.96	.34	.94	.11	.94	.11

distances, from the pitch circle, of the circles on which the centers lie.

For the 12-tooth gear, as previously stated, the flanks are radial; hence, the infinite values in Table IV, denoted by ∞.

The manner of using the odontograph table is shown by an example in Fig. 30, which shows the layout of the teeth of a 16-tooth gear, 2 diametral pitch. For this gear, the following data are readily obtained: Pitch diameter $= \frac{n}{p} = \frac{16}{2} = 8$ inches; addendum $= \frac{1}{p} = \frac{1}{2}$ inch; root $= \frac{9}{8p} = \frac{9}{16}$ inch.

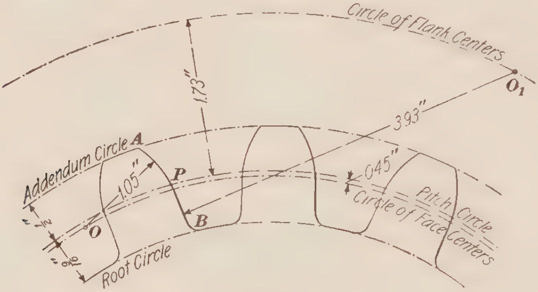

FIG. 30

From these dimensions, the addendum, pitch, and root circles can be drawn. From Table IV, the distance for faces is .09 inch, and that for flanks is 3.46 inches, which divided by 2, the diametral pitch, gives, respectively, .045 inch and 1.73 inches; hence, the circle of face centers is drawn .045 inch *inside* the pitch circle, and the circle for flank centers 1.73 inches *outside* the pitch circle. The radii in Table IV are 2.10 and 7.86, and these divided by 2 give 1.05 inches and 3.93 inches, respectively, for face radius and flank radius. Let P be chosen as a pitch point. Then with $OP = 1.05$ inches, locate the center O on the circle of face centers and describe the arc PA; and with $O_1P = 3.93$ inches locate O_1 on the circle of flank centers and describe the arc PB. At the bottom of the space put in a fillet, as usual.

The name *three-point odontograph* is sometimes given to this system, from the fact that the arc constructed by it passes through three points on the true cycloidal curve.

BEVEL GEARING

34. Relation of Bevel to Spur Gearing.—It has been shown that motion may be transmitted by rolling cylinders when the axes are parallel, or by rolling cones when the axes intersect. If rolling cylinders are used as pitch surfaces and are provided with teeth, spur gears are the result; if frustums of rolling cones are used as pitch surfaces and provided with teeth, the gears so formed are called **bevel gears.** Bevel gears are used to transmit motion from one shaft to another making an angle with the first. In case the shafts are at right angles, and the gears have equal pitch diameters, they are known as **miter gears.**

35. Generation of Bevel-Gear Teeth.—In the generation of cycloidal tooth outlines for spur gears, generating circles were caused to roll on pitch circles. The tooth surfaces, however, are produced by the rolling of generating cylinders on pitch cylinders. The generation of tooth surfaces for bevel gears is accomplished, in a similar manner, by the rolling of a generating cone in contact with the pitch cone.

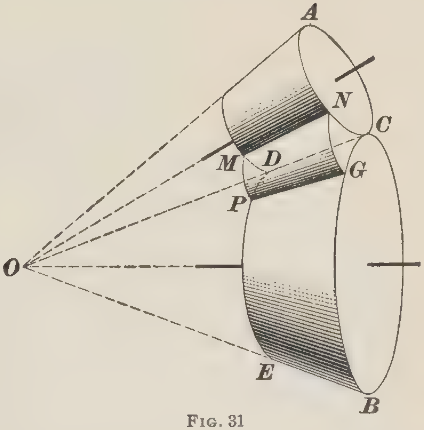

FIG. 31

In Fig. 31, let COB represent a pitch cone, the part $CDEB$ being the pitch surface of a bevel gear, and let AOC be the generating cone. If the generating cone is supposed to describe the tooth surface $MNGP$ by rolling on the pitch cone, the line NG representing the outer edge of the tooth will lie on the surface of a sphere whose radius is ON. The point N that describes this line is always at a fixed distance from the center O; hence, every point in the line NG is equally distant from O, and, as all points in a spherical surface are equally distant from a point within called the center, it follows that NG must lie on a spherical surface. The curve NG, therefore, may be called a **spherical epicycloid,** and by rolling the generating cone on the concave surface of the pitch cone a **spherical hypocycloid** is obtained. A **spherical involute** may be obtained by unwrapping an imaginary flexible sheet from a base cone lying within the pitch cone. A point on the edge of this sheet will describe a curve on the surface of a sphere, and this curve will be a spherical involute.

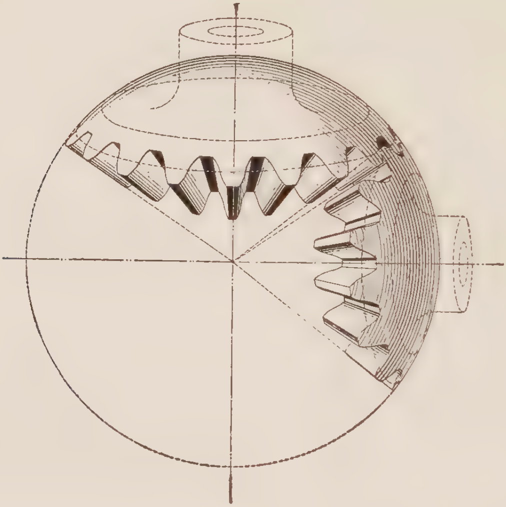

Fig. 32

To be theoretically exact, therefore, the tooth curves for a bevel gear should be traced on the surface of a sphere, as shown in Fig. 32. This method is not a practical one, however, and in practice is replaced by what is known as **Tredgold's approximation,** which is much simpler and is universally used.

By this method, the tooth curves are drawn on cones tangent to the spheres at the pitch lines of the gears, as shown in Fig. 33. The process is simply to develop or unwrap the surfaces of the cones, the unwrapped surfaces being represented by *A B C* and *C D E* in the figure. The length of the arc *A B C* is equal to the length of the pitch circle *A′ C*, and the length of the arc *C D E* is equal to that of the pitch circle *C E′*. The gear-teeth are then drawn on the unwrapped surfaces, precisely as for spur gears of the same pitch and diameter.

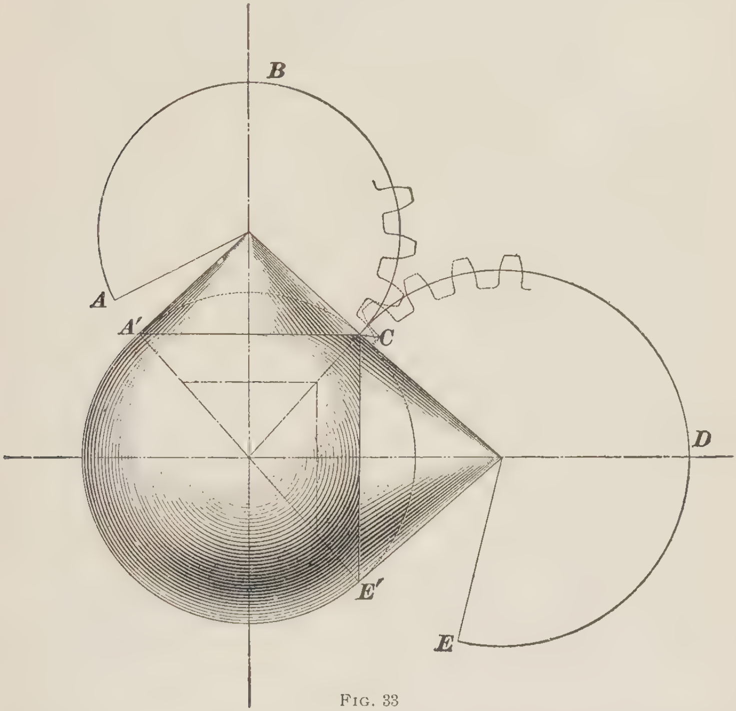

Fig. 33

The teeth, as laid out by Tredgold's method, will vary somewhat from the shape of the spherical teeth. But although the tooth curves may not be exactly the same as the curves on the sphere, the difference is so slight that it has no appreciable effect on the uniformity of the motion transmitted.

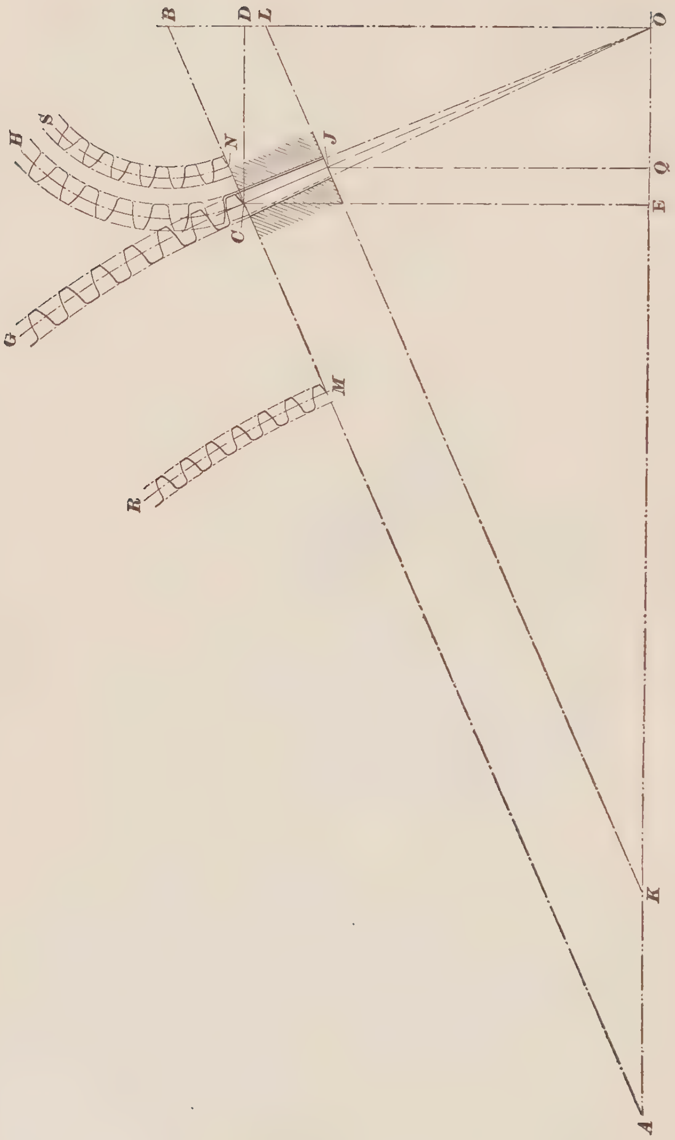

FIG. 34

36. Laying Out Bevel Gears.—Let OA and OB, Fig. 34, be the axes of the pitch cones, intersecting at O, and assume OC to be the line of contact of the two cones. Then, if CD is the radius of one cone, CE is the radius of the other, and the velocity ratio is the ratio $CD : CE$.

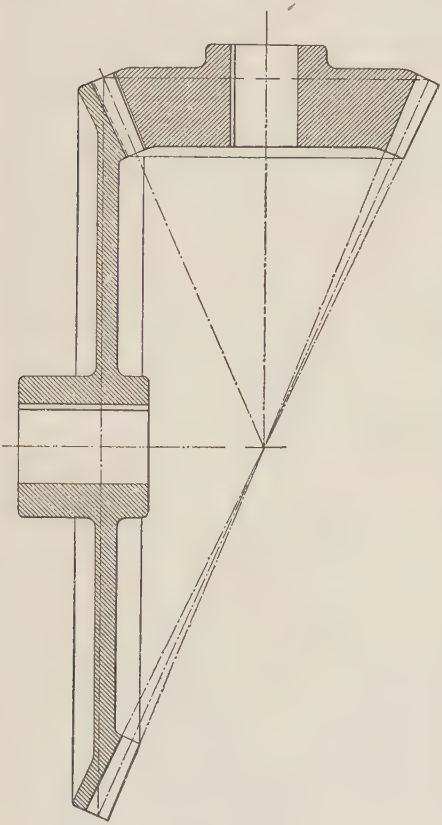

FIG. 35

Through C a line is drawn at right angles to OC, cutting the axes in the points A and B. With A and B as centers and AC and BC as radii, circular arcs CG and CH are drawn. Suppose, now, that CJ is chosen as the length of the rolling conical frustum, or, what is the same thing, the length of the face of a tooth. Through J draw a line parallel to AB, cutting OA and OB in K and L, respectively. Then, with A as a center, and $AM = KJ$ as a radius, draw the arc MR, and with B as a center, and $BN = LJ$ as a radius, draw the arc NS. The four arcs CG, CH, MR, and NS are then considered as pitch circles of spur gears, and the tooth outlines are drawn according to the methods already given.

Let n_a = the number of teeth in the gear whose axis is OA; then, as in spur gearing, the diametral pitch for the outside diameter of the cone is

$$p = \frac{n_a}{\text{diameter}} = \frac{n_a}{2CE}$$

From this value of p, the addendum, the root, and the radii for the circular arcs of the tooth outline are obtained. Similarly, for the inside radius, $p = \frac{n_a}{2\,J\,Q}$; and from this value of p, the teeth on the pitch circles $M\,R$ and $N\,S$ are constructed. In making wooden patterns for cast gears, the teeth may be laid out on strips of thin sheet metal, and these strips may then be fastened to the ends of the rolling frustums. The spaces in the metal strips will form guides at the ends of the teeth, so that the wood in the spaces may be cut away accurately. In Fig. 35 is shown a section of the gears laid out in Fig. 34.

37. Bevel-Gear Angles.—In Fig. 36, $O\,C$ is an element of the pitch cone, $O\,F$ marks the top and $O\,E$ the bottom of the working portion of a tooth. The angles a, b, c, e, f, and g are named as follows: a is the center angle; b, the face angle; c, the cutting angle; e, the cutting decrement, or angle of bottom; f, the face increment, or angle of top; g, the angle of edge.

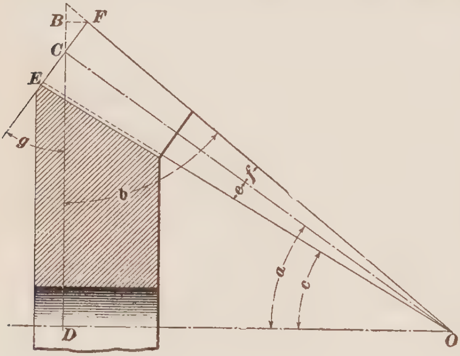

FIG. 36

In turning up a gear-blank to size, it is necessary to know the outside diameter, face angle, and, also, the angle of edge The face angle b is found by subtracting the sum of the angles a and f from 90°. The angle of edge g is equal to the center angle a. The center angle is determined from the ratio between the outside pitch radii $C\,D$ and $O\,D$ of the two gears. Thus, $C\,D$ and $O\,D$ represent the outside pitch radii of the two gears, and $\tan a = \frac{C\,D}{O\,D}$. But $C\,D$ and $O\,D$ are proportional to the number of teeth in their respective gears;

hence, the number of teeth may be substituted for the pitch radii. Having the value of the tangent, the corresponding angle may easily be found from a table of natural tangents. The cutting angle c is found by subtracting the angle e from the angle a. The angles e and f may be obtained by trigonometry, as follows: Let r be the outside pitch radius CD, Fig. 36. Then, $\tan f = \frac{CF}{OC} = \frac{CF}{CD} \times \frac{CD}{OC}$. But $\frac{CF}{CD} = \frac{CF}{r}$ and $\frac{CD}{OC} = \sin a$. Substituting these values,

$$\tan f = \frac{CF}{r} \sin a \qquad (1)$$

Now, CF is the addendum, and when diametral pitch is used, $CF = \frac{1}{p} = \frac{2r}{n}$, where n equals the number of teeth. Hence, for diametral pitch, formula **1** reduces to

$$\tan f = \frac{\frac{2r}{n}}{r} \sin a = \frac{2}{n} \sin a \qquad (2)$$

In terms of the circular pitch, the addendum $CF = .3 p_1$ and consequently, when the circular pitch is used, formula **1** becomes

$$\tan f = \frac{.3 p_1}{r} \sin a \qquad (3)$$

Having determined the value of $\tan f$, the corresponding angle may readily be found by reference to a table of natural tangents.

In cutting bevel gears, the clearance is made constant for the entire length of the tooth, and the cutting angle is therefore taken at the working depth of the tooth, allowance for clearance being made on the cutter. Then $CE = CF$, and it follows that the angle e must equal the angle f.

The outside diameter is equal to $2\,BD$. But,

$$2\,BD = 2\,(BC + CD) \qquad (4)$$

The angle BCF = angle g = angle a. Then $\frac{BC}{CF} = \cos BCF = \cos a$, or $BC = CF \cos a$. But, CF is the addendum which is equal to $\frac{2r}{n}$ for diametral pitch, and $.3p_1$ for circular pitch. Hence, for diametral pitch, $BC = \frac{2r}{n}\cos a$; and for circular pitch, $BC = .3p_1 \cos a$. The radius $CD = r$. Substituting these values, formula **4** becomes, for diametral pitch,

$$\text{outside diameter} = 2\left(\frac{2r}{n}\cos a + r\right) \qquad (5)$$

and for circular pitch,

$$\text{outside diameter} = 2\left(.3p_1 \cos a + r\right) \qquad (6)$$

The following example will show how to calculate the various angles:

EXAMPLE.—Suppose that two bevel gears have 70 teeth and 30 teeth, respectively, and that the diametral pitch at the outer ends of the teeth is 3; find the center, face, and cutting angles, the angles of top, bottom, and edge, and the outside diameter, for each gear.

SOLUTION.—Since the shafts are at right angles, use the above formulas, referring also to Fig. 36. Tan $a = \frac{CD}{OD} = \frac{70}{30} = 2.33333$, from which $a = 66° 48'$ for the larger gear. For the smaller gear, $a = 90° - 66° 48' = 23° 12'$. Ans.

By formula **2**, tan $f = \frac{2}{70} \times \sin 66° 48' = \frac{2}{70} \times .91914 = .02626$; from which $f = 1° 30'$. Hence, $e = 1° 30'$, also, and these angles are the same for both bevel gears. Ans.

For the large gear, $a + f = 66° 48' + 1° 30' = 68° 18'$. Hence, b for this gear is equal to $90° - 68° 18' = 21° 42'$; also, $c = a - e = 66° 48' - 1° 30' = 65° 18'$, while $g = a = 66° 48'$. Ans.

For the small gear, $a + f = 23° 12' + 1° 30' = 24° 42'$. Hence, b for this gear is equal to $90° - 24° 42' = 65° 18'$; also, $c = a - e = 23° 12' - 1° 30' = 21° 42'$, and $g = a = 23° 12'$. Ans.

For larger gear, using formula **5**, outside diameter $= 2\left(\frac{2r}{n}\cos a + r\right)$. Now, $r = \frac{1}{2}\left(\frac{n}{p}\right) = \frac{1}{2}\left(\frac{70}{3}\right) = \frac{70}{6}$; $n = 70$; $\cos a = \cos 66° 48' = .39394$. Then, outside diameter $= 2\left(\frac{2 \times 70}{6} \times \frac{1}{70} \times .39394 + \frac{70}{6}\right) = 23.6$ in., nearly. Ans.

For the smaller gear, $r = \frac{1}{2}\left(\frac{n}{p}\right) = \frac{1}{2}\left(\frac{30}{3}\right) = 5$; $n = 30$; $\cos a = \cos 23° 12' = .91914$. Then, outside diameter $= 2\left(\frac{2 \times 5}{30} \times .91914 + 5\right) = 10.61$ in. Ans.

SPIRAL AND WORM-GEARING

38. Spiral Gearing.—The name **spiral gearing** is given to a system of gearing in which the wheels have cylindrical pitch surfaces, as in spur gears, but in which the teeth are not parallel to the axes. Each tooth winds helically like a screw thread, hence spiral gearing is sometimes called **screw gearing.**

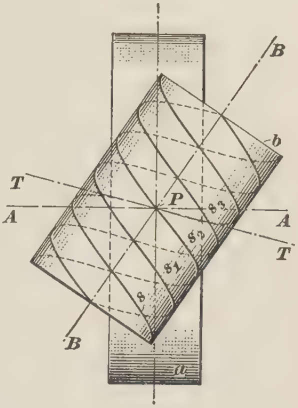

FIG. 37

In Fig. 37, suppose *a* and *b* to be two cylinders having axes *A A* and *B B*, respectively. The cylinders may be made to touch at a single point *P*. Let a helix *s* be traced on the surface of *b*, passing through *P*, and let *T P T* be the tangent to the helix at *P*. Now, through *P*, let a helical line be traced on the cylinder *a*, having such an angle that its tangent is likewise *T P T*. If, now the helix on *b* be replaced by a projection similar to a screw thread, and that on *a* by a groove made to fit this projection, a rotation of *b* about the axis *B B* will cause a rotation of *a* about the axis *A A*. In order that the motion may be continuous, successive grooves on *a* must be so spaced as to engage properly with the thread on *b*.

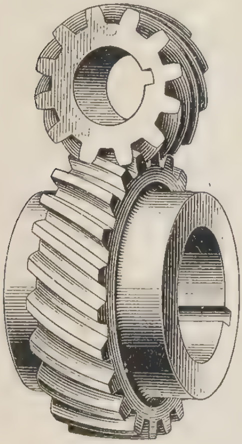

FIG. 38

39. Relation of Spiral to Worm-Gearing.—The cylinder *b* may be provided with only one thread or with several threads. Thus, in Fig. 37, the helix *s* may be used alone, or the helixes s_1, s_2, and s_3 may be added. If from one to three helixes are employed, the projections are called threads rather than teeth; the cylinder and thread (or threads) together are called a **worm,** and the other cylinder is called the **worm-wheel.**

When each cylinder has several helixes, as shown in Fig. 38, the term *tooth* is used instead of *thread;* that is, it is said to be a **spiral gear,** having a certain number of teeth. It must be understood, however, that the number of teeth is the number of helixes wound around the cylindrical pitch surface, and that a worm with a single thread is in reality a spiral gear with one tooth.

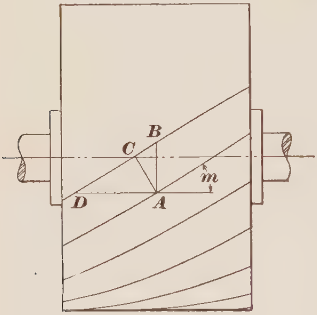

FIG. 39

40. Pitches in Spiral Gearing.—The distance between corresponding points on two consecutive teeth, measured around the cylindrical pitch surface at right angles to the axis, is the **circular pitch,** as shown at AB, Fig. 39. This is also sometimes called the **circumferential pitch.** The distance measured at right angles to the teeth, as AC, is called the **normal pitch.** The distance measured parallel to the axis, as AD, is the **axial pitch.** Let these pitches be denoted by p_1, p_n, and p_a, respectively. If m denotes the angle between the helix and a line on the pitch surface parallel to the axis, then $m = BAC$, and $AC = AB \cos BAC$; that is,

$$p_n = p_1 \cos m \qquad (1)$$

The number of teeth n on the gear is equal to the circumference of the pitch circle divided by the circular pitch; hence,

$$n = \frac{2\pi r}{p_1} = \frac{2\pi r \cos m}{p_n} \qquad (2)$$

where r denotes the pitch radius.

In a pair of gears that run together, the normal pitches must be the same in both, but the circular pitches may or may not be the same. If the axes of the two gears are at right angles, the circular pitch of one must be equal to the axial pitch of the other, and vice versa.

41. Computation of Spiral Gears for Given Velocity Ratio.

Let n_a and n_b = number of teeth in *a* and *b*, Fig. 40, respectively;
r_a and r_b = radii of pitch cylinders of *a* and *b*;
N_a and N_b = speeds of *a* and *b*, in revolutions per minute;
m_a = angle of teeth of *a* with axis of *a*;
m_b = angle of teeth of *b* with axis of *b*;
e = angle between axes of *a* and *b*;
h = distance between axes of *a* and *b*.

In the case of spiral gears, the revolutions per minute of the shafts are inversely as the numbers of teeth, that is,

$$\frac{N_a}{N_b} = \frac{n_b}{n_a}$$

The radii r_a and r_b, however, have no direct influence on the velocity ratio of the gears, and within certain limits may be made of any length.

From Fig. 40, angle m_a + angle m_b = angle *e*. The relative magnitudes of angles m_a and m_b depend on the location of the tangent *TPT*. Having fixed this tangent and the angles m_a and m_b, the radii r_a and r_b may be obtained as follows:

From formula 2 of Art. 40,

$$n_a = \frac{2\pi r_a \cos m_a}{p_n} \qquad (1)$$

and

$$n_b = \frac{2\pi r_b \cos m_b}{p_n} \qquad (2)$$

Dividing,

$$\frac{n_b}{n_a} = \frac{N_a}{N_b} = \frac{r_b \cos m_b}{r_a \cos m_a} \qquad (3)$$

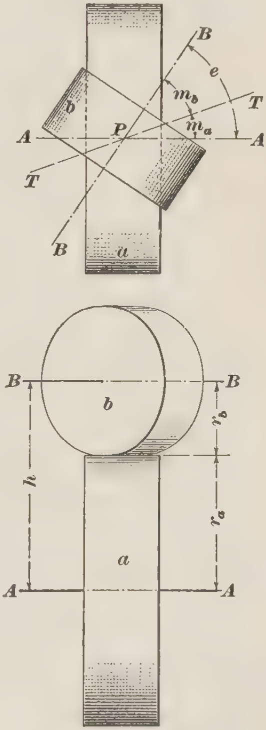

FIG. 40

It will be seen from this formula that the values of r_a and r_b may be assumed if desired and the corresponding values of m_a and m_b calculated. In practice, however, the method given above is generally preferred.

The speed ratio $\frac{N_a}{N_b}$ being given, and angles m_a and m_b having been decided on, the ratio $\frac{r_b}{r_a}$ can be found. Further, the sum $r_a + r_b = h$, the distance between the axes is known, and from the two equations thus at hand the pitch radii r_a and r_b may be calculated. An example will show the method.

Example.—A pair of spiral gears with axes at 75° are to transmit a velocity ratio of $2\frac{1}{2}$ to 1, and the distance between their axes is 14 inches; find the pitch radii.

Solution.—Any number of solutions may be obtained, since it is possible to divide 75° into two angles in an infinite number of ways. Suppose, first, $m_a = m_b = 37\frac{1}{2}°$; then $\frac{N_a}{N_b} = 2.5 = \frac{r_b \cos 37\frac{1}{2}°}{r_a \cos 37\frac{1}{2}°}$, or $\frac{r_b}{r_a} = 2.5$. But, $r_a + r_b = 14''$; whence $r_b = 10''$ and $r_a = 4''$. Ans.

For a second solution, let $m_a = 30°$ and $m_b = 45°$; then

$$2.5 = \frac{r_b \cos 45°}{r_a \cos 30°} = .817\,\frac{r_b}{r_a}, \text{ or } \frac{r_b}{r_a} = \frac{2.5}{.817} = 3.06.$$

Solving, $r_a = 3.45''$ and $r_b = 10.55''$. Ans.

WORM-GEARS

42. Threads of Worms and Teeth of Worm-Wheels.

When the angle m of the helix, Fig. 39, is so great that the tooth becomes a thread winding entirely around the pitch cylinder, the combination is called a **worm** and **worm-wheel**. Usually, the axes of worm-gears are at right angles. The screw may have a single thread or two or three threads With a single thread, the velocity ratio of the gears is equal to the number of teeth in the worm-wheel; thus, if the wheel has 40 teeth, 40 turns of the worm-shaft are required for 1 turn of the wheel.

Fig. 41 shows a worm and worm-wheel. It will be noticed that in the longitudinal section, taken through the worm, the threads appear to be like involute rack teeth. The worm is usually made in a screw-cutting lathe, and as it is easier to turn the threads with straight sides, it is better that they should be of the involute form. Involute teeth, then, should be used on the wheel, and should be of a pitch to correspond with the threads on the worm.

The circular-pitch system is used for worm-gearing because lathes are seldom provided with the correct change gears for cutting diametral pitches. The circular pitch is not so inconvenient, however, in the case of worm-gearing as with spur gearing. If the diameter of the worm-wheel should come in awkward figures, the diameter of the worm can be made such that the distance between centers will be the dimension desired. The circular pitch of the gear must be equal to the axial pitch of the worm.

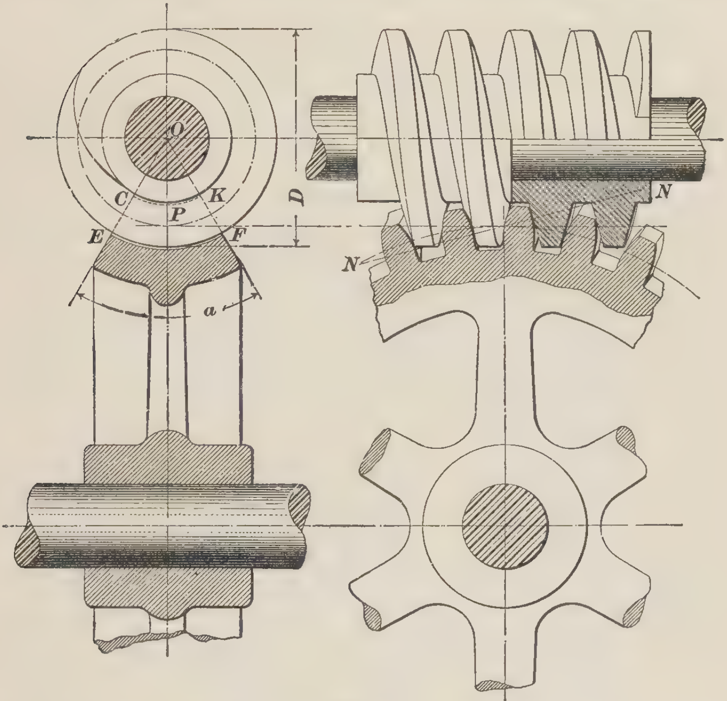

FIG. 41

43. Close-Fitting Worm and Wheel.—To make a close-fitting wheel, a worm is made of tool steel and then fluted and hardened like a tap. It is almost a duplicate of the worm to be used, being of a slightly larger diameter to allow for clearance. This cutter, or **hob,** is placed in mesh with the worm-wheel, on the face of which notches have been cut deep enough to receive the points of the teeth of the hob. The hob is then made to drive the wheel and is dropped deeper into it at each revolution of the latter until the teeth are finished.

Fig. 41 represents a close-fitting worm and wheel. The pitch circles are in contact at P. The outside diameter D of the worm may be made four or five times the pitch. The arcs CK and EF are drawn about O and limit the addendum and root of the wheel teeth, the distance between them being the whole depth of the teeth. Clearance is allowed just as in spur gearing. The angle a is generally taken as either 60° or 90°. The whole diameter of the wheel blank can be obtained by measuring the drawing.

The object of hobbing a wheel is to get more of the bearing surface of the teeth on the worm-thread, making the outline of the teeth something like the thread of a nut.

GEAR TRAINS AND CAMS

GEAR TRAINS

1. Use of Trains.—Motion may be transmitted between two parallel shafts by a single pair of gear-wheels or by a single pair of pulleys and a belt. However, it is often preferable to replace the single pair by a series of such gears or pulleys, especially if the required velocity ratio is great. Thus, in Fig. 1, it would be possible to transmit motion from shaft *A* to shaft *F* by means of two pulleys only, but to give the desired velocity ratio, pulley *a* would be very large and pulley *f* very small. By interposing the intermediate shafts *B* and *E* and the pulleys *b*, *c*, *d*, and *e*, the velocity ratio is obtained with pulleys that do not vary greatly in size. In the same way, motion may be transmitted by a single pair of gears, but it is often better to introduce intermediate gears to avoid inconvenient sizes.

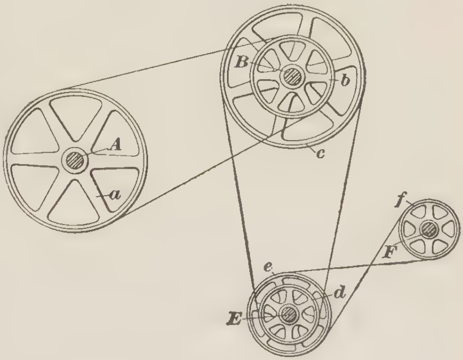

FIG. 1

In general, a series of pulleys, gear-wheels, worms and worm-wheels, etc., interposed between the driving and driven shafts, is called a **train of mechanism.** If all the members are spur gears, the train is called a **gear train** or train of

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gears; with bevel gears for members, it is called a **bevel-gear train.**

2. Velocity Ratio of Two Wheels.—In Fig. 2, let *a* and *b* be two gear-wheels turning on the shafts *A* and *B*, and let *P* be the pitch point or point of contact of the rolling pitch circles. Evidently, since these pitch circles roll together, points on their circumferences have the same velocity. Let r_a denote the radius of the wheel *a*, and N_a the speed of *a*, in revolutions per minute, frequently written R. P. M. Then a point on the circumference of *a* travels, in 1 minute, $2\pi r_a N_a$ feet, and a point on the circumference of *b* travels $2\pi r_b N_b$ feet. Hence, $2\pi r_a N_a = 2\pi r_b N_b$,

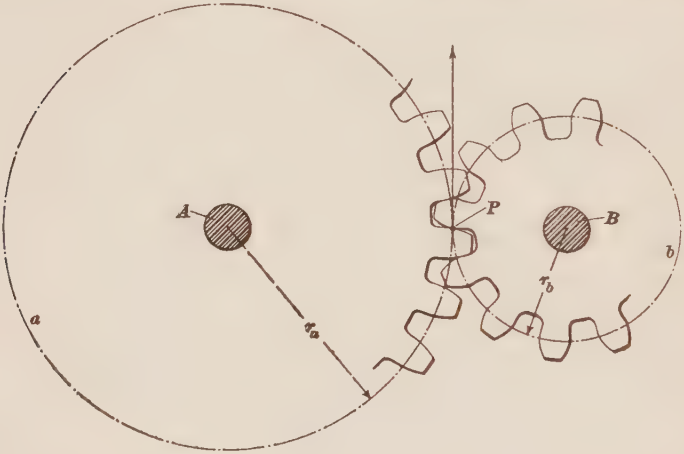

Fig. 2

or
$$r_a N_a = r_b N_b \qquad (1)$$
whence
$$\frac{N_a}{N_b} = \frac{r_b}{r_a} \qquad (2)$$

That is, the revolutions per minute of two wheels running together are inversely as their radii.

The radii of the wheels are directly proportional to the diameters and to the number of teeth; hence it is allowable to use the ratio of either the radii, diameters, or numbers of teeth, as is most convenient. Furthermore, in any one train, the radii of one pair, the diameters of another pair, and the numbers of teeth in a third pair, may be used.

3. Velocity Ratio of a Train of Gears.—Suppose that there is a train, as shown in Fig. 3, in which the pairs are a and b, c and d, and e and f. From formula **2** of Art. **2**, $\frac{N_a}{N_b} = \frac{r_b}{r_a} = \frac{D_b}{D_a}$, where r and D denote, respectively, the radius

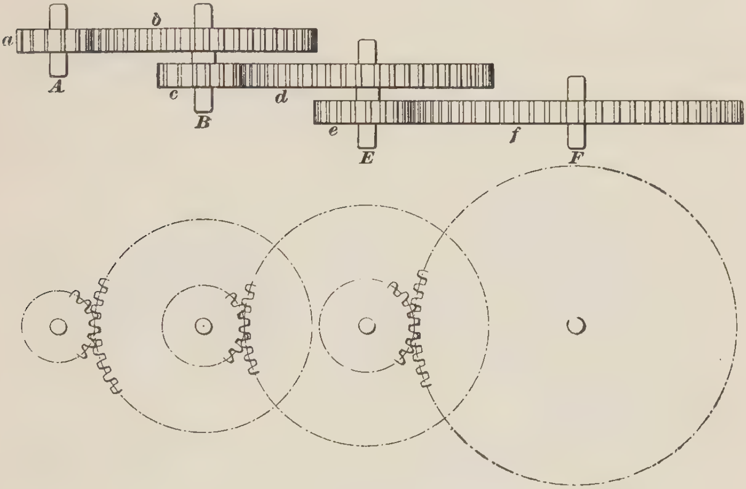

Fig. 3

and the diameter of the gear. For the second pair, $\frac{N_c}{N_d} = \frac{r_d}{r_c} = \frac{D_d}{D_c}$; and for the third pair, $\frac{N_e}{N_f} = \frac{r_f}{r_e} = \frac{D_f}{D_e}$. Multiplying together the left-hand and the right-hand members, respectively, of these three equations, and placing the products equal to one another,

$$\frac{N_a \times N_c \times N_e}{N_b \times N_d \times N_f} = \frac{r_b \times r_d \times r_f}{r_a \times r_c \times r_e} = \frac{D_b \times D_d \times D_f}{D_a \times D_c \times D_e}$$

But gears b and c are fixed to the same shaft, as are gears d and e, hence $N_b = N_c$ and $N_d = N_e$; therefore the first fraction reduces to $\frac{N_a}{N_f}$, which is the ratio between the speeds of the first and last shafts. Therefore the preceding formula may be expressed thus:

$$\frac{\text{R. P. M. of the driving shaft}}{\text{R. P. M. of the driven shaft}} = \frac{\text{product of radii, diameters, or number of teeth of all the followers}}{\text{product of radii, diameters, or number of teeth of all the drivers.}}$$

EXAMPLE.—In Fig. 4 is shown a train of six wheels on four shafts. Four of the wheels are gear-wheels, and two are pulleys connected by a belt. Let d_1, d_2, etc. denote the drivers, and f_1, f_2, etc. the followers, N the number of revolutions of d_1, and n the number of revolutions of f_3. Suppose that the pulley D_1 is 40 inches in diameter, and f_1 35 inches; d_2 has 54 teeth, and f_2 60 teeth; d_3 is 1 foot in diameter, and f_3 2 feet. What is the speed of f_3 if $N = 100$ revolutions per minute?

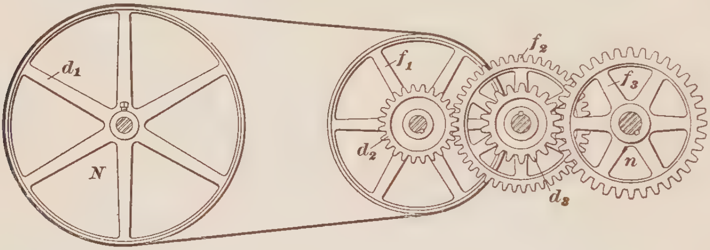

FIG. 4

SOLUTION.—The product of the drivers × the speed of the driving shaft = the product of the followers × speed of the driven shaft; that is, $100 \times 40 \times 54 \times 1 = n \times 35 \times 60 \times 2$. Hence,

$$n = \frac{100 \times 40 \times 54 \times 1}{35 \times 60 \times 2} = 51\tfrac{3}{7} \text{ R. P. M.} \quad \text{Ans.}$$

The drivers and followers may be arranged in any order without changing the result; that is, they may be interchanged among themselves. It should be noticed, however, that if the diameter of one driver be given in inches, the diameter of its follower must also be given in inches.

4. Direction of Rotation.—Axes connected by gear-wheels rotate in opposite directions. Hence, in a train consisting solely of external gear-wheels, if the number of axes be odd, the first and last wheels will revolve in the same direction; if the number be even, they will revolve in opposite directions.

It is evident that in the case of a pinion working in an internal gear the two wheels will turn in the same direction.

5. Idlers.—In the train in Fig. 3, shaft B carries two gears, one being driven by gear a and the other driving gear d. Sometimes, however, only one intermediate gear is used, serving both as a driver and a follower. Such a wheel is called an **idler**, or *idle wheel*, and while it affects the relative direction of rotation of the wheels it is placed between, it does not affect their velocity ratio.

The following examples will illustrate this statement, and show a few ways in which idlers are used. In Fig. 5 is shown one method of arranging the change gears on the end of an engine lathe for changing the speed of the lead screw, which is used to feed the tool in screw cutting. The driver d_1 receives motion from the lathe spindle, and the follower f_2 drives the lead screw. The middle wheel, which is an idler, acts both as a driver and follower, or as d_2 and f_1.

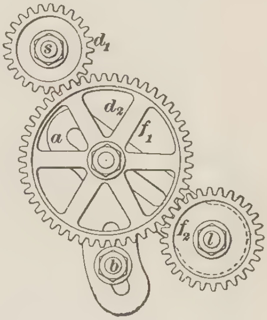

FIG. 5

Let N_1 and N_2 represent, respectively, the number of revolutions of d_1 and f_2 and let the letters d_1, d_2, f_1, and f_2 be used to denote either the radii, diameters, or numbers of teeth of the wheels, as well as the wheels themselves. Then, $\frac{N_2}{N_1} = \frac{d_1 \times d_2}{f_1 \times f_2}$. But, as d_2 and f_1 represent the same wheel, they have the same value, and $\frac{N_2}{N_1} = \frac{d_1}{f_2}$, or $N_2 = \frac{N_1 d_1}{f_2}$. That is, the speed of f_2 is exactly the same as though no idler were used.

To change the speed of the lead screw, a different size of wheel is put on in place of d_1 or f_2, or both are changed. The idler turns on a stud clamped in the slot in the arm a.

This stud can be moved in the slot to accommodate the different sizes of wheels on *l*. To bring the wheel in contact with the gear on *s*, the arm is swung about *l* until in the right position, when it is clamped to the frame by the bolt *b*.

The way in which an idler changes the direction of rotation is shown in Fig. 6, which is a reversing mechanism, sometimes placed on the headstock of a lathe for reversing the feed. Here the idler *I* is in contact with gears *d* and *f*, making three axes, an odd number, so that *d* and *f* turn in the same direction. Wheels *I* and *I'*, however, are pinned to the plate *p*, which can swing about the axis of *f*, and they remain always in contact. Moreover, as plate *p* swings about its center gear, *I* must necessarily remain in contact with gear *f*. If the lower end of the handle *h* be drawn upwards, the plate will be turned clockwise about the shaft of *f*, by means of the pin working in the slot in the lever *l*. The two idlers will take the dotted positions shown, and *d* will drive *f* through both of them. The number of axes will then be even, so that *d* and *f* will turn in opposite directions.

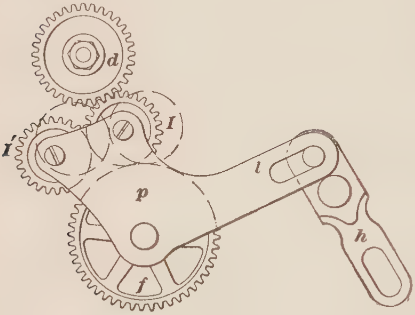

Fig. 6

ENGINE-LATHE GEAR TRAINS

THE BACK-GEAR TRAIN

6. Some of the best examples of gears in trains are to be found in engine lathes. Fig. 7 shows the headstock of an engine lathe, the spindle *S* turning in bearings, as shown, and having a face plate *p* and a center on the right end for placing the work. The lead screw *l*, used in screw cutting, is connected with the spindle by the train of gears on the left, which will be described later. The **back gears** F_1 and D_2, on the shaft $m\,n$, have been drawn above the spindle for convenience of illustration, instead of back of it, where they are really placed.

It is important to keep the cutting speed within limits that the tool will safely stand. For turning work of different diameters and materials, therefore, the spindle must be driven at different speeds. This is accomplished by means of the cone pulley c driven by a similar pulley on the countershaft by means of a belt, and by means of the back gears.

The gear F_2 is fastened to the spindle S and always turns with it. The cone c, however, is loose on the spindle, but can be made to turn the latter by means of a lug, or catch, operated by a nut under the rim of F_2. When the nut is moved out from the center, the lug engages with a slot in the large end of the cone. The cone will then turn with the spindle, and as many changes of speed may be had as there are pulleys on the cone. As ordinarily constructed, however, the cone alone does not give a range of speed great enough to include all classes of work, nor is the belt power sufficient for the larger work and heavier cuts. It makes the lathe more compact and satisfactory to construct the cone for the higher

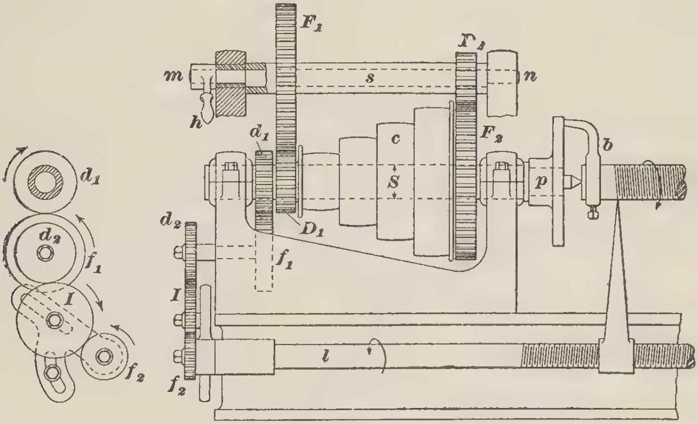

Fig. 7

speeds and lighter work, and to obtain the speeds for the heavier work by means of back gears. Referring to the figure, it will be seen that the back gears are connected by the sleeve *s*, and so form one rigid piece. The gear F_2 meshes with D_2, and D_1, which is loose on the spindle but fastened to the cone, meshes with F_1. To get the slower speeds, the nut mentioned before is moved in toward the center of F_2, disengaging the gear from the cone, which is now free to turn on the spindle. Hence, if the back gears are in mesh with the gears on the spindle, the belt will drive the spindle at a slower speed through the cone and the train D_1, F_1, D_2, F_2.

The back gears cannot remain in gear when the cone and gear F_2 are connected; if they do, the lathe will not start, or teeth will be broken out of the wheels. To provide for throwing them in and out of gear, as required, the rod on which are the back gears and sleeve *s* is provided with eccentric ends at *m* and *n*, fitting in bearings in the frame. By turning the rod part way round by means of the handle *h*, the gears can be thrown either in or out of gear.

A lathe is spoken of as running **back-geared** when the back gears are in, and as being in single gear when they are out of gear. This arrangement, or a modification of it, is used on upright drills, boring mills, milling machines, and other machine tools.

EXAMPLE.—The four steps of the cone pulley of a lathe, as in Fig. 7, have diameters of 15, 12, 9, and 6 inches, respectively, and the cone on the countershaft is the same as that on the lathe. Suppose the gears D_1, F_1, D_2, and F_2 to have, respectively, 30, 108, 24, and 84 teeth. The countershaft makes 105 revolutions per minute. Determine the eight speeds of the spindle that may be obtained.

SOLUTION.—With the back gear thrown out, the following speeds are obtained:

1. With the belt on the first pair of steps, the spindle speed $= \frac{15}{6} \times 105 = 262\frac{1}{2}$ R. P. M.

2. With the belt on the second pair, the speed $= \frac{12}{9} \times 105 = 140$ R. P. M.

3. With belt on third pair, the speed $= \frac{9}{12} \times 105 = 78\frac{3}{4}$ R. P. M.

4. With belt on fourth pair, the speed $= \frac{6}{15} \times 105 = 42$ R. P. M.

The velocity ratio of the train is $\frac{30 \times 24}{108 \times 84} = \frac{5}{63}$. Hence, with the back gear thrown in, the speeds for the various pairs of steps will be those given above multiplied by $\frac{5}{63}$. Thus,

5. $262\frac{1}{2} \times \frac{5}{63} = 20\frac{5}{6}$ R. P. M.
6. $140 \times \frac{5}{63} = 11\frac{1}{9}$ R. P. M.
7. $78\frac{3}{4} \times \frac{5}{63} = 6\frac{1}{4}$ R. P. M.
8. $42 \times \frac{5}{63} = 3\frac{1}{3}$ R. P. M.

THE SCREW-CUTTING TRAIN

7. Screw Cutting.—The engine lathe is frequently used for screw cutting. Referring to Fig. 7, the screw-cutting mechanism is driven from the gear d_1, which is fastened to the spindle; this connects with the lead screw l, through the gears f_1, d_2, the idler I, and the gear f_2. To cut a screw thread, the work is placed between the centers of the lathe and made to turn with the face plate by the dog b clamped to the end of the work. The spindle runs toward the operator, or clockwise, as looked at from the outer end of the headstock, and the carriage, tool post, and tool, all of which are here represented by the pointer, are moved by the lead screw along the lathe bed parallel to the axis of the spindle.

In cutting a right-hand thread, the tool must move from right to left, and from left to right in cutting a left-hand thread.

Suppose that the lead screw has a left-hand thread, as shown in Fig. 7; then, to move the tool from right to left, the lead screw must turn counter-clockwise, regarded from the outer end of the headstock; to move the tool from left to right, it must turn clockwise. Hence, to cut a right-hand thread, the spindle and lead screw must turn in opposite directions and the number of axes in the train connecting them must be even; while to cut a left-hand screw, the spindle and lead screw turn in the same direction and the number of axes in the train must be odd. If the lead screw has a right-hand thread, these movements must be reversed.

In Fig. 7, the lead screw has a left-hand thread, and there are four axes in the train. To cut a left-hand screw, another axis must be added and with this form of lathe provision is made for a second fixed stud, on which is placed a second idler between wheels d_2 and I.

An arrangement frequently adopted is shown in Fig. 8. The driving gear d_1 on the spindle and the follower f_1 on the stud do not mesh directly together, but the idlers I_1 and I

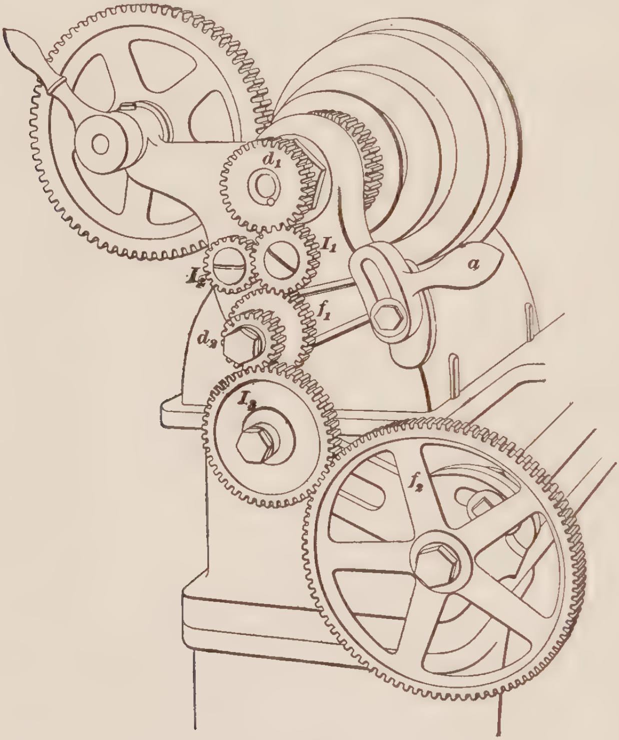

Fig. 8

are interposed as explained in connection with Fig. 6. These two gears are always in mesh, and rotate on studs carried by an arm of the lever a, which is pivoted on the same stud as d_2 and f_1. Otherwise, the train is the same as that of Fig. 7. When d_1 drives f_1 through the single idler I_1 there are five axes in the train, but when the lever a is moved downwards d_1 drives I_2, I_2 drives I_1, and I_1 drives f_1;

there are thus six axes in the train. Hence, by merely shifting the lever a, the number of axes may be made odd or even as desired. A right-hand lead screw is generally used with this train; hence, to cut a right-hand thread, I_2 is thrown out of gear, and to cut a left-hand thread it is thrown in gear. With a left-hand lead screw, the reverse would be true.

8. Gears for Cutting Thread of Given Pitch.—The pitch of the thread cut depends on two things, namely, the pitch of the lead screw and the velocity ratio of the train connecting the lead screw and the spindle.

Let s = number of threads per inch on the lead screw;
t = number of threads per inch to be cut.

Then, while the carriage moves 1 inch, the spindle must make t turns and the lead screw s turns; hence, for a given travel of the carriage,

$$\frac{\text{the turns of spindle}}{\text{the turns of lead screw}} = \frac{t}{s}$$

Since the spindle and lead screw are, respectively, the driving and driven shafts, and following the rule of Art. 3,

$$\frac{t}{s} = \frac{\text{product of numbers of teeth of all followers}}{\text{product of numbers of teeth of all drivers}}$$

With the train of Fig. 7 or that of Fig. 8, therefore,

$$\frac{t}{s} = \frac{f_1 \times f_2}{d_1 \times d_2}$$

When the lathe has a stud, as in Fig. 7, gears d_1 and f_1 remain unchanged, and the different pitches required are obtained by different combinations of the gears d_2 and f_2. The constant ratio $\frac{f_1}{d_1}$ may be denoted by K; then

$$\frac{t}{s} = K\frac{f_2}{d_2} \qquad (1)$$

or

$$f_2 = \frac{t}{Ks} d_2 \qquad (2)$$

EXAMPLE.—If the lead screw, Fig. 7, has six threads per inch, d_1 has 30 teeth, and f_1 60 teeth, find proper gears d_2 and f_2 to cut four, five, six, seven, eight, nine, and ten threads per inch.

SOLUTION.— $K = \frac{f_1}{d_1} = \frac{60}{30} = 2$; $s = 6$. Then, using formula **2**,

For four threads, $f_2 = \frac{4}{2 \times 6} d_2 = \frac{4}{12} d_2$

For five threads, $f_2 = \frac{5}{12} d_2$

For six threads, $f_2 = \frac{6}{12} d_2$, etc.

On the stud, place a gear d_2 having a number of teeth that is some multiple of 12; say, 72 teeth; then $f_2 = \frac{t}{Ks} d_2 = \frac{t}{2 \times 6} \times 72 = 6t$. Hence, the number of teeth in f_2, the gear to be placed on the lead screw, is six times the number of threads per inch to be cut. To cut four threads, a 24-tooth gear is required on the screw; for five threads, a 30-tooth gear; for six threads a 36-tooth gear, etc.

A number of other arrangements are possible. Thus, with an 84-tooth gear on the stud, the gear on the lead screw must have $\frac{84}{2 \times 6} t = 7t$ teeth, and with a 96-tooth gear on the stud, $f_2 = \frac{96}{2 \times 6} t = 8t$. Ans.

In designing a set of gears for a lathe, the number of separate gear-wheels should be as small as can be found to cut the desired range of threads. The change gears should have involute teeth, so that the distances between centers may vary slightly without affecting the constancy of the velocity ratio.

9. Compound Gearing.—Lathes are frequently designed so that the screw-cutting train can be compounded. The single idler I_3, Fig. 8, is replaced by two gears d_3 and f_3, Fig. 9, which are fastened together and turn about the axis s. Gear f_3 is driven by d_2, and d_3 acts as the driver for f_2, the gear on the lead screw. The general formula for this arrangement is $\frac{t}{s} = \frac{f_1 \times f_3 \times f_2}{d_1 \times d_3 \times d_2}$, or

$$\frac{t}{s} = K \times \frac{f_3}{d_3} \times \frac{f_2}{d_2} \qquad (1)$$

The ratio $\frac{f_3}{d_3}$ is usually kept constant for several pitches, the gears f_2 and d_2 being changed. Denoting this ratio by K_1, $\frac{t}{s} = K K_1 \frac{f_2}{d_2}$, or

$$f_2 = \frac{t}{K K_1 s} d_2 \qquad (2)$$

The ratio $K = \frac{f_1}{d_1}$ is usually either 2 or 1, and the ratio $K_1 = \frac{f_3}{d_3}$ is usually either 4 or 2, or $\frac{1}{4}$ or $\frac{1}{2}$, according as f_3 or d_3 is the larger of the two gears.

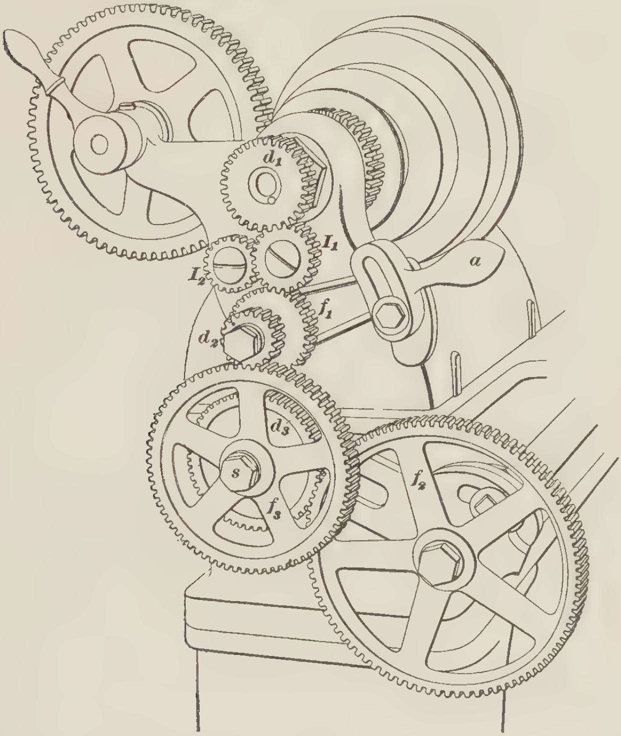

FIG. 9

The following example illustrates the application of these principles:

EXAMPLE.—In a lathe with compound gearing, gears d_1 and f_1 are equal, gear d_3 has 30 teeth, gear f_3 has 36 teeth, and the lead screw has five threads per inch. Find a set of change gears, the smallest to have 24 teeth and the largest 64 teeth, to cut from four to sixteen threads per inch, using as small a number of gears in the set as possible.

Solution.—From formula **1**, $\frac{t}{5} = 1 \times \frac{36}{30} \times \frac{f_2}{d_2}$, or $\frac{t}{5} = \frac{36}{30} \times \frac{f_2}{d_2}$. Hence, $f_2 = \frac{t}{5} \times \frac{30}{36} \times d_2 = \frac{t}{6} d_2$. That is, to cut four threads per inch, $f_2 = \frac{4}{6} d_2$; to cut five threads, $f_2 = \frac{5}{6} d_2$; to cut six threads, $f_2 = \frac{6}{6} d_2$, and so on for the remaining threads. Since d_2 is always multiplied by a fraction whose denominator is 6, let d_2 be chosen as some multiple of 6, say 48. Then,

For four threads, $f_2 = \frac{4}{6} \times 48 = 32$ teeth
For five threads, $f_2 = \frac{5}{6} \times 48 = 40$ teeth
For six threads, $f_2 = \frac{6}{6} \times 48 = 48$ teeth
For seven threads, $f_2 = \frac{7}{6} \times 48 = 56$ teeth
For eight threads, $f_2 = \frac{8}{6} \times 48 = 64$ teeth

It is evident that for nine threads, with a 48-tooth gear on the stud, there would need to be a 72-tooth gear on the lead screw. But the largest gear is to have only 64 teeth. Hence, the size of d_2 is changed to 24 teeth, and then,

For nine threads, $f_2 = \frac{9}{6} \times 24 = 36$ teeth
For ten threads, $f_2 = \frac{10}{6} \times 24 = 40$ teeth
For eleven threads, $f_2 = \frac{11}{6} \times 24 = 44$ teeth
For twelve threads, $f_2 = \frac{12}{6} \times 24 = 48$ teeth
For thirteen threads, $f_2 = \frac{13}{6} \times 24 = 52$ teeth
For fourteen threads, $f_2 = \frac{14}{6} \times 24 = 56$ teeth
For fifteen threads, $f_2 = \frac{15}{6} \times 24 = 60$ teeth
For sixteen threads, $f_2 = \frac{16}{6} \times 24 = 64$ teeth

Therefore, the change gears required are those having 24, 32, 36, 40, 44, 48, 52, 56, 60, and 64 teeth, respectively, or eleven gears in all, since there must be two 48-tooth gears in cutting six threads per inch.

EXAMPLES FOR PRACTICE

1. In a lathe geared as in Fig. 7, the wheels d_1, f_1, and d_2 have 18, 24, and 16 teeth, respectively, and the lead screw has six threads per inch; what must be the number of teeth in f_2 to cut eight threads per inch? Ans. 16 teeth

2. In a lathe like that in Fig. 7, f_2 has 20 teeth, d_2 15 teeth, and the ratio K has a value of $\frac{3}{2}$; how many threads per inch must the lead screw have to cut twelve threads per inch? Ans. 6 threads

3. In the lathe of Fig. 9, suppose that the values of K and K_1 are 2 and 1, respectively, and that d_2 has 24 teeth; find how many teeth f_2 must have, in order to cut ten threads per inch. assuming the lead screw to have six threads per inch. Ans. 20 teeth

EPICYCLIC TRAINS

10. In the trains discussed thus far, all the wheels turn on fixed axes, that is, axes forming a part of the frame or fixed link of the machine. There are trains, however, in which a wheel (or several wheels) turns on an axis that itself moves relative to the frame. In particular, there are cases in which the axis revolves about another fixed axis; a train containing such a combination is called an **epicyclic train.**

11. Principle of Combined Rotations.—In Fig. 10, a wheel w is pinned at B to an arm a that rotates about the fixed point A. The wheel w turns about the pin B, which moves in the circular path s. With the arm and wheel in the first position, a line BC of the wheel coincides with AB, the center line of the arm. Suppose that the arm moves to a new position AB'; then, if the wheel were rigidly fastened to the arm, the line BC would be at $B'C'$ in the line AB'. If, however, the wheel has in the meantime turned on the pin B, BC will have taken some new position $B'C''$. Let $B'C''$ be prolonged to meet BC in E; then the angle $BAB' = m_1$ = angle through which the arm has turned; angle $C'B'C'' = m_2$ = angle through which the wheel has turned on pin B, that is, relative to the arm; angle $C''EC = m_3$ = angle through which the wheel has turned from its original position.

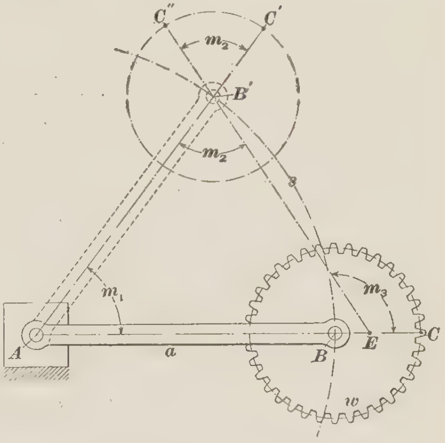

FIG. 10

The angle m_3 is an exterior angle of the triangle AEB', and it can be proved by geometry that angle m_3 = angle m_1

+ angle m_2. That is, the angle through which the wheel has turned relative to the fixed frame, to which the arm *a* is attached at *A*, is equal to the sum of the angle through which the arm has turned and the angle through which the wheel has turned relative to the arm.

It is necessary to pay attention to the direction of the rotation; the above rule holds good only when the arm and wheel turn in the same direction. If, however, a counter-clockwise rotation is considered as + and a clockwise rotation as −, and the angles are given their proper signs, the rule holds good in all cases. The following examples illustrate this principle:

1. The arm turns through 40° counter-clockwise, and the wheel turns on its pin through 60°, also counter-clockwise. Then $m_3 = 40° + 60° = 100°$, the angle through which the wheel has turned in a fixed plane.

2. The arm turns 110° counter-clockwise, and the wheel turns on the pin 45° clockwise. Then, the turning of the wheel in a fixed plane is $m_3 = +110° - 45° = +65°$.

3. While the arm makes a complete turn (i. e., $m_1 = 360°$), the wheel makes three and one-half complete turns relative to the arm, both turns being counter-clockwise. Then, relative to the fixed link, the wheel makes $+1 + 3\frac{1}{2} = +4\frac{1}{2}$ turns.

4. The arm makes two turns clockwise, and at the same time the wheel makes seven turns counter-clockwise relative to the arms. Relative to the fixed link, then, the wheel makes $N = -2 + 7 = +5$ turns, that is, five turns counter-clockwise.

5. The arm makes one turn, and the wheel is fixed to the arm so that it cannot turn on the pin *B*. In this case the wheel makes one revolution relative to the fixed plane, though it does not turn on its axis. Referring to Fig. 10, if the wheel is fixed to the arm, $m_2 = 0$ and $m_3 = m_1$; that is, the wheel turns through the same angle as the arm.

6. The arm makes one turn counter-clockwise, and at the same time the wheel makes one turn clockwise relative to the arm. Relative to the fixed plane, the wheel makes

$+1-1=0$ turns; that is, it has no motion of rotation, but has only a circular translation.

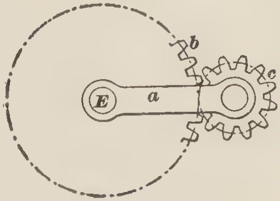

FIG. 11

12. Train of Two Wheels. In Fig. 11, an arm *a* joins the two gears *b* and *c* and holds them in mesh. Suppose that gear *b* is fixed and the arm *a* is turned counter-clockwise about the axis *E*. Then the gear *c* will turn on its axis; that is, relative to the arm, and the case is similar to that shown in Fig. 10. According to the principle stated in Art. **11**,

turns of wheel c = turns of arm a + turns of c relative to a

If the arm is fixed and the wheel *b* is given a complete turn, the wheel *c* will make $\frac{b}{c}$ turns, where *b* and *c* denote the number of teeth, the radii, or the diameters of the wheels *b* and *c*. It follows that when *b* is held fixed and the arm makes one turn, the wheel *c* makes $\frac{b}{c}$ turns relative to the arm, just as it did when the arm was fixed. If the arm turns counter-clockwise, *c* turns counter-clockwise on its axis; hence, the total number of turns of *c* relative to the fixed plane is $1+\frac{b}{c}$.

A convenient way of arriving at this result is the following: First, assume the arm to be held fixed and give the wheel *b* one negative turn, that is, a clockwise turn. As a result, the wheel *c* is given $\frac{b}{c}$ positive turns. Second, assume the wheels and arm to be locked together so as to form a rigid body and give the whole system one positive turn; the first step gives wheel *b* one negative turn, the second a positive turn, and as a result wheel *b* is brought back to its first position. As regards its arm, the result of the two steps is to give it one positive turn; and, as regards the wheel *c*, it is given $+\frac{b}{c}$ turns by the first, and + one turn by the second step, or $1 + \frac{b}{c}$ turns as a whole. It appears, therefore, that these separate motions give the same result as the single counter-clockwise turn of arm *a* with *b* fixed. The results may be tabulated as follows:

	Wheel *b*	Arm *a*	Wheel *c*
Arm fixed	-1	0	$+\frac{b}{c}$
Wheels locked	$+1$	$+1$	$+1$
Result	0 (fixed)	$+1$	$1 + \frac{b}{c}$

The algebraic sums in the third line give the total motion of each link, and the result may be interpreted thus: With wheel *b* fixed, one turn of arm *a* causes $1 + \frac{b}{c}$ turns of wheel *c*.

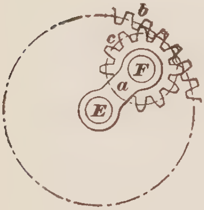

Fig. 12

13. Annular Two-Wheel Train. Suppose that the fixed gear is annular, as shown in Fig. 12. In this case, if the arm is fixed, a clockwise rotation of *b* about its axis *E* will cause $\frac{b}{c}$ clockwise turns of *c* about its axis *F*. Applying the method of successive motions,

	Wheel *b*	Arm *a*	Wheel *c*
Arm fixed	-1	0	$-\frac{b}{c}$
Wheels locked . . .	$+1$	$+1$	$+1$
Result	0 (fixed)	$+1$	$1 - \frac{b}{c}$

That is, with the annular wheel *b* fixed, one counter-clockwise turn of the arm causes $1 - \frac{b}{c}$ clockwise turns of wheel *c*. By the same method, it may be found that one clockwise turn of the arm causes $\frac{b}{c} - 1$ counter-clockwise turns of the wheel *c*.

EXAMPLE 1.—In Fig. 11, suppose that *b* has 60 teeth and *c* 24 teeth. If the arm *a* makes 1 counter-clockwise turn, how many turns does *c* make, and in what direction?

SOLUTION.—

	b	ARM *a*	*c*
Arm fixed	-1	0	$+\frac{60}{24}$
Wheels locked . . .	$+1$	$+1$	$+1$
	0	$+1$	$+1+\frac{60}{24}\ (=3\frac{1}{2})$

Wheel *c* makes $+3\frac{1}{2}$ turns; that is, $3\frac{1}{2}$ counter-clockwise turns. Ans.

EXAMPLE 2.—In Fig. 12, let *b* and *c* have 60 and 24 teeth, respectively; if the arm is given 1 clockwise turn, how many turns will *c* make?

SOLUTION.—

	b	ARM *a*	*c*
Arm fixed	$+1$	0	$+\frac{60}{24}$
Wheels locked . . .	-1	-1	-1
	0	1	$\frac{60}{24}-1\ (=1.5)$

Wheel *c* makes $+1\frac{1}{2}$ turns. Ans.

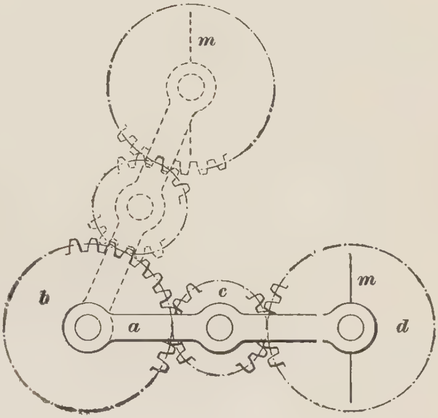

FIG. 13

14. Higher Epicyclic Trains.—In Fig. 13, the arm *a* carries an idler *c* and the wheel *d*. By the addition of the idler, the wheel *d* is caused to turn in a direction opposite to that of the arm, as in the annular wheel train, Fig. 12. Applying the usual method of analysis,

	b	ARM *a*	*c*	*d*
Arm fixed . . .	-1	0	$+\frac{b}{c}$	$-\frac{b}{c}\times\frac{c}{d}\left(=-\frac{b}{d}\right)$
Wheels locked .	$+1$	$+1$	$+1$	$+1$
Result	0	$+1$	$1+\frac{b}{c}$	$1-\frac{b}{d}$

With the arm fixed, one negative turn of *b* gives $\frac{b}{c}$ positive turns of the idler c, and for each turn of c, wheel d makes $\frac{c}{d}$ turns in the opposite direction, that is, clockwise; hence, while b makes one turn and c makes $\frac{b}{c}$ turns, d must make $\frac{b}{c} \times \frac{c}{d} = \frac{b}{d}$ turns. The idler, therefore, has no effect on the number of turns of wheel d.

An interesting result is obtained when b and d are of the same size. In this case, $1 - \frac{b}{d} = 1 - 1 = 0$, which means that wheel d does not rotate at all but has merely a motion of translation. Thus, as shown in the figure, a vertical line m remains vertical for all positions of the wheel.

15. In Fig. 14, the arm a carries the two wheels c and d, which are fixed to the same shaft, and the wheel e. Wheels b and c mesh together, as do wheels d and e. With the arm fixed, let wheel b be given a clockwise turn. Then wheel c, and wheel d which is fast to it, must make $\frac{b}{c}$ counter-clockwise turns. For each turn of d, e makes $\frac{d}{e}$ turns; hence, while d makes $\frac{b}{c}$ turns, e makes $\frac{b}{c} \times \frac{d}{e}$ turns. The motions when b is fixed and the arm is turned are:

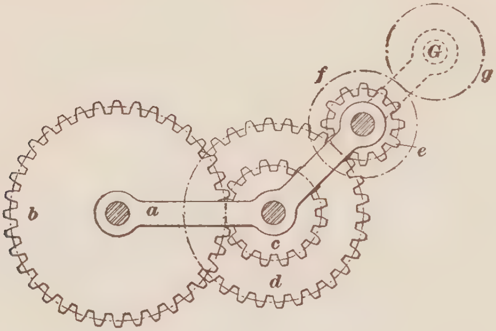

FIG. 14

	b	ARM a	c AND d	e
Arm fixed . . .	-1	0	$+\frac{b}{c}$	$-\left(\frac{b}{c}\times\frac{d}{e}\right)$
Wheels locked .	$+1$	$+1$	$+1$	$+1$
Result	0	$+1$	$1+\frac{b}{c}$	$1-\frac{b}{c}\times\frac{d}{e}$

The same analysis may be applied to a train with any number of wheels. Thus, suppose that a second wheel f is fixed to wheel e, Fig. 14, and that this gears with a wheel g that turns about a third moving axis G. For this arrangement, the analysis gives,

	b	ARM a	c AND d	e AND f	g
Arm fixed	-1	0	$+\frac{b}{c}$	$-\frac{b}{c}\times\frac{d}{c}$	$+\frac{b}{c}\times\frac{d}{e}\times\frac{f}{g}$
Wheels locked	$+1$	$+1$	$+1$	$+1$	$+1$
Result	0	$+1$	$+\left(1+\frac{b}{c}\right)$	$1-\frac{b}{c}\times\frac{d}{e}$	$1+\frac{b}{c}\times\frac{a}{e}\times\frac{f}{g}$

16. Reverted Trains.—In Fig. 15 is shown a train in which the last wheel e turns on the stationary axis of the fixed wheel b, and is driven through the gears c and d, of which c engages with b and d with e. Such a train is called a **reverted train.** The analysis is as follows: Keeping the arm a fixed, a clockwise rotation of b causes $\frac{b}{c}$ counter-clockwise turns of c and d (c and d are fixed together), and for each turn of d, e makes $\frac{d}{e}$ clockwise turns, or $\frac{b}{c}\times\frac{d}{e}$ turns for one turn of b. Hence,

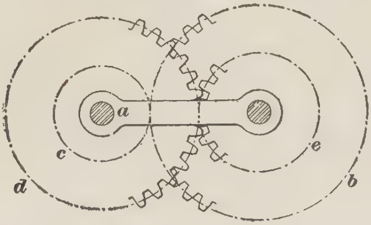

FIG. 15

	ARM a	b	c AND d	e
Arm fixed . . .	0	-1	$+\frac{b}{c}$	$-\frac{b}{c}\times\frac{d}{e}$
Wheels locked .	$+1$	$+1$	$+1$	$+1$
Result . . .	$+1$	0	$1+\frac{b}{c}$	$1-\frac{b}{c}\times\frac{d}{e}$

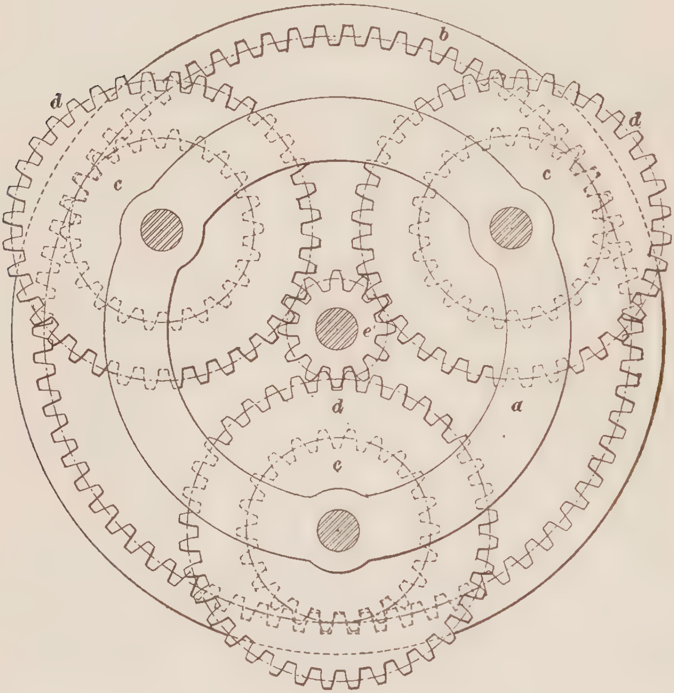

FIG. 16

That is, for each positive turn of the arm, wheel e makes $1-\frac{b}{c}\times\frac{d}{e}$ turns. If 1 is greater than $\frac{b}{c}\times\frac{d}{e}$, e turns in the same direction as the arm, but if 1 is less than $\frac{b}{c}\times\frac{d}{e}$, it turns in the opposite direction. This reverted train may be used to give a slow motion to the wheel e. Suppose that the wheels have the following numbers of teeth:

b, 41; *c*, 39; *d*, 40; *e*, 40. Then $1 - \frac{b}{c} \times \frac{d}{e} = 1 - \frac{41 \times 40}{39 \times 40} = 1 - \frac{41}{39} = -\frac{2}{39}$. That is, 39 turns of the arm *a* are required to give 2 turns of the wheel *e*.

17. Annular Reverted Train.—In Fig. 16 is shown the application of an annular reverted train. The annular wheel *b* forms part of the fixed frame of the machine. A frame *a* has pinned to it three pair of gears, *c d*, *c d*, *c d*, of which *c*, *c*, *c* engage the fixed annular wheel *b*, and *d*, *d*, *d* engage with the small pinion *e*, which is the driver of the train. Several revolutions of the pinion *e* are required to produce one turn of the frame *a*, and the train thus gives the desired mechanical advantage.

EXAMPLE.—In Fig. 16, let *b* have 42; *c*, 11; *d*, 29; and *e*, 12 teeth. What is the velocity ratio, that is, how many turns of the driving shaft are required for 1 turn of the frame *a*?

SOLUTION.—Applying the usual analysis,

	FRAME *a*	*b*	*c* AND *d*	*e*
Frame *a* fixed . .	0	-1	$-\frac{b}{c} = -\frac{42}{11}$	$+\frac{b}{c} \times \frac{d}{e} = \frac{42}{11} \times \frac{29}{12}$
Wheels locked . .	$+1$	$+1$	$+1$	$+1$
Result	1	0	$1 - \frac{42}{11}$	$1 + \frac{42}{11} \times \frac{29}{12}$

Velocity ratio $= 1 + \frac{42}{11} \times \frac{29}{12} = \frac{225}{22} = 10.23$; that is, 10.23 turns of the shaft are required for 1 turn of *a*. Ans.

EXAMPLES FOR PRACTICE

1. In Fig. 12, suppose that *b* has 60 teeth and *c* 12 teeth. If the arm *a* makes 5 clockwise turns, how many turns does *c* make, and in what direction? Ans. 20 turns, counter-clockwise

2. Let *b* and *d*, Fig. 13, each have 30 teeth. If *a* makes 2 clockwise revolutions, and *b* 1 counter-clockwise turn, how many turns will *d* make, and in what direction? Ans. 1 turn, counter-clockwise

3. Suppose that the gears *b*, *c*, *d*, and *e*, Fig. 14, have 80, 40, 50, and 20 teeth, respectively. If *a* makes 50 turns counter-clockwise, how many turns does *e* make, and in what direction?
Ans. 200 turns, clockwise

4. In Fig. 15, let *b*, *c*, *d*, and *e* have 45, 15, 35 and 25 teeth, respectively. If *a* turns 30 times clockwise, how many turns does *e* make, and in what direction? Ans. 96 turns, counter-clockwise

REVOLVING GEAR TRAINS

18. Differential Back Gears.—Upright drills for metal work are sometimes provided with arrangements for increasing the range of the speeds and driving power that are different from the back gears explained in connection with the engine-lathe train. Fig. 17 shows one arrangement for this purpose. The cone pulley *c* is loose on the shaft *s*. A casting *d*, also loose on the shaft, has teeth on the inside, thus forming an annular gear. A plate *p* carrying the small gear, or pinion, *I* is fast to the shaft. On the left-hand end of the pulley hub is another gear *f*, which is fast to the hub of the cone. The action is as follows: A pin, on which there is a collar and nut, is clamped in the slot *l* in the plate *d*. The pin projects through *d*, so that when it is placed in the inner end of the slot it will engage with a corresponding slot in the plate *p*. When it is lowered, however, the pin disengages from the plate, but the collar can be made to fall in a slot in the arm *a*, which is a part of the frame. With the pin in the former position, gears *d*, *I*, and *f* are locked together, so that the shaft must turn with the cone. With the pin in the latter position, gear *d* is locked to the frame and cannot turn while gear *f* still turns with the cone. Therefore, the plate *p* is driven by the train consisting of the wheels *f*, *I*, and *d*.

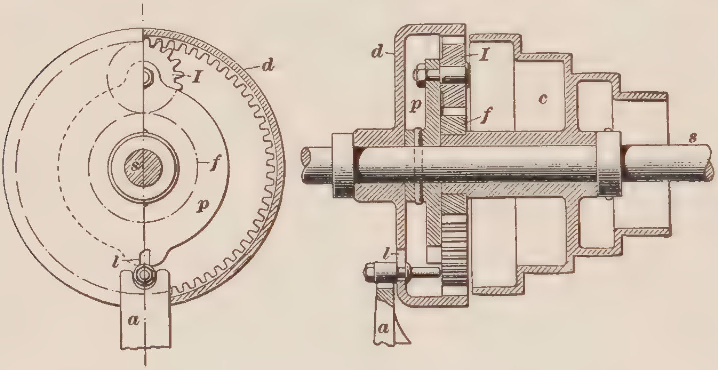

FIG. 17

Consequently, *p* and the shaft *s* attached to it turn more slowly than the cone.

19. Automobile Gearing.—In Fig. 18 is shown a revolving bevel-gear train as applied to automobile construction. The engine shaft *s* carries the bevel gear *b* and the flywheel *w*, both of which are keyed to it. The second bevel wheel *d* can turn on shaft *s* and carries two sheaves, *f* and *g*. By means of a band brake *k*, the wheel *d* may be held stationary; or by means of a clutch *h*, the sheave *g* may be fastened to wheel *w* so that *d* is fast to the shaft *s*. A yoke *m* attached to the sprocket wheel *n* turns freely on the shaft *s* and carries the intermediate bevel gears *c* and *e*. The shaft runs in bearings in the fixed frame *a*. The following combinations can be obtained:

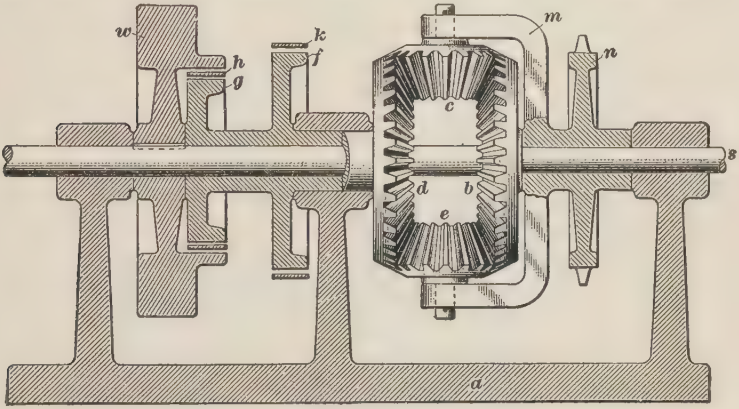

FIG. 18

1. Neither brake *k* nor clutch *h* is applied. The engine drives shaft *s* and wheel *b*, wheels *c* and *e* act as idlers, and *d* turns with the same speed as *b*, but in the opposite direction. No motion is imparted to the sprocket wheel *n*.

2. The brake *k* is applied, thus holding wheel *d* fixed. In this case, the yoke *m* turns one-half as fast as the wheel *b* driven by the shaft; that is, two revolutions of the shaft are required for one revolution of the sprocket wheel. With this combination, the automobile is driven at low speed.

3. The brake *k* is released and the clutch *g* is applied. The wheel *d* is thus locked to the shaft, and therefore *d*, *b*, yoke *m*, and sprocket wheel *n* all turn as one rigid piece keyed to the shaft. Each turn of the shaft gives a revolution of the sprocket wheel *n*, and the automobile is driven at high speed.

4. With the clutch *h* in gear, the brake *k* is applied, thus locking the wheel *w* with the frame *a*; this stops the automobile.

REVERSING MECHANISMS

20. Clutch Gearing.—A reversing mechanism often used, especially when the reversal must take place automatically, is shown in Fig. 19. It consists of three bevel gears, of which the driver *d* is fast on the driving shaft *s*. The gears f_1 and f_2 are continually in contact with *d*, and are loose on shaft *m n*. A sleeve *c* can move endwise on *m n*, but is compelled to turn with it by a key *a* in the shaft. When in the position shown, the notches in *c* engage with f_1, and *m n* turns; when the sleeve is in mid-position, *m n* will not turn; and when thrown to the right, it engages with f_2 and the direction of rotation is reversed.

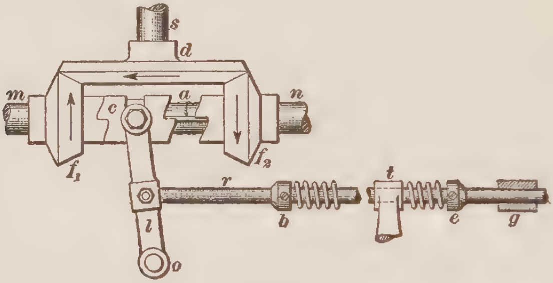

FIG. 19

If the reversal is to be automatic, some provision must be made to insure that after sleeve *c* has been pushed out of contact on one side, it will be thrown in contact with the other gear. One way of doing this is shown in the figure. The lever *l* is pivoted at *o*, and at the other end is forked to embrace *c*, a roller on each prong running in the groove shown. On the rod *r*, which is pivoted to *l* at one end and slides in a guide *g* at the other end, are two collars *b* and *e*, held in place by setscrews. Helical springs are also placed on *r* against the inside of the collars. Suppose that the tappet *t*, which is free to slide on *r*, is given a motion to the right through mechanism connected with f_1, but not shown in the figure. When it reaches the spring, it will compress it until the pressure of the spring on *e* is sufficient to overcome the resistance of the clutch. Further movement of *t* will move *c* to the right until free of f_1, when the energy stored in the spring will suddenly throw *c* in contact with f_2. The time at which reversal occurs can be adjusted by changing the positions of the collars *b* and *e*.

21. Quick-Return Clutch Gear.—Sometimes it is desirable to have a slow motion in one direction with a quick return. Fig. 20 shows a method that may be used for this purpose. The driver d_1 is made cup-shaped so as to allow a smaller driver d_2 to be placed inside. For the slow motion, d_2 drives f_2, and, for the quick return, d_1 drives f_1. The change of direction is effected by shifting the clutch on the shaft so as to engage alternately with f_1 and f_2.

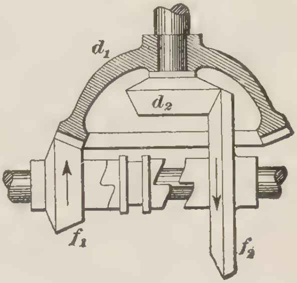

FIG. 20

22. Reversing by Belts.—In many machines, especially those running at high speed, a reversal of the motion is effected by belts rather than gears. Two belts are generally employed—one open and one crossed. Sometimes, these belts are made to shift alternately from a tight to a loose pulley, and in other cases they are arranged to drive two pulleys on the same shaft in opposite directions, either of the pulleys being thrown in or out by means of a friction clutch, as, for example, in lathe countershafts.

23. Planer Reversing Mechanism.—Of the first class, just mentioned, Fig. 21 affords an illustration as applied to a planer operating on metal. The table h is driven forwards and backwards by the rack and gearing shown. The work to be planed is clamped to the table h, and during the forward, or cutting, stroke a stationary tool removes the metal. As no cutting takes place on the return stroke, the table is made to run back from two to five times as fast as when moving forwards. There are three pulleys on shaft n, of which t is the tight or driving pulley, l_1 and l_2, shown in the top view, being loose on the shaft. There are two belt shifters g and k, in the shape of bell-cranks, pivoted at a and b, respectively. A crossed belt, moving in the direction shown by the arrow, runs from a small pulley on the countershaft, and is guided by the shifter g. This belt drives the table during the cutting stroke. The other belt, which is open, runs over a large pulley on the countershaft, and is guided by the shifter k.

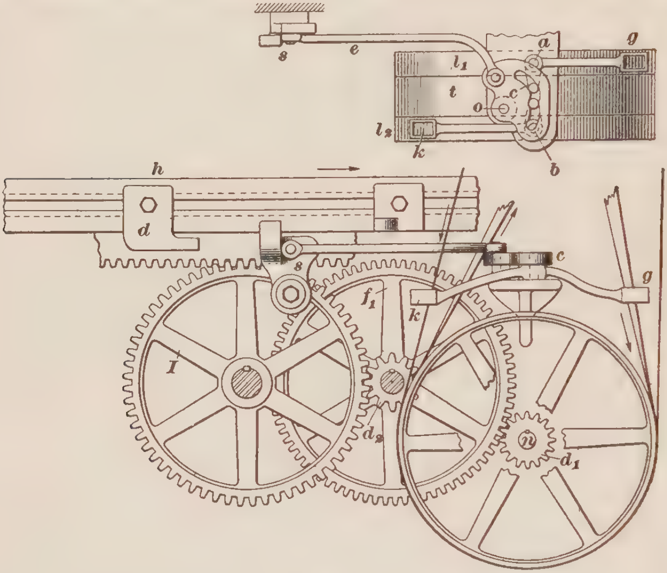

Fig. 21

The short arms of the shifters carry small rollers working in a slot in a cam-plate c, which is pivoted at o. The ends of this slot are concentric about o, but one end has a greater radius than the other. As shown in the figure, the two shifters are in mid-position and the belts are on the loose pulleys. Suppose, however, the rod *e* to be pulled to the left. Shifter *k* will not move, because its roller will continue to be in the same end of the slot, which is concentric about *o*; but the roller on *g* will be pulled to the left by passing from the end of the slot that is of larger radius to the end of the smaller radius. The crossed belt will, therefore, be shifted to the tight pulley, which will cause the table to run forwards until the dog *d* strikes the rocker *s*, thus throwing the cam in the other direction. The effect of this will be to first shift the crossed belt from the tight to the loose pulley, and then to shift the open belt to the tight pulley; one motion follows the other, so as to decrease the wear and tear of the belts. The length of the stroke of the table can be varied by changing the position of the dogs, which are bolted to a **T** slot on the edge of the table.

The table is driven as follows: The belt pulleys drive gear d_1, which is keyed to the same shaft; d_1 drives f_1, which in turn drives d_2, keyed to the same shaft; d_2 drives gear I, which in turn drives the table by means of a rack underneath it. The circumferential speeds of I and d_2 are evidently the same, and the speed of the table is the same as the circumferential speed of the gear d_2. The velocity ratio of the gearing is generally expressed in terms of the number of feet traveled by the belt to 1 foot passed over by the table.

Suppose the belt pulleys to be 24 inches in diameter, and to make N revolutions while the table travels 1 foot. Let the diameter of d_1 be 3 inches; of f_1, 26 inches, and of d_2, 4 inches. The circumference of d_2 is $12\frac{1}{2}$ inches, nearly, so that for every foot traveled by the table, d_2 and f_1 will turn $\frac{12}{12\frac{1}{2}} = \frac{24}{25}$ times. The number of turns made by the belt pulleys for each foot passed over by the table is, therefore, $N \times 3 = \frac{24}{25} \times 26$, or $N = \frac{24 \times 26}{75} = 8.32$. Hence, for 1 foot of travel of the table, the belt travel is $8.32 \times \frac{24}{12} \times 3.1416 = 52\frac{1}{4}$ feet. That is, the planer is geared to run $52\frac{1}{4}$ to 1.

CAMS AND CAM TRAINS

24. Cams.—A **cam** may be defined as a machine part so shaped that by its motion it imparts a definite motion to some other part with which it is in direct contact. The cam is the driving part and is used to impart a motion that could not easily be obtained by other means, such as by link work or gear trains. Cams have the disadvantage of small contact surface and correspondingly rapid wear, and as a general rule are to be avoided if other devices can be obtained that will do the work as well.

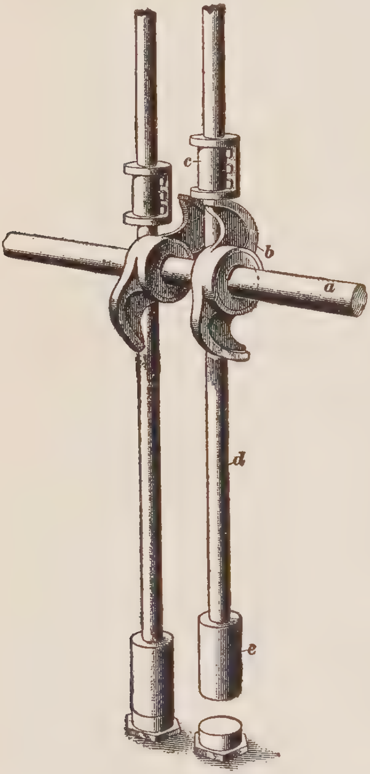

Fig. 22

A **cam train** is usually a mechanism of three links, the fixed link, the cam, and the follower. An example is seen in the stamp mill for ore crushing, as shown in Fig. 22. The rotating shaft *a* carries the cam *b*, which acts on the piece *c*. This piece is attached to the rod *d* that carries the stamp *e*. The rod is guided by a framework, not shown in the figure. It will be seen that the train has three links—the fixed framework, the shaft and attached cam, and the stamp.

ROTARY CAMS

25. Fig. 23 represents a common application of the **plate cam.** The cam *c* is supposed to turn clockwise, as indicated by the arrow, about the axis *b*, and to transmit a variable motion through the roller or follower *f* and the lever *l* to the rod *p*. The lever swings on the axis O_0 and the roller moves up or down on the arc *0–6*, as the cam revolves. The roller is held in contact with the cam by its own weight and that of the lever and the rod.

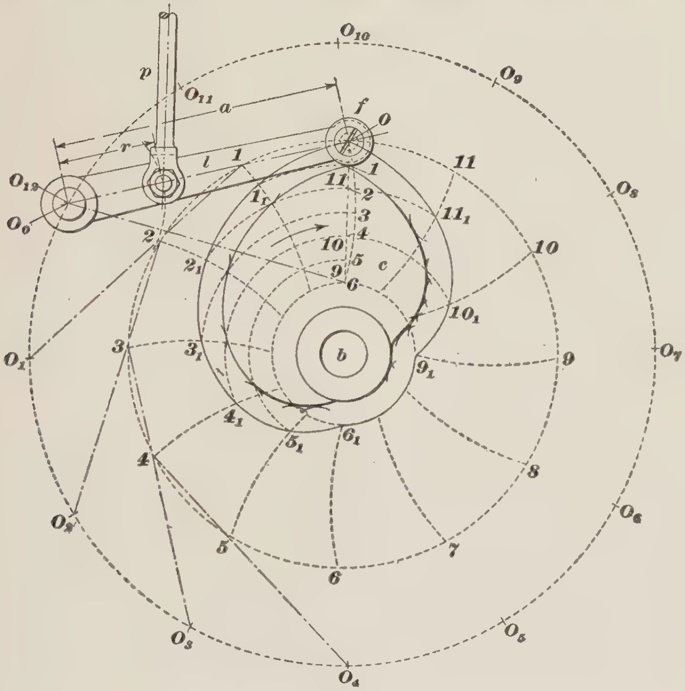

FIG. 23

Suppose that the locations of the cam shaft *b* and the rod *p* are known, and that the cam shaft is to rotate uniformly clockwise and impart motion to the rod, so that during the first half of every revolution it will move uniformly downwards, during the next quarter-turn remain stationary, and during the last quarter return to its former position with a uniform motion.

In order to determine the outline of the cam on which the circumference of the roller bears, it is necessary to find an outline that will give the center of the roller the required motion. Then, by placing the point of the compasses at different places on this outline, and striking arcs inside of it, with radii equal to the radius of the roller, the curve for the actual cam can be drawn, the curve being tangent to the arcs and parallel to the first outline.

In this case, the roller f moves in an arc directly over the center of b. Knowing the distance the rod f is to move, it is necessary to choose the point O_0 and the throw of the cam, that is, the distance that f is to move, so that the movement of p will be to the movement of f as r is to a.

Now, with O_0 as a center and a radius equal to a, describe the arc *0–3–6*, in which the center of the roller is to move, and mark the highest and lowest points *0* and *6* of the roller. The lower point should not be near enough the shaft to allow the roller to strike the hub of the cam.

It evidently makes no difference with the relative motions of the cam and roller whether the cam turns clockwise, and the lever remains with its axis at O_0, or whether the cam is assumed to be stationary, and the lever and roller to move counter-clockwise in a circle about the center b. This latter assumption will be adopted in drawing cams.

With b as the center, draw a circle through O_0 and space it into a number of equal parts, say twelve, and number the divisions around to the left. Now, assume the lever l to move around the axis b in a counter-clockwise direction. It will take positions O_1–*1*, O_2–*2*, O_3–*3*, etc. Hence, using these several points on the outer circle as centers, and with radii equal to a, the length of the lever, describe a series of arcs corresponding to the original arc *0–3–6*. Number these arcs *1*, *2*, *3*, etc., to correspond with the numbers on the outer circle. During the first half-turn of the cam, or, what is the same thing, while the lever is moving from its position at O_0

to O_6 on the outer circle, the center of the roller must move uniformly from its outer to its inner position. Hence, draw the chord of the original arc *0–3–6*, and divide it into six equal parts, numbering them toward the center as shown. Then, with *b* as a center, describe an arc through point *1*, intersecting arc *1* in point 1_1. Now, sweep arcs through the other points, getting 2_1, 3_1, 4_1, etc., which are all points in the path of the center of the roller. From O_6 to O_9 on the original circle, the center of the roller remains at a constant distance from *b*; hence, 6_1 and 9_1 must be connected by a circular arc. From O_9 to O_{12}, the points are found as before by dividing the chord *0–6* into three equal parts and numbering them as shown, the numbers running outwards.

Finally, draw the path of the center of the roller through points 1_1, 2_1, 3_1, 4_1, etc.; then draw the outline for the cam itself parallel to it, as explained at first. This is easily done by setting the dividers to describe a circle whose radius shall be the same as that of the roller *f*. Next, with various points on the curve $0–1_1–2_1–3_1$ . . . 11_1 as centers (the more, the better), describe short arcs as shown. By aid of an irregular curve, draw a curve that will be tangent to the series of short arcs; it will be the required outline of the cam, and will be parallel to the curve $0–1_1–2_1–3_1$ . . . 11_1.

The question sometimes arises in designing cams of this nature, whether it is the chord *0–6* or the arc *0–6* that should be divided to give the roller the proper outward and inward motions. For all practical purposes either way is sufficiently exact, but neither is quite correct, though it is better to space the chord. The exact way would be to draw the rod *p* and the roller in the different positions desired, and then design the cam to meet the roller at these points.

26. Quick-Drop Cam.—The cam shown in Fig. 24 differs in principle from the preceding one only in that the roller moves in a straight line, passing to one side of the center *b* of the shaft. Let it be required to design a cam of this nature to turn clockwise, as indicated by the arrow, which will cause the roller *a* and rod *p* to rise with a uniform motion to a distance h during two-thirds of a revolution. When the roller reaches its highest point, it is to drop at once to its original position, and to remain there during the remainder of the revolution. Assume the distance from the center b to the center line of p to be equal to r. With r as a radius, describe a circle about b, as shown. The center line of the rod will be tangent to this circle in all positions. With the same center and a radius equal to bA_0, A_0 being the extreme outward position of the roller, describe the outside circle A_0–4–8–A_0. Divide this circle into some convenient number of equal parts, the number depending on the fraction of a revolution required for the different periods of motion. Since the roller is to rise during two-thirds of a revolution, it is well to use twelve divisions as before, thus giving $\frac{2}{3} \times 12 = 8$ whole divisions for the first period.

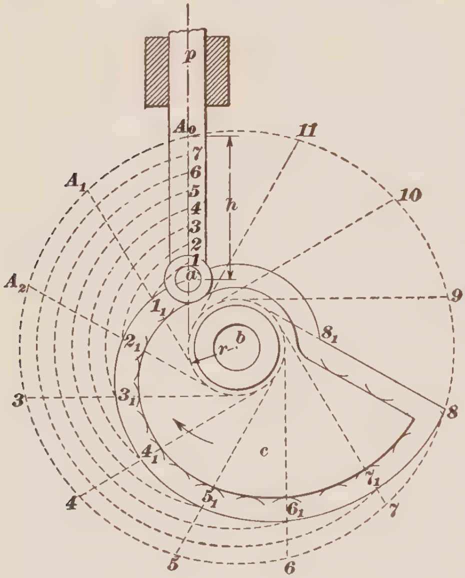

FIG. 24

Now, proceeding as before, by assuming the rod to move about the cam to the left, its positions when at the points of division A_1, A_2, etc., will be represented by drawing lines through these points, and tangent to the inner circle, whose radius is r. The points a and A_0 are the two extreme positions of the roller. Divide the line aA_0 into eight equal parts, numbering them from the inside outwards, since the first movement of the roller is outwards. With b as a center, draw concentric arcs through these points, intersecting the tangents at 1_1, 2_1, 3_1, etc. At point 8 on the outer circle, the roller drops along the line $8-8_1$, the point 8_1 being determined by drawing an arc about *b* with a radius *b a*. From point 8_1 back to *a*, the rod is at rest. The true cam outline is now to be found, as was done in the last example, by striking small arcs from points on the curve $a-1_1-2_1-3_1-4_1-5_1-6_1-7_1-8-8_1$ as centers, and radii equal to the radius of the cam-roller *a*. This cam can rotate clockwise only in driving the rod *p*.

27. Cam for Stamp Mill.—The cam shown in Fig. 25 is a modification of that in Fig. 24, and is a form that is generally used in stamp mills where ore is crushed. The axis *O* of the cam lies at a distance *r* from the axis *B C* of the stamp rod. To construct the cam outline, a circle *c* is first drawn about *O* as a center with a radius *r*. This circle is evidently tangent to *B C* at *G*. The vertical movement or stroke of the stamp rod is *A B*, *A* being the lowest and *B* the highest point of contact with the cam. This length of stroke *A B* is made equal to one-half the length of the circumference of the circle *c*, and the point *A* is located at a distance above *G* equal to one-fourth the circumference of *c*, so that *B* is at a distance from *G* equal to three-fourths the circumference of *c*.

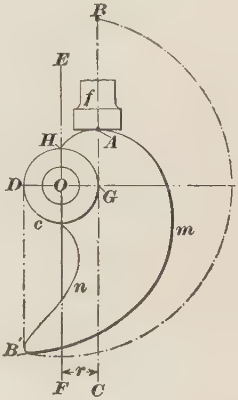

Fig. 25

Through the point *A*, then, the curve *m* is constructed as an involute of the circle *c*. The final point *B′* is found by drawing a circle, with *O* as a center and *O B* as a radius, cutting the curve *m* at *B′*. It will be seen that *B′* is vertically below *D* and that *B′ D* is equal to three-fourths the circumference of the circle *c*. For the radius *G A* of the curve *m* is equal to one-fourth the circumference of *c*, that is, equal to the arc *G H*, since the curve *m* is formed by the unwinding of a string from the circle *c*, starting at *H*. Hence, when the string has been unwound from the arc *H G D*, or three-fourths the circumference, the radius DB', representing the length of the string at that instant, must be equal to the arc HGD.

The inner curve n may be chosen at random, but it must lie wholly within the space between m and $B'D$. For an involute curve the tangent to the base circle c is normal to the curve m, and it follows that in any position of the cam the tangent at the point lying on BC above G is perpendicular to BC. Hence, the point of contact of the cam and the collar f always lies on BC. As actually constructed, two of these cams are placed opposite each other, as shown in Fig. 22.

28. Cams With Tangential Followers.—It is sometimes necessary to construct a cam to act tangentially on a flat surface as a follower. The follower may have a motion of translation, as EF moving along XY in Fig. 26, or it may turn about a fixed axis, as CD swinging about C in Fig. 27. For the first case, the method of designing the cam is as follows: Let the line EF represent the flat surface of the

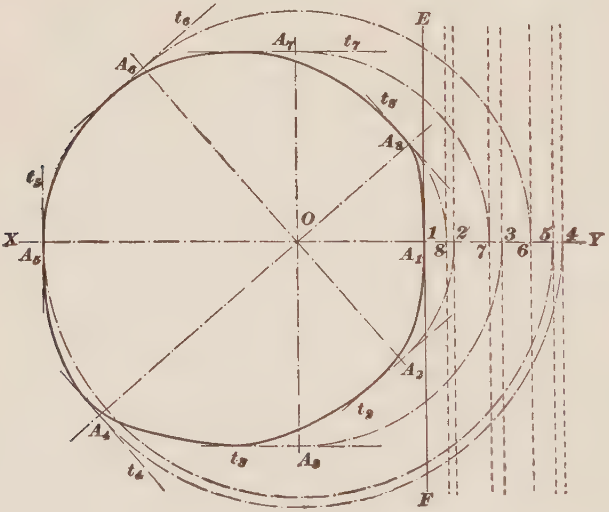

FIG. 26

follower, O the center of rotation of the cam, and XY the line of motion of the follower. Suppose that the follower is to take the positions *2*, *3*, *4*, etc. when the radii OA_2, OA_3, OA_4, etc. of the cam coincide, respectively, with the line OY. With O as a center and $O2$, $O3$, $O4$, etc. as radii, strike arcs cutting corresponding radii in the points A_2, A_3, A_4, etc. Through A_2 draw a line t_2, making the same angle with OA_2 as the follower EF makes with OX. In the figure, EF is at right angles to OX; hence t_2 is at right angles to OA_2. This is the usual arrangement. Likewise, draw lines t_3, t_4, t_5, etc. through A_3, A_4, A_5, etc. and perpendicular, respectively, to OA_3, OA_4, OA_5, etc. Draw a smooth curve tangent to the lines t_2, t_3, t_4, etc., which will be the desired cam outline.

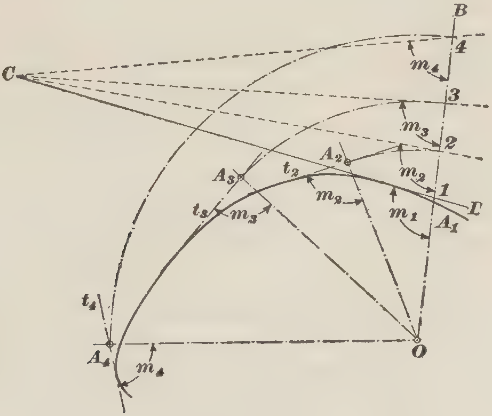

FIG. 27

Sometimes it will be found that the outline cannot be drawn without making some part of it concave. In this case, the conditions required cannot be met and some of them must be changed. All parts of the outline of a cam acting tangentially on a straight follower must be convex, for a concave part cannot become tangent to the follower.

In Fig. 27 is shown the construction for a follower that oscillates about a fixed axis. This axis is denoted by C, and $C1$, $C2$, $C3$, and $C4$ are follower positions corresponding to the radii OA_1, OA_2, etc. of the cam. That is, when OA_2

coincides with the line OB, the follower will be at $C2$, and so on. With O as a center and radius $O2$ draw an arc cutting the first radius at A_2, and through the point A_2 draw the line t_2, making the angle m_2 with OA_2, equal to the angle that $C2$ makes with OB. Likewise, locate A_3 and draw t_3, making the angle m_3 with OA_3; and locate A_4 and draw t_4. A curve tangent to the lines $C1$, t_2, t_3, t_4, etc., provided that it has no concave part, is the required cam outline.

29. Harmonic-Motion Cams.—If a cam is required to give a rapid motion between two points, without regard to the kind of motion, its surface should be laid out so as to gradually accelerate the roller from the start and gradually retard it toward the end of its motion, in order that the movement may be as smooth and free from shocks as possible. For this reason, cams are frequently designed to produce harmonic motion—that is, a motion of the follower like that of the steam-engine piston with a very long connecting-rod or a slotted cross-head. The diagram in Fig. 28 shows how harmonic motion is plotted. Let AC be the stroke line. Then the semicircle ABC, drawn on AC as a diameter, is divided into a number of equal arcs by the points a, b, c, d, etc. If perpendiculars are dropped from these points, the points *1*, *2*, *3*, *4*, etc. are obtained on the stroke line. The spaces between the latter points represent the distances traversed by the moving point in equal time intervals. It will be seen that the distances increase from points *1* to *5*, and decrease from *5* to *9*.

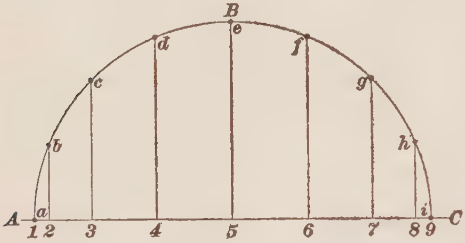

FIG. 28

To apply this motion to the two cams discussed in Arts. **25** and **26**, simply lay off the distances *1–2*, *2–3*, *3–4*, etc. on the chord *0–6*, in Fig. 23, or the line $A_0 a$ in Fig. 24, in place of the equal spaces used in these figures. Fig. 29 shows a convenient method of spacing. On the chord *0–6* as a diameter draw the semicircle *0 d 6*, divide it into a suitable number of equal parts, and project these divisions by straight lines on the chord *0–6*. Through the points of intersection *1*, *2*, *3*, *4*, etc., and with *B* as a center, describe the arcs $1\text{–}1_1$, $2\text{–}2_1$, $3\text{–}3_1$, etc., and complete the cam outline, as explained in Art. 25.

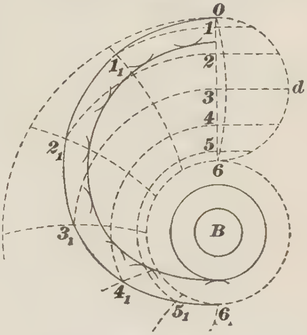

FIG. 29

30. Positive-Motion Cams. The cams thus far considered can drive the follower in one direction only, making a spring or weight necessary to keep the two in contact after the follower has reached its extreme position. If, however, the cam-plate extends beyond the roller and a groove is cut in it for the roller to run in, the motion of the roller is positive in both directions.

The word **positive**, when applied to a mechanism, has a different meaning from any heretofore given to it. A mechanism so constructed that nothing short of actual breakage of some one of its parts can keep it from working properly when motion is imparted to one of the links that operates it is called a **positive mechanism**, or a **positive gear** when speaking of valve gears, and the motion produced is called a **positive motion**. Those mechanisms that depend for their operation on the raising or lowering of a weight, that is, on gravity or the action of a spring, are termed **non-positive** or **force-closed mechanisms**. Non-positive mechanisms, although extensively used, are of a lower order of mechanical excellence than positive mechanisms, and, other things being equal, a positive motion should be chosen when designing a mechanism, since a non-positive one will refuse to work if the weight or the part operated by the spring should get caught.

A common way of providing for a positive return of the follower is shown in Fig. 30. The cam-plate has a groove in which the roller of the follower is held. The roller is forced from the center by the inner side of the groove, and toward the center by the outer side. With this arrangement, the grooves must have a width slightly greater than the roller diameter, so that the roller will not be in contact with both faces of the groove at the same time.

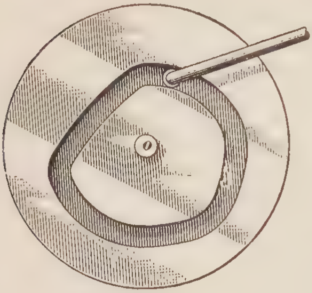

FIG. 30

SLIDING AND CYLINDRICAL CAMS

31. Sliding Cams.—A **sliding cam** consists of a plate sliding within fixed guides and carrying a curved face that acts on the follower. The design of a sliding cam for any desired motion of the follower is a simple process. In Fig. 31, suppose that the follower is to occupy successively the positions *2*, *3*, *4*, etc., when the points *2'*, *3'*, *4'*, etc. of the driving plate coincide with the point *1'* on the line *EF* of the motion of the follower. Erect perpendiculars through the points *1'*, *2'*, *3'*, etc., and through the points *1*, *2*, *3*, etc. draw lines parallel to the direction of motion of the plate. The

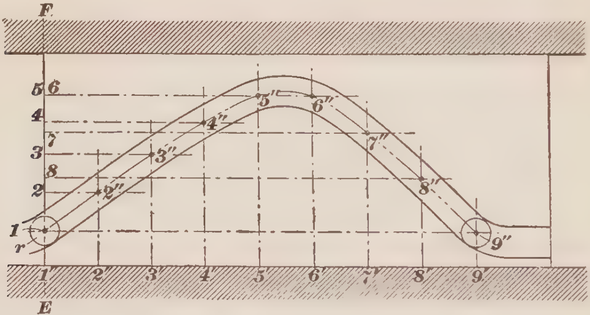

FIG. 31

intersections $1''$, $2''$, $3''$, etc. are points on the cam outline. If the follower has a roller r, the actual outline is obtained by drawing arcs, as shown in Figs. 23 and 24. To render the motion positive, a second face is provided, the two forming a groove whose width is slightly greater than the diameter of the roller.

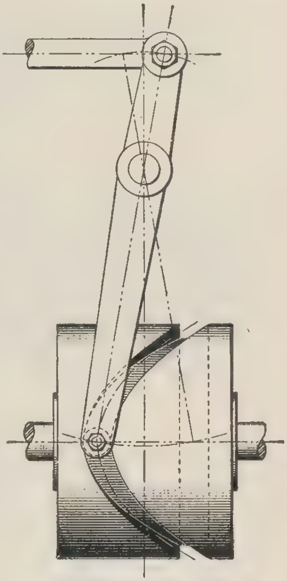

FIG. 32

32. Cylindrical Cams.—The **cylindrical cam** is equivalent to a sliding cam. Thus, if the plate in Fig. 31 is wrapped on a right cylinder and this cylinder is rotated on its axis, the follower is driven in precisely the same way as when the plate is given a motion of translation. Fig. 32 shows a cylindrical cam, consisting of a cylinder in which is cut a groove to receive the roller of the follower.

33. Cams for Screw Machines.—Cams are largely used for operating the feed mechanisms and turrets on automatic screw machines. A typical arrangement is shown in Fig. 33, in which the cam, consisting of a metal strip a attached to a cylinder b, gives an endwise motion in one direction to a rod c through a roller d that is connected with the rod. The cam revolves continuously, and the rod is returned to its normal position by means of a spring not

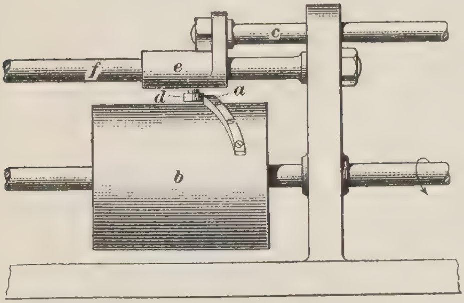

FIG. 33

shown in the illustration. The roller *d* is kept in contact with the cam by this spring. The cam moves a sleeve *e* to which both the roller *d* and the rod *c* are attached. This sleeve is guided parallel to the axis of the cam by means of a rod *f*, on which it slides. The strip *a* is fastened to the cylinder at such an angle, and in such a position, as to give the required horizontal movement to the rod *c*.

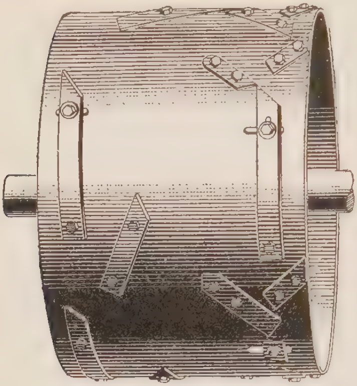
FIG. 34

By arranging a series of strips on the surface of the cylinder, as shown in Fig. 34, and by using additional rollers, several movements may be given to the rod, or to more than one rod, during each revolution of the cylinder.

RATCHET MECHANISMS

34. Ratchet Gearing.—In the ordinary gear train, the office of the wheel is to transmit motion. In the ratchet train, on the other hand, the office of one of the parts, called the **pawl** or **click**, is to prevent the relative motion of two other parts. Two types of ratchet trains are shown in Figs. 35 and 36; each consists of a fixed frame *a*, pawl *b*, and ratchet wheel *c*. In the first, the wheel *c* may be turned counterclockwise relative to the frame *a*, but not in the reverse direction; this mechanism has many applications. It is used

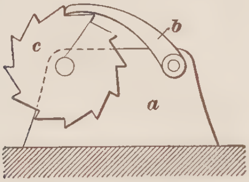

FIG. 35

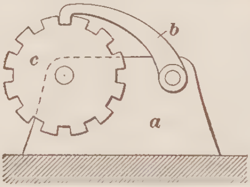

FIG. 36

in winches to prevent the load from running down, and in ratchet braces and drills. The ratchet of Fig. 35 is called a *running ratchet*, and that shown in Fig. 36 is called a *stationary ratchet*. In the stationary ratchet, relative motion of *a* and *c* in either direction is impossible so long as the pawl is in action.

In Fig. 37 is shown a *frictional ratchet*. Rollers or balls are placed between the ratchet wheel and the outer ring. When the ring is turned in one direction, the rollers wedge between the wheel and the ring, causing them to bind and preventing a relative motion between them. Motion in the other direction releases them, and permits a free relative rotation.

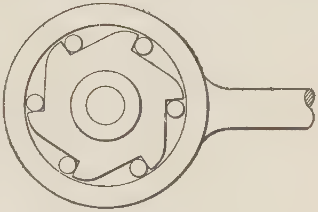
FIG. 37

35. Ratchets for Feed Mechanisms.—Running ratchets are frequently used in the feed mechanisms of shapers, planers, and other machine tools of this class; the arrangement for this purpose is shown in Fig. 38. The pawl *b* is carried at the end of the arm *c*, which is caused to oscillate by a rod not shown. The wheel *a* has radial teeth, and the pawl, which is made symmetrical, can occupy either position *b* or *b′*. In order that the pawl may be held firmly against the ratchet wheel, its axis is provided with a small triangular piece, shown dotted, against which presses a flat end presser, always urged upwards by a spring, also shown dotted. Whichever position *b* may be in, it will be held against the wheel.

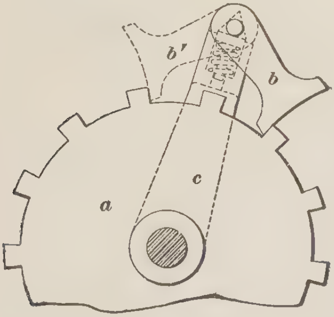

FIG. 38

Ratchet wheels for feed mechanisms must be so arranged that the feed can be easily adjusted. This is often done by changing the swing of the lever *c*, Fig. 38, which is usually connected by a rod with a vibrating lever having a definite angular movement at the proper time for the feed to occur. This lever is generally provided with a **T** slot, in which the pivot for the rod can be adjusted by means of a screw and nut. By varying the distance of the pivot from the center of motion, either one way or the other, the swing of the arm *c* can be regulated.

Another method of adjusting the motion is shown in Fig. 39. The wheel turns on a stationary shaft or stud *o*, and the end of the shaft is turned to a smaller diameter than the rest and is threaded, thus forming a shoulder against which an adjustable shield *s* can be clamped by the nut *n*. Back of the wheel is the arm *c*, also loose on the shaft, carrying the pawl *b*, which latter should be of a thickness equal to that of the wheel, plus that of the shield. The teeth of the wheel may be made of a shape suitable to gear with another wheel, to which the feed-motion will then be imparted, or another wheel back of and attached to the one shown could be used for the purpose.

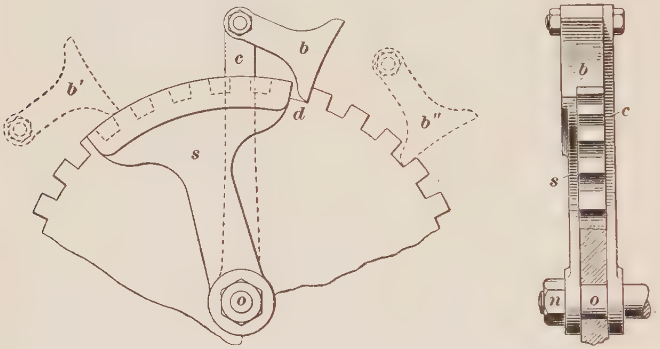

Fig. 39

The extreme left-hand position of *b* is shown at *b*′. Here the pawl rides on the shield and does not come in contact with the teeth of the wheel. When the pawl comes to the right-hand edge of the shield, however, it will drop into contact at *d*; and, if *b*″ is the extreme right-hand position, the wheel will be turned through a space corresponding to 3 teeth. If the shield should be turned to the right, the wheel will be moved a smaller number of teeth each time; if it should be turned to the left, a feed of four or more teeth, up to the full capacity of the stroke, can be obtained; with the shield in its mid-position, it will carry the pawl during the whole swing of the arm and there will be no feed.

36. Ratchet and Screw.—Where ratchets are employed in the feed-motions of machine tools, they are made to operate a screw, which, in turn, drives the head carrying the tool.

EXAMPLE.—A ratchet having 80 teeth is attached to the end of a screw having six threads per inch. If the pawl is set to move the ratchet 3 teeth for every stroke of the arm, how much feed will the tool have, supposing it to be moved directly by the screw?

SOLUTION.—One turn of the screw will move the tool $\frac{1}{6}$ in. But for each stroke the ratchet and, hence, the screw move $\frac{3}{80}$ turn, and the tool will travel $\frac{3}{80} \times \frac{1}{6} = \frac{1}{160} = .00625$ in. Ans.

PULLEYS AND BELTING

BELT GEARING

KINEMATICS OF FLEXIBLE GEARING

1. Transmission by Belts, Ropes, and Chains.—The transmission of motion and power between shafts that are some distance apart may be effected by belts, ropes, or chains running over suitable pulleys. Belts form a convenient means of transmitting power, but owing to their tendency to stretch and slip on the pulleys, they are not suitable where an exact velocity ratio between the shafts is required. For driving machinery, however, this freedom to stretch and slip is an advantage, since it prevents shocks that are liable to occur when a machine is thrown suddenly into gear, or when there is a sudden fluctuation in the load.

For transmitting considerable power between shafts at some distance apart, ropes are frequently used instead of belts. The advantages of rope driving are lightness, low first cost, smooth running, flexibility, and ease of repair. With rope drives, the shafts connected are generally run at high speeds. Chain gearing is used when positive driving is essential, and also for the transmission of large powers. Ordinarily, with chain drives the motion is slow, although recently chains of special forms have been devised for high-speed transmission, and these are being used quite extensively in place of belts.

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VELOCITY RATIO IN BELT AND ROPE DRIVES

2. Velocity Ratio of a Pair of Pulleys.—Let the pulleys a and b, Fig. 1, be connected by a belt or rope, and let D_a and D_b denote their respective diameters, and N_a and N_b their respective velocities, expressed in revolutions per minute. The speed of a point on the circumference of either pulley, assuming that the belt neither slips nor stretches, is the same as the speed of the belt. Then v, the speed of the belt, in feet per minute, is equal to the product of the circumference of the pulley, in feet, and the revolutions per minute. That is, $v = \pi D_a N_a$ and $v = \pi D_b N_b$; hence, $\pi D_a N_a = \pi D_b N_b$, and

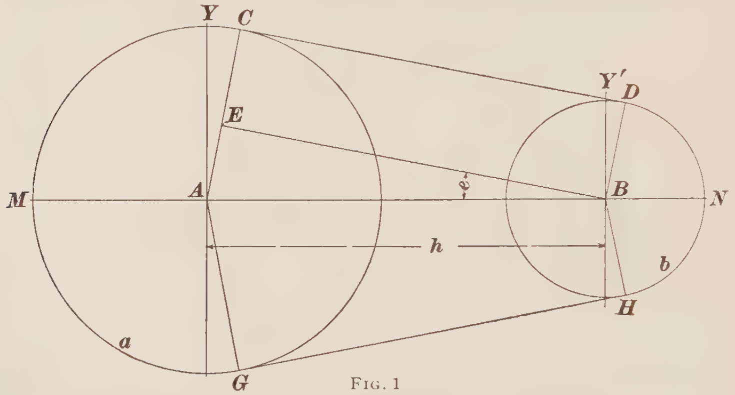

FIG. 1

$$D_a N_a = D_b N_b \qquad (1)$$

or

$$\frac{N_a}{N_b} = \frac{D_b}{D_a} \qquad (2)$$

That is, *the speeds or numbers of revolutions of two pulleys connected by belt are inversely proportional to their diameters.*

In formulas **1** and **2**, the diameters may be taken in either feet or inches, but the same unit must in every case be used for both diameters.

EXAMPLE 1.—A pulley 30 inches in diameter, making 210 revolutions per minute, drives a second pulley 14 inches in diameter; how many revolutions per minute does the latter pulley make?

SOLUTION.—From formula **1**, $30 \times 210 = 14 \times N_b$, whence

$$N_b = \frac{30 \times 210}{14} = 450 \text{ R. P. M.} \quad \text{Ans.}$$

EXAMPLE 2.—The driving pulley of a machine is 1 foot in diameter and must make 750 revolutions in 5 minutes; what size pulley should be used on the driving shaft, if its speed is 143 revolutions per minute?

SOLUTION.—In all examples of this kind, the speeds and diameters must be reduced to the same units. 750 revolutions in 5 minutes = 750 ÷ 5 = 150 R. P. M.; 1 ft. = 12 in. Hence, $12 \times 150 = D_b \times 143$, whence

$$D_b = \frac{12 \times 150}{143} = 12.6 \text{ in., nearly. Ans.}$$

3. Influence of Belt Thickness.—The formulas in Art. **2** are based on the assumption that the thickness of the belt is so small in comparison with the pulley diameter that it can be neglected. Actually, only the center line of the belt or rope has a constant speed v; hence, if t denotes the thickness of belt or rope, the effective radii are $r_a + \frac{1}{2}t$ and $r_b + \frac{1}{2}t$, while the effective diameters are $D_a + t$ and $D_b + t$, respectively. Formula **1**, Art. **2**, therefore becomes

$$N_a(D_a + t) = N_b(D_b + t) \qquad (1)$$

and formula **2**, Art. **2**, becomes

$$\frac{N_a}{N_b} = \frac{D_b + t}{D_a + t} \qquad (2)$$

EXAMPLE.—If, in the first example of Art. **2**, the belt thickness is $\frac{3}{8}$ inch, how many revolutions per minute does the 14-inch pulley make, taking account of this thickness?

SOLUTION.—From formula **1**, $N_a(D_a + t) = N_b(D_b + t)$, whence, $210 \times (30 + \frac{3}{8}) = N_b \times (14 + \frac{3}{8})$ and $N_b = 210 \times \frac{30\frac{3}{8}}{14\frac{3}{8}} = 444$ R. P. M., nearly. Ans.

Hence, by taking belt thickness into account, the speed of the driven pulley is reduced from 450 to 444 revolutions per minute, or about 1.3 per cent. In practice, however, the thickness of the belt is usually neglected.

4. Velocity Ratio in Compound System of Belts. In Fig. 2 is shown a compound system of belts in which there are six pulleys, a, b, c, d, e, and f. Denoting the diameters by D_a, D_b, etc. and the revolutions per minute by N_a, N_b, etc., for the three pair a and b, c and d, e and f, the following relations exist: $\frac{N_a}{N_b} = \frac{D_b}{D_a}$, $\frac{N_c}{N_d} = \frac{D_d}{D_c}$, and $\frac{N_e}{N_f} = \frac{D_f}{D_e}$. Multiplying together the left-hand and right-hand members, respectively, of these three equations, and placing the products equal to each other, $\frac{N_a \times N_c \times N_e}{N_b \times N_d \times N_f} = \frac{D_b \times D_d \times D_f}{D_a \times D_c \times D_e}$.

Now, pulleys *b* and *c* are fixed to the same shaft and make the same number of revolutions per minute; hence, $N_b = N_c$, and for the same reason $N_d = N_e$. Canceling these equal quantities in the fraction, it becomes

$$\frac{N_a}{N_f} = \frac{D_b \times D_d \times D_f}{D_a \times D_c \times D_e} \qquad (1)$$

or $N_a \times D_a \times D_c \times D_e = N_f \times D_b \times D_d \times D_f$ (2)

Considering separately the three pair connected by the three belts, pulleys *a*, *c*, and *e* may be taken as the drivers, and pulleys *b*, *d*, and *f* as the followers; then formula **1** may be expressed thus:

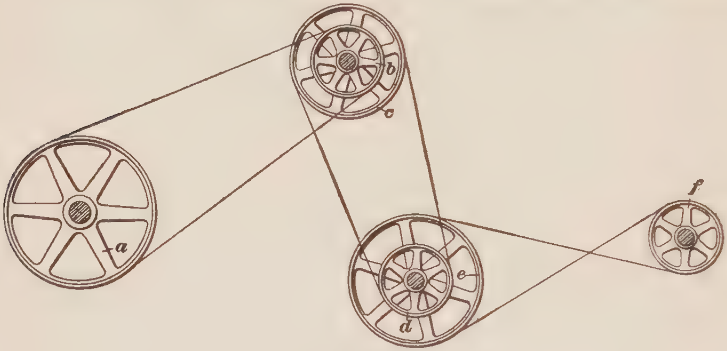

FIG. 2

$$\frac{\textit{R. P. M. of first driver}}{\textit{R. P. M. of last follower}} = \frac{\textit{product of diameters of all followers}}{\textit{product of diameters of all drivers}}.$$

and formula **2** thus: *The speed of the first driver multiplied by the product of the diameters of all the drivers is equal to the speed of the last follower multiplied by the product of the diameters of all the followers.*

The thickness of the belt may be taken into account by increasing all pulley diameters by an amount equal to that thickness.

EXAMPLE.—An emery grinder is to be set up to run at 1,200 revolutions per minute. The countershaft has pulleys 20 and 8 inches in diameter. The pulley on the grinder is 6 inches in diameter and is belted to the 20-inch pulley on the countershaft. The line shaft runs at 180 revolutions per minute and carries a pulley that is belted to the 8-inch pulley on the countershaft. Calculate the diameter of the line shaft pulley.

SOLUTION.—From formula **2**, $180 \times D \times 20 = 1{,}200 \times 8 \times 6$, whence

$$D = \frac{1{,}200 \times 8 \times 6}{180 \times 20} = 16 \text{ in.} \quad \text{Ans.}$$

5. Pulley Diameters.—When the speeds of the first and last shafts are given, and the diameters of all the pulleys are to be found, the following rule may be applied:

Rule.—*Divide the higher speed by the lower. If two pulleys are to be used, this quotient will be the ratio of their diameters.*

If four pulleys are required, find two numbers whose product is equal to this quotient. One of these numbers will be the ratio of the diameters of one pair of pulleys, and the other number will be the ratio for the other pair.

EXAMPLE.—It is required to run a machine at 1,600 revolutions per minute, the driving shaft making 320 revolutions per minute; what size pulleys are required: (*a*) when two pulleys are used; (*b*) when four pulleys are used?

SOLUTION.—(*a*) $1{,}600 \div 320 = 5$. The two pulleys must, therefore, be in the ratio of 5 to 1, the driving pulley being five times as large as the driven pulley, since the latter has the greater speed. Then assume diameters of 30 and 6 in. Ans.

(*b*) $2\frac{1}{2} \times 2 = 5$. One pair of pulleys may have the ratio of $2\frac{1}{2}$ to 1, and the other pair, 2 to 1. Assume diameters of 25 and 10 in. for one pair, and of 12 and 6 in. for the other pair. Ans.

6. Direction of Rotation.—When a belt connecting two pulleys forms a single loop, it is called an **open belt.** If, however, it forms a double loop, like a figure 8, it is called a **crossed belt.** Thus, in Fig. 2, the pulleys *a* and *b* are connected by an open belt, as are the pulleys *c* and *d*, while a crossed belt connects the pulleys *e* and *f*. *Pulleys connected by open belts turn in the same direction, and those connected by crossed belts in opposite directions.* If several belts are used between the first driver and last follower, these will turn in the same direction if there is no crossed belt, or if there is an even number of crossed belts, but in opposite directions if the number of crossed belts is odd.

EXAMPLES FOR PRACTICE

1. A driving pulley is 54 inches in diameter, and the driven pulley, which runs at 112 revolutions per minute, is $2\frac{1}{2}$ feet in diameter; what is the speed of the driving shaft? Ans. 62.22 R. P. M.

2. The flywheel of an engine running at 180 revolutions per minute is 8 feet 5 inches in diameter; what should be the diameter of the pulley that it drives if the required speed of the latter is 600 revolutions per minute? Ans. $30\frac{5}{16}$ in., nearly

3. A machine is to be belted through a countershaft, so as to run at 1,200 revolutions per minute, the speed of the driving shaft being 120 revolutions per minute; find three ratios that could be used for two pair of pulleys.

$$\text{Ans.}\left\{\begin{array}{lll} 5 & : 1 \text{ and } 2 & : 1 \\ 4 & : 1 \text{ and } 2\frac{1}{2} & : 1 \\ 3\frac{1}{3} & : 1 \text{ and } 3 & : 1 \end{array}\right.$$

4. An emery grinder is to be set to run at 1,400 revolutions per minute; the countershaft has pulleys 30 and 8 inches in diameter; the pulley on the grinder is 7 inches in diameter; what size pulley should be used on the line shaft, its speed being 185 revolutions per minute? Ans. $14\frac{1}{8}$ in.

LENGTHS OF OPEN AND CROSSED BELTS

7. Length of an Open Belt.—In the discussion of the lengths of belts the following symbols will be used:

R = radius of larger pulley;
r = radius of smaller pulley;
s = sum of radii of the two pulleys, $R + r$;
d = difference between radii of the two pulleys, $R - r$;
e = angle, in degrees, of straight part of belt with line joining centers of shafts;
h = distance between shafts;
l = total length of belt.

In Fig. 1, $MCDN$ represents half the length of the belt. This distance is made up of three parts, MC, CD, and DN, so that

$$\frac{1}{2}l = MC + CD + DN \qquad (1)$$

In the figure, the radii AC and BD are drawn perpendicular to the common tangent CD, and hence they are parallel. Furthermore, BE is drawn parallel to DC, and $BECD$ is therefore a rectangle. Hence, BE is equal to CD and makes the same angle e with MN as would CD if it were extended to meet MN. Also, $CE = DB = r$. But, $AE = AC - CE$, and since $AC = R$ and $CE = r$, $AE = R - r$. Now, the angles YAC and $Y'BD$ have their sides respectively perpendicular to the sides of the angle e. Hence, by geometry, angle YAC = angle $Y'BD$ = angle e. And since the angles MAY and $Y'BN$ are right angles, it follows that the angle $MAC = 90° + e$, and the angle $DBN = 90° - e$.

The circumference of the larger pulley is $2\pi R$, and the arc MC bears the same ratio to the whole circumference that the angle MAC bears to 360°. In other words, $MC : 2\pi R = 90 + e : 360$, or the arc $MC = 2\pi R \times \frac{90 + e}{360} = \pi R \times \frac{90 + e}{180}$. By the same course of reasoning, the arc $DN = \pi r \times \frac{90 - e}{180}$. Also, $CD = EB = AB \cos e = h \cos e$. Substituting these values of MC, CD, and DN in formula **1**,

$$\begin{aligned}\tfrac{1}{2}\,l &= \pi R \times \frac{90 + e}{180} + h \cos e + \pi r \times \frac{90 - e}{180}\\ &= \pi R \tfrac{90}{180} + \pi R \frac{e}{180} + h \cos e + \pi r \tfrac{90}{180} - \pi r \frac{e}{180}\\ &= \frac{\pi}{2}(R + r) + \pi (R - r)\frac{e}{180} + h \cos e\\ &= \frac{\pi}{2} s + \pi d \frac{e}{180} + h \cos e\end{aligned}$$

Then,
$$l = \pi s + \pi d \frac{e}{90} + 2h \cos e \qquad (2)$$

Since $\frac{AE}{AB} = \frac{R - r}{h} = \frac{d}{h} = \sin e$, the angle e may readily be found from a table of natural sines by taking the angle corresponding to the value that equals $\frac{d}{h}$. This value of e, in degrees, is substituted in formula **2**.

EXAMPLE.—In Fig. 1, let the pulleys have diameters of 20 and 12 inches, respectively, and let the distance between the centers of the shafts be 4 feet; calculate the length of the open belt required.

Solution.— $R = 10$, $r = 6$, $d = 10 - 6 = 4$, $s = 10 + 6 = 16$, and $h = 48$ in. To find the angle e, $\sin e = \frac{d}{h} = \frac{4}{48} = .0833$, whence $e = 4° 47' = 4.8°$, nearly, and $\cos e = .9965$. Substituting these values in formula **2**,

$$l = 3.1416 \times 16 + 3.1416 \times 4 \times \frac{4.8}{90} + 2 \times 48 \times .9965$$
$$= 146.6 \text{ in.} = 12 \text{ ft. } 2.6 \text{ in.} \quad \text{Ans.}$$

8. When the angle e is comparatively small, the following formula gives a sufficiently exact approximation to the length of an open belt:

$$l = \pi s + 2h\left(1 + \frac{d^2}{8h^2}\right)$$

The greater the difference in radii and the smaller the distance between the pulleys, the less exact is the formula.

Example.—Solve the example of Art. **7** by the approximate method.

Solution.— $l = 3.1416 \times 16 + 2 \times 48 \times \left(1 + \frac{4^2}{8 \times 48^2}\right) = 146.35$ in. Ans.

The difference between the two results is only $\frac{1}{4}$ inch.

9. Length of a Crossed Belt.—Referring to Fig. 3,

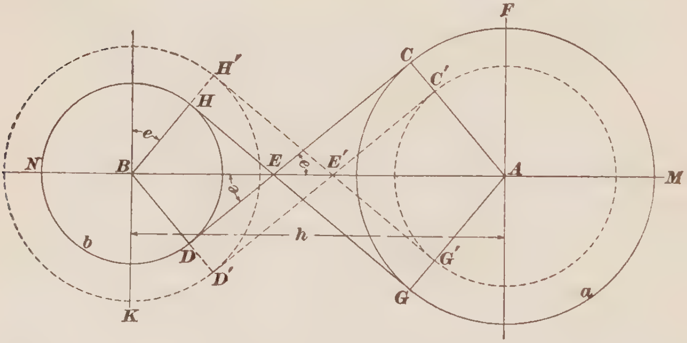

Fig. 3

it appears that for the half length of the crossed belt

$$\tfrac{1}{2} l = \text{arc } MC + CD + \text{arc } DN \qquad (1)$$

The angles CEA and DEB are each equal to e, and the radii AC and BD are drawn perpendicular to CD. Hence, the angles CAF and DBK are each equal to e, since their sides are perpendicular, respectively, to the sides of CEA

and DEB. Then, the angle $MAC = MAF + CAF = 90° + e$, and the angle $NBD = NBK + DBK = 90° + e$. Therefore, by the same reasoning as in the case of the open belt, the arc $MC = \pi R \times \frac{90 + e}{180}$, and the arc $DN = \pi r \times \frac{90 + e}{180}$. Also, $CD = CE + ED$. But $CE = AE \cos CEA = AE \cos e$, and $ED = EB \cos DEB = EB \cos e$. Then, substituting these values, $CD = AE \cos e + EB \cos e = (AE + EB) \cos e = h \cos e$.

Next, substituting the values of MC, CD, and DN in formula **1**, $\frac{1}{2} l = \pi R \times \frac{90 + e}{180} + h \cos e + \pi r \frac{90 + e}{180} = \pi (R + r) \frac{90 + e}{180} + h \cos e$, from which

$$l = 2\pi(R + r) \frac{90 + e}{180} + 2 h \cos e \qquad \textbf{(2)}$$

or

$$l = \pi s\left(1 + \frac{e}{90}\right) + 2 h \cos e \qquad \textbf{(3)}$$

The angle e may be determined from the triangles ACE and BDE, in which $AC = AE \sin e$ and $BD = EB \sin e$. Adding, $AC + BD = (AE + EB) \sin e$, or $(R + r) = s = h \sin e$, whence

$$\sin e = \frac{s}{h} \qquad \textbf{(4)}$$

Example.—In the example of Art. **7**, calculate the length of the belt if it is a crossed belt.

Solution.— $R = 10$; $r = 6$; $d = 10 - 6 = 4$; $s = 10 + 6 = 16$; and $h = 48$. By formula **4**, $\sin e = \frac{s}{h} = \frac{16}{48} = .33333$, from which $e = 19° 28' = 19.47°$, and $\cos e = .9428$. Using formula **3**,

$$l = 3.1416 \times 16 \left(1 + \frac{19.47}{90}\right) + 2 \times 48 \times .9428 = 151.65 \text{ in.,}$$

or 12 ft. $7\frac{11}{16}$ in., very nearly. Ans.

EXAMPLES FOR PRACTICE

1. Two pulleys have diameters of 36 inches and 20 inches, respectively, and the distance between their centers is 128 inches; find the length of an open belt to connect them. Ans. 344.5 in.

2. What length of crossed belt is required to connect two pulleys of 42 inches and 18 inches diameter, respectively, if the distance between shaft centers is 8 feet? Ans. 295.7 in.

3. Using the formula of Art. **8**, find the length of an open belt to connect two pulleys 30 inches and 36 inches in diameter, respectively, the distance between centers being 12 feet. Ans. 391.7 in.

CONE PULLEYS

10. Variable Speed.—**Speed cones** are used for varying the speed of a shaft or other rotating piece driven by a belt. Figs. 4 and 5, respectively, show continuous cones and

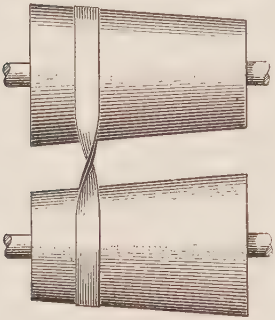

FIG. 4

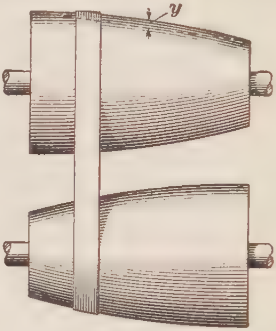

FIG. 5

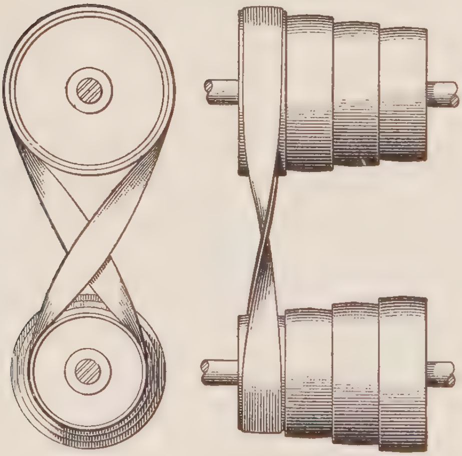

FIG. 6

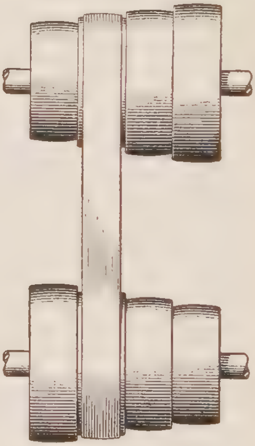

FIG. 7

conoids, the former being suitable for crossed and the latter for open belts. The speed of the driven shaft can be raised gradually by shifting the belt. Figs. 6 and 7 show sets of stepped pulleys. As a flat belt always tends to run to the larger end of a conical pulley, continuous cones or conoids require special provision for keeping the belt at any desired point on the pulley. For this reason, stepped cones, rather than continuous cones, are generally used. Whenever possible, it is desirable to have both pulleys alike, so that they can be cast from one pattern.

11. Continuous Cones for Crossed Belts.—Let a and b, Fig. 8, represent two speed cones of the same size, having the diameters of the large and small ends equal to D and d, respectively. The driving cone a has a uniform speed of N revolutions per minute, while the speed of b is n_1 or n_2 revolutions per minute, according as the belt runs at the small or large end. It is assumed that pulley b is to have a range of speeds between n_1 and n_2 revolutions, n_1 being the greater.

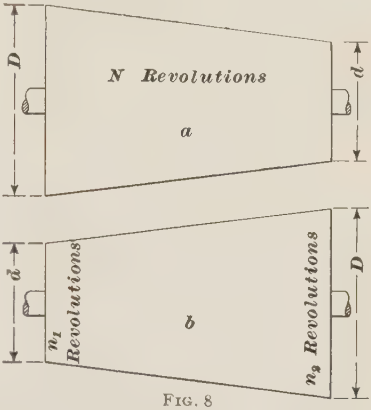

FIG. 8

Since a and b are to be the same size, a certain relation must exist between N and n_1 and n_2.

From formula **1**, Art. **2**, $ND = n_1 d$, or $D = \frac{n_1 d}{N}$, and $Nd = n_2 D$, or $D = \frac{N d}{n_2}$. Placing the two values of D equal to each other, $\frac{n_1 d}{N} = \frac{N d}{n_2}$, or

$$N = \sqrt{n_1 n_2} \qquad (1)$$

That is, *N must equal the square root of the product of n_1 and n_2.*

Having determined N, the relation between D and d can be found. Since $N = \frac{n_1 d}{D}$ and $N = \frac{n_2 D}{d}$, $\frac{n_1 d}{D} = \frac{n_2 D}{d}$, or $n_1 d^2 = n_2 D^2$. Hence,

$$D = d\sqrt{\frac{n_1}{n_2}} \qquad \textbf{(2)}$$

That is, *the large diameter is equal to the smaller diameter multiplied by the square root of the quotient of* n_1 *divided by* n_2.

EXAMPLE.—Two continuous speed cones are to be designed to give a range of speed between 100 and 700 revolutions per minute; they are to be alike in all respects. What must be the speed of the driving shaft and the large diameter of the cones, assuming the small diameter to be 4 inches?

SOLUTION.—From formula **1**, $N = \sqrt{n_1 n_2} = \sqrt{100 \times 700} = \sqrt{70{,}000} = 264.57$ R. P. M. Ans.

From formula **2**,

$$D = d\sqrt{\frac{n_1}{n_2}} = 4\sqrt{\frac{700}{100}} = 4 \times 2.646 = 10.584 \text{ in. Ans.}$$

12. Cone Pulleys for Crossed Belts.—Speed cones, to be properly designed, should have their diameters at different points so proportioned that the belt will always have the same length, when tightly drawn, whatever its position. With crossed belts, this condition is easily fulfilled, as it is only necessary to make the sum of the corresponding diameters or radii the same for all pairs of steps, or, in the case of the continuous cones, for all belt positions. The truth of this statement may be shown as follows: In Fig. 3, suppose the belt to run on the pulleys shown by dotted lines, the sum of whose radii is equal to the sum of the radii of the pulleys a and b. Then, since it is assumed that $AG + BH = AG' + BH'$, it follows, by subtracting $AG' + BH$ from both members of the equation, that $AG - AG' = BH' - BH$, or $GG' = HH'$, and it can be proved by geometry that $G'H'$ is parallel to GH and equal to it in length. Likewise $C'D'$ is equal to and parallel to CD. Then the right triangles CEA and $C'E'A$ have the angle CAE in common. Also, the angles ACE and $AC'E'$ are equal, since each is a right angle. Now, the sum of the three angles of a triangle is equal to two right angles, and since the triangles CEA and $C'E'A$ have two angles of the one equal to two angles of the other, it follows that their third angles are equal, or $CEA = C'E'A$. Therefore, with the belt in the position shown by the dotted lines, the sum s, the angle e, and the distance h are the same as for the position shown by full lines. Consequently, the length of belt as found by formula **3**, Art. **9**, will be the same in both cases. Hence, *for crossed belts, the sum of the corresponding diameters of two speed cones should be the same at all points.*

Example.—The steps of a cone pulley have diameters of 7, $8\frac{3}{8}$, 10, and 12 inches, respectively, and the diameter of the step on the other cone corresponding to the 10-inch step is $8\frac{3}{4}$ inches; calculate the diameters of the remaining steps for a crossed belt.

Solution.—Sum of diameters $= 10 + 8\frac{3}{4} = 18\frac{3}{4}$ in. Since this is to be the same for all pairs, the diameters of the remaining steps are as follows:

For the 7-in. step . . . . $18\frac{3}{4} - 7 = 11\frac{3}{4}$ in. Ans.
For the $8\frac{3}{8}$-in. step . . . . $18\frac{3}{4} - 8\frac{3}{8} = 10\frac{3}{8}$ in. Ans.
For the 12-in. step . . . . $18\frac{3}{4} - 12 = 6\frac{3}{4}$ in. Ans.

13. Cone Pulleys for Open Belts.—When an open belt is used, the sum of the diameters does not remain the same. In order that the belt shall run equally tight in all positions or on all steps, the sum of the corresponding diameters $D + d$ is the smallest when $D - d$ has its greatest value, that is, when the diameters are most unequal, and is greatest when $D = d$, that is, when the diameters are equal. The difference between cones for open and for crossed belts is shown in Figs. 4 and 5. With the crossed belt, the sides of the cone are straight, while with the open belt the sides bulge near the middle sections, showing that the sum $D + d$ is greater at the center than at the ends. The distance y, Fig. 5, must be added to each radius at the middle section to make the cones of Fig. 4 suitable for an open belt.

14. Numerous formulas are given for finding corresponding step diameters, but they are approximate, and are, moreover, not easy to apply. The following **graphic method** is very accurate, is easy to use, and is generally considered preferable to methods making use of calculations. It is known as *Burmester's method.*

Let $A Z$, Fig. 9, be a horizontal line, and $A W$ a line at 45° with $A Z$. From A, lay off $A B$ on $A W$ to represent to some scale the distance h between the axes of the cones. From B, lay off $B C$ at right angles to $A B$ and make $B C = \frac{1}{2} A B$. Then, with $A C$ as a radius and A as a center, draw the arc $C D E$. Now, draw any line $X X$ parallel to $A Z$ and cutting $A B$ at O. Locate on the arc $C D E$ any points, as N_1, N_2, N_3, etc., to the right of O, and from these draw the vertical lines $N_1 P_1$, $N_2 P_2$, etc., to cut the line $X X$ in the points P_1, P_2, P_3, etc., and the line $A W$ in the points M_1, M_2, M_3, etc. Now assume that $P_1 N_1$ represents the radius of a step to the same scale that $A B$ represents the distance h; then $O P_1$, or the equal length $P_1 M_1$, represents to the same scale the radius of the step that is the mate to the step $P_1 N_1$. Similarly, $P_2 N_2$ and $Q P_2$, $P_3 N_3$ and $O P_3$, $P_4 N_4$ and $O P_4$ are corresponding radii of steps.

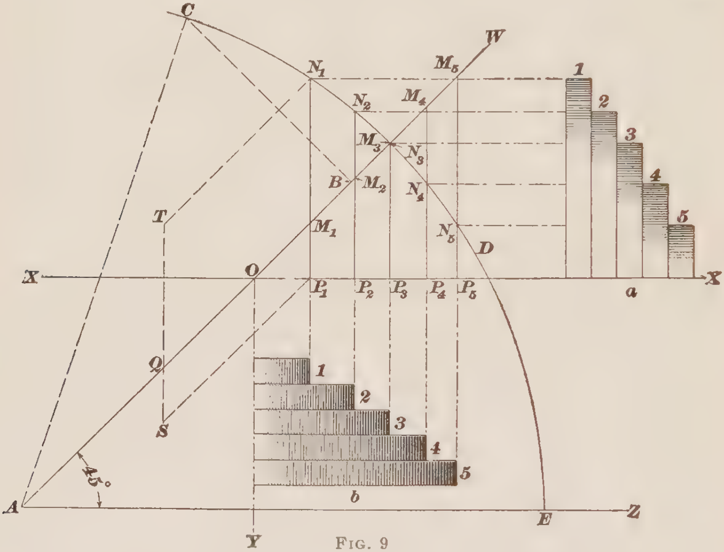

FIG. 9

If $O X$ be taken as the axis of one stepped cone, the steps may be found by projecting horizontally from N_1, N_2, etc. The half cone thus obtained is shown at *a*. The other cone *b* may be laid off on the vertical line *O Y* as an axis by projecting vertically from the points P_1, P_2, P_3, etc. The corresponding steps on *a* and *b* have the same numbers; thus *2* goes with *2*, *3* with *3*, and so on.

The actual sizes of the cones depend on the location of the axis *X X*. If this line is raised so that *O* is nearer *B*, the radii of all steps on both cones will be smaller; and conversely, if *O* is taken nearer *A*, all radii will be greater.

Usually the radii of one pair of steps are given or assumed, and in this case the axis *X X* can be located as follows: Take any point *Q* on *A B* and lay off *Q S* vertically downwards to represent the smaller radius *r*; then from the point *S* lay off *S T* upwards to represent the larger radius *R*. These radii are drawn to the same scale as *A B* is drawn to represent *h*. Now through *S* and *T* draw lines parallel to *A B*, and through N_1, where the line from *T* cuts the arc *C D E*, draw a vertical line cutting the line from *S* in P_1; then through P_1 draw the axis *X X* cutting *A W* in *O*.

15. Frequently, the steps of cones are to be given diameters such that definite velocity ratios, fixed beforehand, will be given as the belt is shifted to different pairs of steps. An example will show how this may be done. Suppose that each cone is to have five steps; that the cones are to be alike, so that they may be cast from the same pattern; and that the velocity ratios for the first two pair are 4 and 2, respectively. The distance *h* is known or assumed. Let it be 40 inches. Since the cones are to be alike and the number of steps is odd, the diameters of the middle steps must be the same; let this diameter be taken as 16 inches.

For an open belt, the other steps are found by the following method: In Fig. 10, make $A B = 40$ inches to some scale, and draw the arc *C D E* as described in connection with Fig. 9. From the intersection *D* drop a vertical line; on this lay off $D P_3$ equal to the radius of the middle step, or $\frac{16}{2}$ inches, to the same scale as *A B*, and through P_3 draw the axis *X X*. On *X X* lay off, from *O*, any convenient length OH, erect a vertical line at H, and then lay off $HK = \frac{1}{4} OH$ and $HL = \frac{1}{2} OH$. Join K and L to O, and let N_1 and N_2 represent the intersections of the lines OK and OL with the arc CDE. Then from N_1 and N_2 drop the perpendiculars $N_1 P_1$ and $N_2 P_2$ on OX. Now OP_1 and $P_1 N_1$ are the first pair of radii, and OP_2 and $P_2 N_2$ the second pair. These pairs give the required velocity ratios 4 and 2; for, since the triangles OP_1N_1 and OHK are similar, the ratio of the radii $R_1 : r_1 = OP_1 : P_1N_1 = OH : HK = 4 : 1$; and since the triangles OP_2N_2 and OHL are similar, $R_2 : r_2 = OP_2 : P_2N_2 = OH : HL = 2 : 1$.

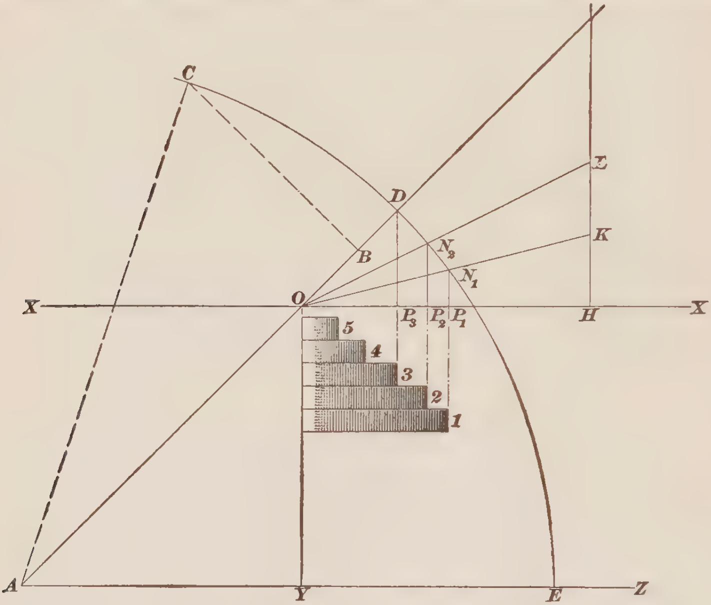

FIG. 10

The lines OP_3 and P_3D, which are equal, give the radii R_3 and r_3 of the middle steps. Projecting P_1, P_2, and P_3 parallel to OY, the steps *1*, *2*, and *3* of the cone *b* are obtained. Now, since the cones are to be alike, step *4* of cone *b* must have the same radius as step *2* of cone *a*, that is, P_2N_2; and step *5* of *b* must have the radius P_1N_1 of step *1* of cone *a*. By measurement, the various radii are as follows:

Steps	*1*	*2*	*3*	*4*	*5*
Cone *a*	$3\frac{1}{16}$ in.	$5\frac{1}{4}$ in.	8 in.	$10\frac{1}{2}$ in.	$12\frac{1}{4}$ in.
Cone *b*	$12\frac{1}{4}$ in.	$10\frac{1}{2}$ in.	8 in.	$5\frac{1}{4}$ in.	$3\frac{1}{16}$ in.

For a crossed belt, the problem is easily solved by a simple calculation. The sum of the radii of the equal middle steps is $R_3 + r_3 = 8 + 8 = 16$ inches, and this sum is constant. That is, $R_1 + r_1 = 16$. But $R_1 = 4\,r_1$ for this pair of pulleys. Then $4\,r_1 + r_1 = 16$, $5\,r_1 = 16$, or $r_1 = 3.2$ inches, and $R_1 = 16 - 3.2 = 12.8$ inches.

Also, $R_2 + r_2 = 16$. But $R_2 = 2\,r_2$ for this pair. Hence, $2\,r_2 + r_2 = 16$, $3\,r_2 = 16$, or $r_2 = 5.33$ inches, and $R_2 = 16 - 5.33 = 10.67$ inches.

If the distance between the axes of the pulleys to be connected by open belt is great, or if, as is sometimes the case, one of the axes is adjustable, the diameters can be calculated as though the belt were crossed. Otherwise, when designed for open belts, they should be laid out as described in Arts. **14** and **15**.

EXAMPLES FOR PRACTICE

1. Two continuous speed cones are required to give a range of speed between 100 and 600 revolutions per minute; assuming the large diameters of the cones to be 14 inches, what must be: (*a*) the small diameters; and (*b*) the speed of the driving shaft? Both cones are to be alike.

$$\text{Ans.}\begin{cases}(a)\ 5.71\text{ in.}\\(b)\ 244.95\text{ R. P. M.}\end{cases}$$

2. In example 1, if the speed of the driving shaft were 260 revolutions per minute, and the slowest speed of the driven cone 140 revolutions: (*a*) what would be the greatest speed of the driven cone? (*b*) what would be the ratio of the large and small diameters of the cones?

$$\text{Ans.}\begin{cases}(a)\ 482.86\text{ R. P. M.}\\(b)\ 1.86 : 1\end{cases}$$

3. The radii of one of two speed cones are 3, $5\frac{1}{2}$, 8, and 10 inches and the radius of the step on the second cone corresponding to the 8-inch step on the first cone is 6 inches. Find the radii of the second cone, the distance between shafts being 30 inches and an open belt being used. Ans. $10\frac{7}{16}$, $8\frac{7}{16}$, 6, and $3\frac{5}{8}$ in.

4. The distance between the axes of two cones is 36 inches and an open belt is used. The two cones are to be alike, and are required to give velocity ratios as follows:

Steps	1	2	3	4	5
Velocity ratio	3	$1\frac{1}{2}$	1	$\frac{2}{3}$	$\frac{1}{3}$

The radii of the first pair are, respectively, 15 inches and 5 inches. Find, by the graphic method, the radii of the other pairs.

	Steps	1	2	3	4	5
Ans.	Cone *a*,	15 in.	$12\frac{15}{32}$ in.	$10\frac{7}{16}$ in.	$8\frac{5}{16}$ in.	5 in.
	Cone *b*,	5 in.	$8\frac{5}{16}$ in.	$10\frac{7}{16}$ in.	$12\frac{15}{32}$ in.	15 in.

5. With the data of example 4, calculate the radii for a pair of cones with a crossed belt.

Ans.	Cone *a*,	15 in.	12 in.	10 in.	8 in.	5 in.
	Cone *b*,	5 in.	8 in.	10 in.	12 in.	15 in.

POWER TRANSMISSION BY BELT

16. The Effective Pull.—In Fig. 11, let *d* and *f* be two pulleys connected by a belt, *d* being the driver and *f* the follower. To avoid undue slipping, the belt must be drawn tight. This will produce tensions in the upper and lower parts, which will be called T_2 and T_1, respectively.

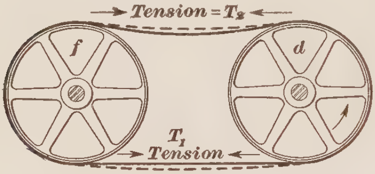

FIG. 11

Suppose the two pulleys to be stationary and that the belt is put on with a certain tension; then, T_1 will be equal to T_2. If, now, the pulley *d* is turned in the direction of the arrow, it tends to stretch the lower part of the belt, increasing its tension still more, while the tension of the upper part is diminished. This goes on until the difference of the tensions is sufficient to start pulley *f*.

The difference $(T_1 - T_2)$ between the tensions in the tight and loose sides of the belt is the force that does the work in transmitting power, and is called the **effective pull.**

If S denotes the speed of a point on the belt, in feet per minute, which is practically the same as the speed of a point on the pulley circumference, the work done per minute is the product of the force $(T_1 - T_2)$ and the distance S through which the point on the belt moves. Hence the work per minute $= (T_1 - T_2)\,S$ foot-pounds. Now, 1 horsepower is the performance of 33,000 foot-pounds per minute; hence, if H is the horsepower transmitted,

$$H = \frac{(T_1 - T_2)\,S}{33{,}000}$$

Example 1.—A pulley 4 feet in diameter is driven at 100 revolutions per minute, and transmits power to another pulley by means of a belt without slip; if the tension on the driving side of the belt is 400 pounds and on the slack side is 100 pounds, what is the horsepower transmitted?

Solution.— $S = 4 \times 3.1416 \times 100 = 1{,}256.6$ ft. per min. From the formula, $H = \frac{(400 - 100) \times 1{,}256.6}{33{,}000} = 11.42$ H. P. Ans.

Example 2.—The diameter of the driving pulley is 36 inches; it makes 150 revolutions per minute and carries a belt transmitting 6 horsepower. What is the effective pull of the belt?

Solution.—Speed of belt, in feet per minute, is $\frac{150 \times 36 \times 3.1416}{12} = 1{,}413.7$. From the formula, $T_1 - T_2 = \frac{33{,}000\ H}{S} = \frac{33{,}000 \times 6}{1{,}413.7} = 140$ lb., nearly. Ans.

17. The Width of the Belt.—A belt should be wide enough to bear safely and for a reasonable length of time the greatest tension that will be put on it; this will be the tension T_1 of the driving side of the belt. As belts are usually laced, or fastened with metallic fasteners, both of which require holes to be punched in the ends, it is customary to use the breaking strength through the lace holes, divided by a suitable factor of safety, as the greatest allowable tension. The average breaking strength for single leather belts, through the lace holes, is 200 pounds per inch of width. This divided by 3, which is a suitable factor of safety for belting, gives $66\frac{2}{3}$ pounds. Thus, in the last example, the tension of the driving side of the belt was assumed to be 400 pounds. Hence, using $66\frac{2}{3}$ pounds as the safe working stress per inch of width, a belt $\frac{400}{66\frac{2}{3}} = 6$ inches wide would be required.

The tension T_1, for any particular case, depends on three things—viz., the effective pull of the belt, the coefficient of friction between the belt and pulley, and the arc of contact of the belt on the smaller pulley. As the equations involving these quantities are somewhat complicated, Table I has been calculated. It will afford a convenient means for finding not only the width of belt for a given horsepower, but the horsepower for a given width as well. In the first column, the arc covered by the belt is stated in degrees, and in the second column in fractional and decimal parts of the circumference covered. The third column gives the greatest allowable values of $(T_1 - T_2)$, or the effective pull, per inch of width, for single leather belts. These values were computed by assuming a value for T_1 of $66\frac{2}{3}$ pounds, and a coefficient of friction of .27. This latter has been found, by experiment, to be a fair value to use for leather belts running over cast-iron pulleys, under conditions met with in practice.

The arc of contact on the smaller pulley of a pair may be very readily found. Let R and r represent the radii of the large and small pulleys, respectively, and let h represent the distance between centers. Then, if the number of degrees in the arc of contact is denoted by a, $\cos\frac{a}{2} = \frac{R - r}{h}$. By substituting the known values of R, r, and h, the cosine of half the angle a is found, and the angle $\frac{a}{2}$ can easily be determined by referring to a table of cosines. This, multiplied by 2, will give the arc of contact, in degrees.

TABLE I

EFFECTIVE PULL OF BELTS

Arc Covered by Belt		Allowable Value of Effective Pull, or $(T_1 - T_2)$ per Inch of Width
Degrees	Fraction of Circumference	
90	$\frac{1}{4} = .25$	23.0
$112\frac{1}{2}$	$\frac{5}{16} = .312$	27.4
120	$\frac{1}{3} = .333$	28.8
135	$\frac{3}{8} = .375$	31.3
150	$\frac{5}{12} = .417$	33.8
$157\frac{1}{2}$	$\frac{7}{16} = .437$	34.9
180 or over	$\frac{1}{2} = .50$	38.1

18. To use the table in finding the width of a single leather belt required for transmitting a given horsepower, the following rule may be applied:

Rule.—*Compute the effective pull of the belt. Divide the result by the suitable effective pull per inch of width, as given in the third column of Table I; the quotient will be the width of belt required, in inches.*

Example.—What width of single belt is needed to transmit 20 horsepower with contact on the small pulley of $\frac{3}{8}$ of the circumference and a speed of 1,500 feet per minute?

Solution.—The effective pull is, from the formula in Art. **16**, $T_1 - T_2 = \frac{33{,}000 \times 20}{1{,}500} = 440$ lb. From Table I, the allowable pull per inch of width is 31.3 lb. Hence the width required is $\frac{440}{31.3} = 14$ in. Ans.

19. Horsepower Transmitted by a Belt.—The process of finding the horsepower that a single belt will transmit must evidently be just the reverse of the preceding. It is, therefore, as follows:

Rule.—*Multiply together a suitable value for the effective pull, taken from Table I, the width of the belt, in inches, and the speed, in feet per minute. The result divided by 33,000 gives the horsepower that the belt will transmit.*

Example.—What horsepower will a 1-inch belt transmit with a speed of 900 feet per minute and an arc of contact of 180°?

Solution.— $T_1 - T_2$, from Table I, $= 38.1$. $38.1 \times 1 \times 900 = 34{,}290$, which, divided by 33,000, gives 1.04 H. P., nearly. Ans.

20. General Rule for Belting.—From the last example, it appears that *a single belt traveling 900 feet per minute will transmit 1 horsepower per inch of width when the arc of contact on the smaller pulley does not vary much from 180°.* This rule may be expressed algebraically as follows:

Let H = horsepower to be transmitted;
W = width of belt, in inches;
S = belt speed, in feet per minute.

Then,

$$H = \frac{WS}{900} \qquad (1)$$

$$W = \frac{900\,H}{S} \qquad (2)$$

$$S = \frac{900\,H}{W} \qquad (3)$$

Example 1.—Two pulleys, 48 inches in diameter, are to be connected by a single belt, and make 200 revolutions per minute; if 40 horsepower is to be transmitted, what must be the width of belt?

Solution.—The belt speed is $\frac{200 \times 48 \times 3.1416}{12} = 2{,}513$ ft. per minute, about. Applying formula **2**, $W = \frac{900 \times 40}{2{,}513} = 14.3$ in. Ans.

A 14-in. belt might safely be used, since the rule gives a liberal width when the pulleys are of equal size.

Example 2.—What size pulleys should be used for a 4-inch belt that is to connect two shafts running at 400 revolutions per minute and transmit 14 horsepower? Both pulleys are to be of the same size.

Solution.—By formula **3**, $S = \frac{900 \times 14}{4} = 3{,}150$ ft. per minute. Since this speed equals the circumference of the required pulley in feet × 400, the circumference of pulley is $\frac{3{,}150 \times 12}{400} = 94.5$ in.; the diameter is $\frac{94.5}{3.1416} = 30$ in. Ans.

21. Double Belts.—**Double belts** are made of two thicknesses of leather cemented together throughout their whole length; they are used where much power is to be transmitted. In Europe it is common practice to rivet as well as cement double belts. As the formulas for single belts are based on the strength through rivet or lace holes, when these are used, a double belt, if twice as thick, should be able to transmit twice as much power as a single belt; and, in fact, more than this, where, as is quite common, the ends of the belt are glued together instead of being laced. Double belts, however, are not usually twice as thick as single belts.

Where double belts are used on small pulleys, the contact with the pulley face is less perfect than it would be if a single belt were used, owing to the greater rigidity of the double belts. More work, also, is required to bend the belt as it runs over the pulley than in the case of the thinner and more pliable belt, and the centrifugal force tending to throw the belt from the pulley also increases with the thickness. Moreover, in practice, it is seldom that a double belt is put on with twice the tension of a single belt. For these reasons, the width of a double belt required to transmit a given horsepower is generally assumed to be about seven-tenths the width of a single belt to transmit the same power. On this basis, formulas **1**, **2**, and **3** of Art. **20** become, for double belts

$$H = \frac{W S}{630} \qquad \textbf{(1)}$$

$$W = \frac{630\,H}{S} \qquad \textbf{(2)}$$

$$S = \frac{630\,H}{W} \qquad \textbf{(3)}$$

EXAMPLES FOR PRACTICE

1. If the effective pull on a belt per inch of width is 50 pounds, and the belt passes over a pulley 36 inches in diameter, which makes 160 revolutions per minute, how wide should the belt be to transmit 12 horsepower? Ans. $5\frac{1}{4}$ in.

2. What width of single belt should be used to transmit 5 horsepower, when the belt speed is 2,000 feet per minute, and the arc of contact on the smaller pulley is 90°? Ans. $3\frac{1}{2}+$ in.

3. Using the general rule, find the horsepower that a 16-inch single belt will transmit, the belt speed being 1,000 feet per minute. Ans. 17.8 H. P.

4. Using Table I, how much power could the above belt be depended on to transmit if the arc of contact on the smaller pulley is $\frac{1}{3}$ of the circumference? Ans. 14 H. P., nearly

5. Required the diameters of pulleys necessary to enable a 10-inch belt to transmit 9 horsepower at 125 revolutions per minute, both pulleys being of the same size. Ans. 2 ft., nearly

6. How much power would the belt in example 3 transmit if the belt were double? Ans. 25.4 H. P., nearly

CARE AND USE OF BELTING

22. The belts most commonly used are made of leather, single and double, but canvas belts covered with rubber are sometimes used, especially in damp places, where the moisture would ruin the leather.

Leather belts are generally run with the hair or grain side next to the pulley. This side is harder and more liable to crack than the flesh side. By running it on the inside the tendency is to cramp or compress it as it passes over the pulley, while if it ran on the outside, the tendency would be for it to stretch and crack. Moreover, as the flesh side is the stronger side, the life of the belt will be longer if the wear comes on the weaker or grain side.

The lower side of a belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then tend to increase the arc of contact, as illustrated by the full lines in Fig. 11. Long belts, running in any direction other than the vertical, work better than short ones, as their weight holds them more firmly to the pulleys.

It is bad practice to use rosin to prevent slipping. It gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt properly cared for persists in slipping, a wider belt or larger pulleys should be used, the latter to increase the belt speed. Belts, to be kept soft and pliable, should be oiled with castor or neat's-foot oil, or other suitable belt dressing. Mineral oils are not good for this purpose.

Tightening or **guide pulleys,** whenever used to increase the length of contact between the belt and pulley or to tighten a belt, should be placed on the slack side, if possible. Thus placed, the extra friction of the guide pulley bearings and the wear and tear of the belt that would result from the greater tension of the driving side are avoided.

23. Guiding Belts.—When a belt is to be shifted from one pulley to another, or must be guided to prevent running off the pulley, the fork or other device used for guiding should be close to the driven pulley, and so placed as to guide the advancing side of the belt.

This principle is sometimes made use of where pulleys have flanges to keep the belt from running off the pulleys. Where constructed with straight flanges, as in Fig. 12, if the belt has any inclination to run on one side, its tendency is to crowd up against the flange as shown at *a*, *a*. When constructed as in Fig. 13, however, with the flanges grooved as at *c*, *c*, the advancing side of the belt will be guided at *b*, just as it reaches the pulley, by contact with the thick portion of the flange, and during the rest of the way will not touch the flange at all.

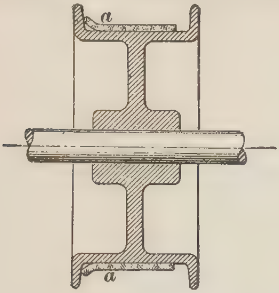

Fig. 12

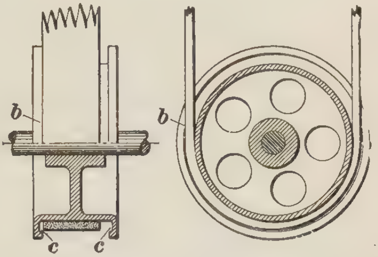

Fig. 13

In Fig. 14 is shown the arrangement of a belt shifter; *g* is the driving pulley, and *t* and *l* are tight and loose pulleys on the driven shaft *c d*; *b* is the shifter, and can be moved parallel to *c d*. The acting surface, or *face*, of *g* is made straight to allow the belt to shift readily, and the faces of *t* and *l* are **crowned**—that is, the diameters of the pulleys are increased slightly toward the centers of the pulley faces—so that the belt will not tend to run off.

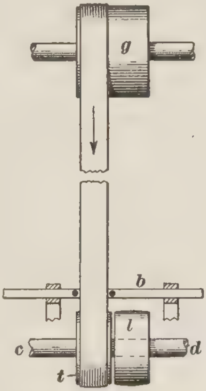

Fig. 14

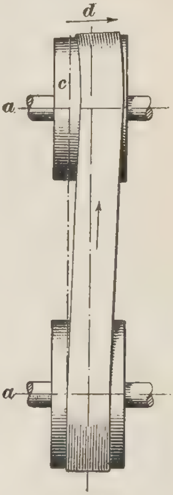

Fig. 15

24. The Climbing of Belts.—In Fig. 15, suppose the shafts *a*, *a* to be parallel, and the pulley *d* to be cone-shaped. The right-hand side of the belt will be pulled ahead more rapidly than the left-hand side, because of the greater diameter and consequent greater speed of that part of the pulley. The belt will, therefore, leave its normal line at *c*, and climb to the high side of the pulley. This tendency is taken advantage of by crowning pulleys in the middle. Each side of the belt then tends to move toward the middle of the pulley; that is, the tendency is for the belt to stay on the pulley.

25. Belt Fastenings.—There are many good methods of fastening the ends of belts together, but lacing is generally used, as it is flexible like the belt itself, and runs noiselessly over the pulleys. The ends to be laced should be cut squarely across, and the holes in each end for the lacings should be exactly opposite each other when the ends are brought together. Very narrow belts, or belts having only a small amount of power to transmit, usually have only one row of holes punched in each end, as in Fig. 16; *a* is the outside of the belt, and *b* the side running next to the pulley. The lace is drawn half way through one of the middle holes, from the under side, as at *1*; the upper end is then passed through *2*, under the belt, and up through *3*, back again through *2* and *3*, through *4* and up through *5*, where an incision is made in one side of the lace, forming a barb that will prevent the end from pulling through. The other side of the belt is laced with the other end, first passing it up through *4*. Unless the belt is very narrow, the lacing of both sides should be carried on at once.

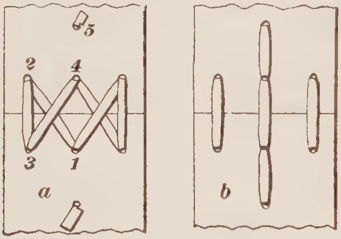

Fig. 16

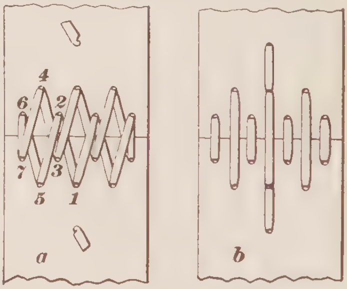

Fig. 17

Fig. 17 shows a method of lacing where double lace holes are used, *b* being the side to run next to the pulley. The lacing for the left side is begun at *1*, and continues through

2, 3, 4, 5, 6, 7, 4, 5, etc. A 6-inch belt should have seven holes, four in the row nearest the end, and a 10-inch belt should have nine holes. The edge of the holes should be at least $\frac{3}{4}$ inch from the sides, and the holes should not be nearer than $\frac{7}{8}$ inch to the ends of the belt. The second row should be at least $1\frac{3}{4}$ inches from the end.

Another method is to begin the lacing at one side instead of in the middle; this method will give the rows of lacing on the under side of the belt the same thickness all the way across.

It is also quite common to cement the ends of a belt instead of lacing them together. To do this, the ends to be cemented are shaved off, with a uniform taper, for a distance about equal to the width of the belt, then coated with cement and laid together, and subjected to pressure until the joint is dry.

BELT CONNECTIONS FOR NON-PARALLEL SHAFTS

26. It very frequently happens that one shaft must drive another at an angle with it. Sometimes this involves the use of guide pulleys, and occasionally guide pulleys must be used to connect parallel shafts, where the shafts are near together, or there is some obstruction in the way. In all such cases *the point at which the center of the belt is delivered from each pulley must lie in the middle plane of the other pulley.*

The middle plane of a pulley is the plane through the center of the pulley, perpendicular to its axis. Unless the shafting is to turn backwards at times, it is immaterial in what direction a belt *leaves* a pulley; but it must always be *delivered* into the plane of the pulley toward which it is running. If it is necessary for a belt to run backwards as well as forwards, it must also *leave* in the plane of the pulley. This principle applies to the guide pulleys as well as to the main pulleys.

27. Shafts at Right Angles.—The common method of connecting shafts at right angles is by means of a **quarter-turn belt,** as shown in Fig. 18. Here, the pulley *d* is the driver, rotating in the direction shown, and the pulley *f* is the follower. The point at which the belt is delivered from the pulley *d* lies in the middle plane of the pulley *f*, that is, in the line *B B*; also, the point at which the belt is delivered from the pulley *f* lies in the middle plane of the pulley *d*, or in the line *A A*. Thus arranged, the belt will run from *d* to *f* without running off either pulley. The reason for this will be made clearer by referring to Fig. 19, in which the belt is represented by *A B* and *C D*, and is traveling in the direction of the arrows. The two shafts being parallel, and the pulleys lying in the same plane, the belt runs from *A* to *B* and from *C* to *D* with no tendency to run off. Let a line *A C* be drawn tangent to the pulleys at the middle points of their faces, where the belt leaves the pulleys. This line may then be considered as an axis around which the pulley *B C* may swing without affecting the action of the belt. For, suppose the pulley *B C* to be swung on the axis *A C* through a right angle, that is, into the position of pulley *f*, Fig. 18. Then, since the point *A* will still lie in the middle plane of

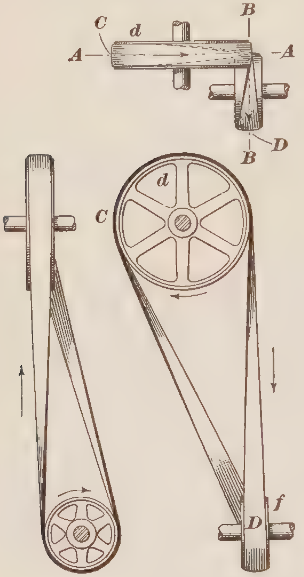

FIG. 18

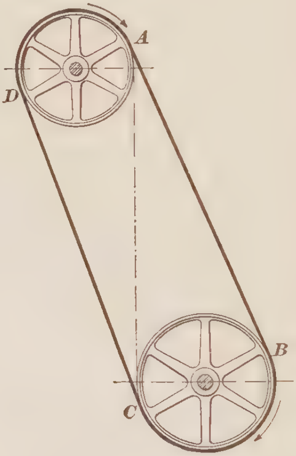

FIG. 19

B C, and the point *C* will still lie in the middle plane of *A D*, the action of the belt will remain unchanged; that is, the center of the belt at the point where it leaves either pulley will lie in the middle plane of the other pulley, and there will be no tendency to run off. If the pulley *B C* be turned through any other angle about *A C*, the required condition will be fulfilled and the belt will run properly.

With the arrangement shown in Fig. 18, however, the pulleys cannot run backwards, because *D*, the point of delivery of *f*, is not in the middle plane *A A*, and *C* is not in the middle plane *B B*.

The following simple method of locating the pulleys for a quarter-turn belt may be used in practice: Locate the pulley *d* and the machine so that the pulleys *d* and *f* will be as near the correct position as can be judged by the eye. Using a plumb-bob, drop a plumb-line from the center of the right-hand side of the pulley *d*, and move the machine until the center of the *back* side of the pulley *f* touches the plumb-line. In case it should not be convenient to move the machine, shift the pulleys instead. If it be desired to run the belt in the direction opposite to that indicated by the arrow in Fig. 18, shift the machine carrying *f* to the left until the center of the *front* side of the pulley *f* touches a plumb-line dropped from the center of the left-hand side of the pulley *d*; that is, from the point *C*.

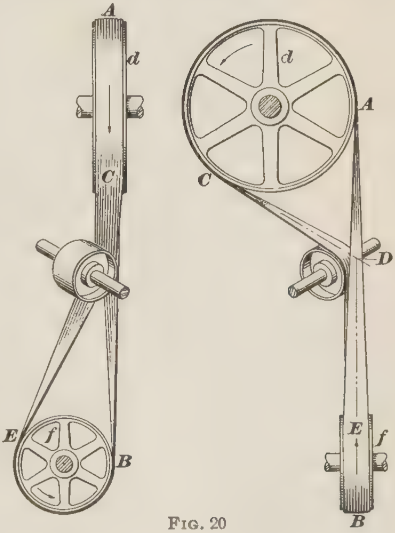

FIG. 20

28. There is this objection to a quarter-turn belt: when the angle at which the belt is drawn off the pulleys is large, the belt is strained, especially at the edges, and it does not hug the pulleys well.

Small pulleys placed quite a distance apart, with narrow belts, give the best results, from which it follows that quarter-turn belts, like the foregoing, are not well suited to transmit much power. Fig. 20 shows how the arrangement can be improved by placing a guide pulley against the loose side of the belt. The driver *d* rotates counterclockwise, thus making *A B* the driving or tight side of the belt. To determine the position of the guide pulley, select some point in the line *A B*, as *D*; draw lines *C D* and *E D*; the middle plane of the guide pulley should then pass through the two lines. Looked at from a direction at right angles to pulley *f*, line *C D* coincides with *A B*; looking at right angles to pulley *d*, line *E D* coincides with *A B*.

29. A third method of connecting two shafts at right angles is shown in Fig. 21. In general, it is to be preferred

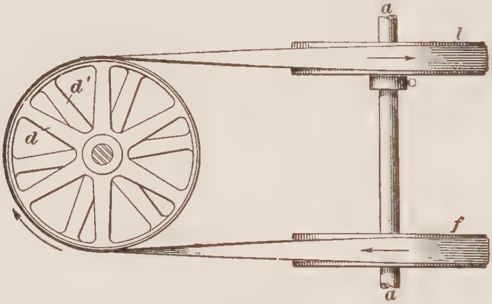

FIG. 21

to the quarter-turn belt. The pulley *d* is the driver. The belt passes around the loose pulley *l* and back around a loose pulley *d'* on the driving shaft behind *d*. It then goes around *f*, which is fast on the shaft *a a*, and finally back again and around *d*. Since the loose pulleys turn in a direction opposite that of their shafts, their hubs should be long. The two pulleys on each shaft must be of the same size. It is evident that either *f* or *d* can be the driver and can run in either direction.

It is to be observed that while a quarter-turn belt can be used with the shafts at an angle other than a right angle, the last arrangement cannot.

In Fig. 22 is shown a method of connecting the shafts when it is not possible to put the follower *f* directly under the driver *d*. The guide pulleys *g*, *g′* must be so placed that the belt will lead correctly from the point *A* into the middle plane of the guide pulley *g′*, from *B* into the middle plane of *d*, and so on around. By twisting the belt at *c*, the same side will come in contact with all the pulleys; this is a desirable arrangement.

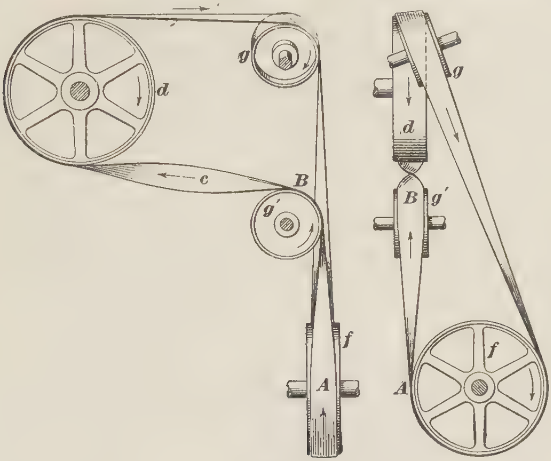

Fig. 22

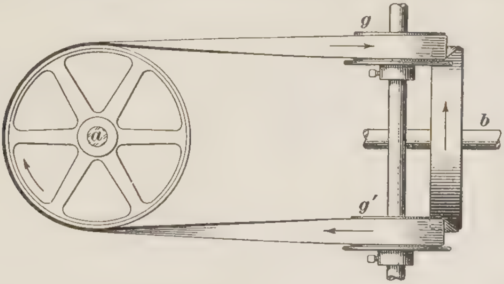

Fig. 23

In the arrangement shown in Fig. 23, the shafts *a* and *b* would intersect, if long enough. Examples occur where shafts running on two sides of a room are to be connected. Guide pulleys *g*, *g′*, like those in the figure, termed **mule pulleys,** are used. As their planes are horizontal, means must be provided to prevent the belt from running off at the bottom. Sometimes this is done by simply crowning the pulleys, and sometimes by putting flanges on the lower sides.

30. Other Examples of Belt Transmission.—Guide pulleys are sometimes used to lengthen the belt between two shafts that are too close together to be connected directly, or it may be that it is not possible to get two pulleys in the same plane. Fig. 24 shows an arrangement of this kind. The diameter of each guide pulley should be equal to the distance between the planes of d and f. With the guide pulleys arranged as shown, the belt will run in either direction. It is more convenient, however, to place the guide pulleys on one shaft. In that case, their axes will be on the line $O O$; g' will be in the line $C K$, and g in the line $A B$. Then the belt will be delivered from d into the middle plane

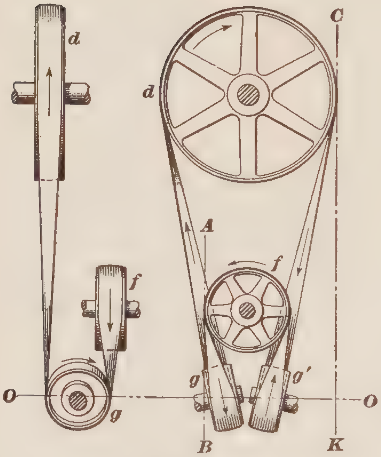

Fig. 24

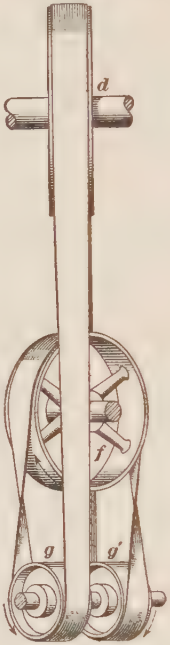

Fig. 25

of g'; and from g' into the middle plane of f. The belt will run in only one direction, however.

A device for connecting two horizontal shafts making an angle with each other is given in Fig. 25. It can be used where a quarter-turn belt will not work successfully. The guide pulleys turn in the same direction, which is a convenience, because they can then be mounted on one shaft, turning in bearings at the ends, and the belt will run in either direction.

MATERIALS OF CONSTRUCTION

IRON AND STEEL

INTRODUCTION

1. Properties of Iron.—A knowledge of the physical properties of the different materials used in machine construction is exceedingly important to the designer and builder of machinery, since it enables him to select the materials best suited for the various parts of machines.

The materials most largely used in machine construction are *iron* and *steel*. **Iron** may be of two kinds—*cast iron* and *wrought iron*. Cast iron can be melted and poured into molds, after which it again solidifies. This process is known as *casting*. Wrought iron can be heated until it becomes plastic, when it may be worked into various shapes, either under a hammer or in a press.

Steel is simply a special form of iron, and may be divided into three classes, known as *machinery steel*, *tool steel*, and *steel castings*. Machinery steel is the ordinary steel used for machine parts, such as shafts; tool steel is less easily worked, but is capable of being hardened and tempered, and is largely used for tools; while the steel used for making castings differs from the other two varieties in that it can be cast in molds.

In order to become familiar with the physical properties of iron and steel, it is desirable to know something of their metallurgy; that is, of the process employed in their manufacture. The methods of producing iron and steel have marked effects on their qualities, while the presence of small

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quantities of other elements, sometimes as impurities, may have still greater effects. One element that has a very important bearing on the properties of iron and steel is carbon. Other substances affecting them in a greater or less degree are sulphur, phosphorus, tungsten, nickel, chromium, and manganese. Small percentages of some of these elements will frequently produce marked changes in the characteristics of the finished product. These characteristics will be taken up later.

IRON

PRODUCTION OF IRON

2. Ores of Iron.—Iron exists in nature as ore, which is a combination of iron and other elements in the form of rock or earth. Frequently, the iron is not distinguishable except by chemical analysis. The only ores from which iron is manufactured in large quantities are those containing the oxides and carbonates of iron, the oxides being the richer in iron. The ore known as *magnetite*, Fe_3O_4, contains about 72 per cent. of iron; that known as *red hematite*, Fe_2O_3, contains about 70 per cent. of iron; while another ore, *brown hematite*, $2Fe_2O_3$, $3H_2O$, contains about 60 per cent. of iron. These constitute the valuable oxide ores. The carbonate ore, *ferrous carbonate*, $FeCO_3$, contains about 48 per cent. of iron.

Magnetite is black and brittle, and is attracted by a magnet, from which characteristic it derives its name. Red hematite varies in color from a deep red to a steel gray, but all varieties make a red streak when drawn across unglazed porcelain. On account of its abundance and the character of the iron it yields, red hematite is the most important of the ores of iron. Brown hematite varies in color from a brownish black to a yellowish brown. The carbonate varies in color from yellow to brown, but the light-colored ore rapidly becomes brown when exposed to the air. This ore is reduced to Fe_2O_3 by roasting and exposure to the air, which drives off the carbon dioxide and water, as well as much of the sulphur and arsenic, when these are present. The carbonate is thus changed to an oxide having the same composition as red hematite.

3. Separation of Iron From Its Ores.—Iron in the metallic form is extracted from the ores by the action of heat. The ores are first heated at a comparatively low temperature, so as to drive off all moisture and volatile matter that may be present. Then they are heated to a comparatively high temperature, in the presence of carbon, C, or carbon monoxide, CO. At this higher temperature, the oxygen of the ore combines with the carbon or carbon monoxide to form carbon dioxide, CO_2, thus leaving the iron chemically free. This process is known as the **reduction** process, since the ores are reduced, or deprived of their oxygen and other non-metallic substances, leaving the metal itself quite free.

The fuels used to produce the high temperatures in the reduction process are wood, soft coal, hard coal, coke, and gas. The heating is done in a furnace lined with some refractory material, such as firebrick, which is not easily affected by great heat. The ores charged into the furnace usually contain silica, alumina, or some other substance that is difficult to remove, except by chemical combination with another substance put in especially for that purpose and known as a **flux.** The flux unites with the impurities in the ore and becomes fluid, when it is known as **slag.** The slag is lighter than the molten iron, and consequently floats on the surface of the iron, from which it may be removed either by tapping off the iron from below and leaving the slag to be drawn later, or by drawing off the slag through a tap hole at the surface of the molten iron, without disturbing the iron.

4. The Blast Furnace.—All iron used for manufacturing purposes is obtained by reducing the ores of iron in a special furnace called a **blast furnace.** Fig. 1 shows the common form of blast furnace used in this work. The furnace itself is shown at *a*. At *b* and *c* are stoves, or devices

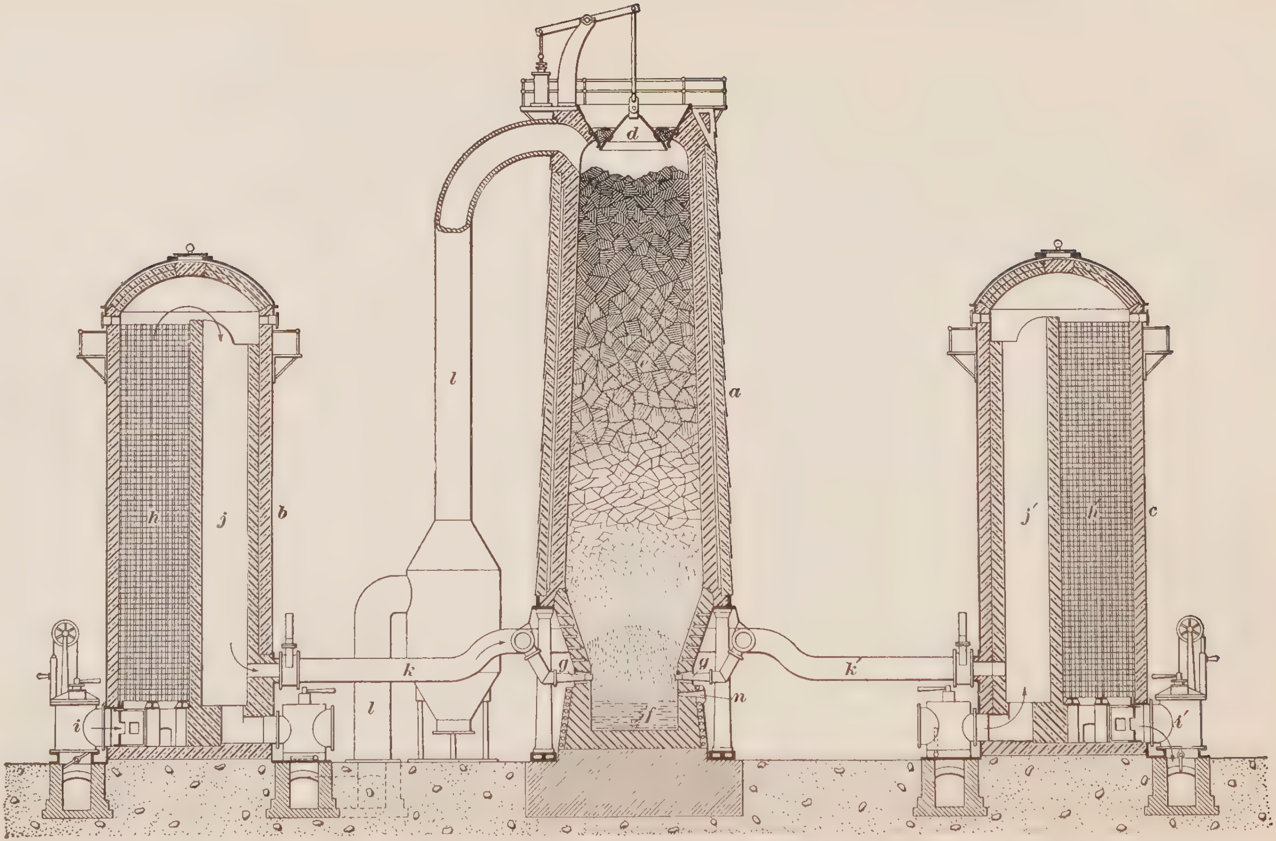

FIG. 1

for heating the air before it enters the furnace. The furnace consists of a slightly tapering shell, built up of iron plates and lined with firebrick. The lower end of this shell is cone-shaped, with the small end of the cone pointing downwards. When the furnace is in operation, this is the region of highest temperatures where most of the melting takes place.

The ore, fuel, and flux are charged into the furnace at the top, through an opening that is closed by the **bell** *d* to prevent the escape of gases. In ordinary operation, the furnace is kept filled with the charge to about the level shown in the figure. The lowest temperatures exist near the top of the furnace, the mass growing gradually hotter as it descends. As fast as the metal at the bottom melts, the charge above settles, being regularly replenished by charging at the top. Thus there is a continual downward movement of the charge.

The low temperatures existing at the top of the furnace, where the fresh ore lies, are sufficient to dry the ore and drive off volatile substances. Sometimes, however, the ore is roasted in a separate furnace before being charged into the blast furnace. This roasting not only serves to drive off volatile matter and moisture, but also oxidizes much of the impurities contained in the ore, since the process is carried on in the presence of a plentiful supply of air.

As the charge in the blast furnace descends into the zone of greater temperatures, the reduction process takes place, and the molten iron collects at the bottom. The slag, which contains much of the impurities and undesirable substances carried in the ore, collects just above the mass of molten iron, since it is of less specific gravity than iron. This is especially advantageous since it protects the iron from the oxidizing effect of the hot blast entering through the **tuyères** at *g*. The slag is drawn off at intervals through the **cinder notch** *n*, which is slightly below the level of the tuyères.

The air blast, by means of which the combustion is hastened and the higher temperatures obtained, is furnished by blowing engines. It enters near the base of the furnace, through the tuyères, at a pressure of from 5 to 15 pounds per square inch. As the blast passes up through the charge, the oxygen of the heated air combines with the carbon of the fuel, forming carbon monoxide, CO, since the amount of oxygen present is insufficient to form carbon dioxide, CO_2. As carbon monoxide is quite combustible, it is conveyed by the pipe *l* from the top of the furnace to one of the reheating stoves, as *b*, instead of being allowed to escape at once into the atmosphere.

5. The stove is an iron shell lined with firebrick and containing a **combustion chamber** *j* and **checkerwork** *h*. This checkerwork is simply a column of firebrick, so arranged as to form large spaces between the bricks. The carbon monoxide from the blast furnace is led into the combustion chamber and additional air is there supplied to cause the CO to burn to CO_2. The hot gases resulting from this combustion pass upwards, and then downwards through the checkerwork, thus heating it, after which they pass out through the opening *i* to the stack.

As soon as the checkerwork in *b* is heated, valves in the pipes are turned so that the gases from the top of the furnace are led to the other stove, *c*. At the same time, the cold-air blast from the blowing engines is allowed to enter at *i*, pass through the checkerwork *h*, become heated by contact, and thence pass out through *j* and the pipe *k* to the furnace tuyères. As soon as the checkerwork becomes comparatively cool, the valves are again changed, and the cold air is driven through the stove *c*, which by this time has become thoroughly heated, and the stove *b* is again heated up. By this arrangement, a hot-air blast is supplied to the furnace, thus saving much heat that would otherwise be wasted, and at the same time insuring better operation. There should be at least three stoves for each furnace, so that, in case one stove should need repairs, two others would be available, enabling the furnace to run without interruption. In some cases, four, or even five, stoves are used. By frequently changing the air blast from one stove to another, a fairly constant blast temperature may be maintained.

6. Pig Iron.—After the blast furnace has been started it is continuous in its operation, the molten iron being drawn off at regular intervals through a tap hole in the bottom of the furnace. In the sand floor that surrounds the base of the furnace and sloping downwards and away from it, a long trench is dug, leading away from the tap hole from which the molten iron is drawn. From this trench, branch trenches are dug at intervals, and these branches lead to numerous smaller trenches or molds about 3 feet long and from 3 to 4 inches wide and deep. When sufficient molten iron has collected in the bottom of the furnace, the blast is shut off, the tap hole is opened, and the iron runs out, filling the trenches and molds. The tap hole is then plugged, and the blast again turned on. The molten iron, on cooling, is known as **pig iron.**

Pig iron contains from 3 to 10 per cent. of impurities, of which the larger part is carbon, although silicon, sulphur, manganese, phosphorus, and other elements may be present. Pig iron is usually classified according to its condition, the impurities it contains, and the purposes for which it is to be used. It is especially valuable because it melts and becomes quite fluid at a temperature of about 2,200° F., which is readily attainable in the foundry cupola. This property of pig iron, combined with its relative cheapness and its extensive use in the manufacture of wrought iron and steel, makes it the most useful form of iron.

CAST IRON

7. The Cupola.—In making iron castings, pig iron is melted in a special form of melting furnace called a **cupola,** and is then poured into molds of sand. After it has solidified, it is known as **cast iron.** The pig iron used in foundry work and melted in the cupola is known as *foundry pig.*

A form of cupola largely used in foundry work is shown in Fig. 2. The outer shell *a* is made of iron plates firmly riveted together and lined with firebrick. The base *b* is made of cast iron, and contains a pair of doors, which are

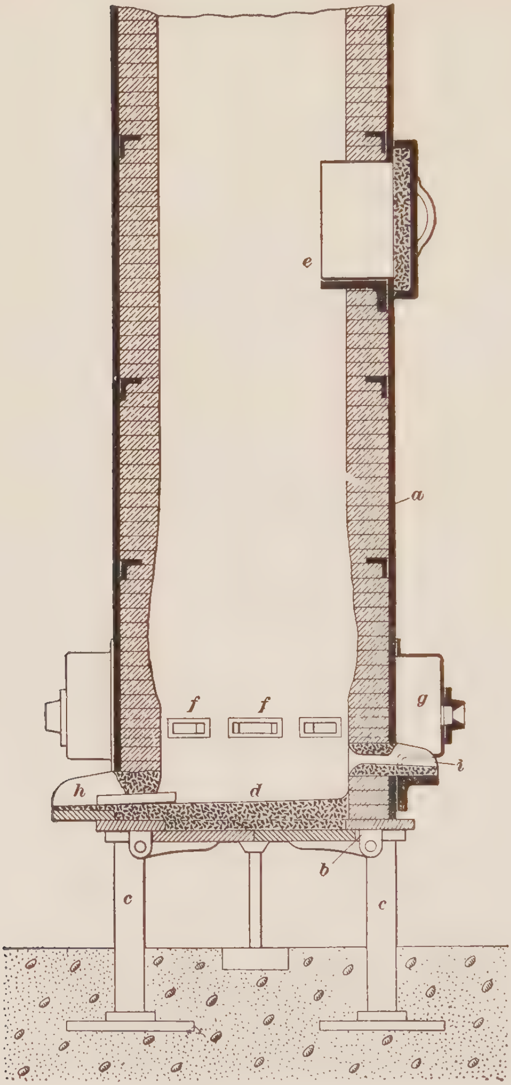

FIG. 2

kept in a closed position by an iron bar or strut. At the end of a run or cast, the strut is knocked out, and the doors, opening downwards, allow all the slag and iron remaining in the cupola to drop out.

The cupola is supported on columns *c*, *c* and has a total height, from the sand bottom *d* to the charging door *e*, of from 3 to 3½ times the inside diameter, the latter varying from 22 to 100 inches. At *f*, *f* are shown the tuyères, which conduct the air blast from the wind belt *g* to the interior of the cupola. The molten iron is drawn off into ladles through the tap hole *h*, and is then carried to the molds, while the slag is removed through the tap hole *i*.

In preparing to operate the cupola, the bottom of the furnace is first covered with sand, which is rammed down hard. A good fire of soft light wood is then built on the sand bottom and coke is thrown in. As soon as the coke has begun to burn well, alternate charges of coke and pig iron are put in through the charging door *e*. The iron is usually broken up before being charged. As the iron in the lower end of the cupola melts, the mass slowly settles, and fresh charges of coke and iron are added at the top. The air blast is furnished by a fan or a blower.

8. Characteristics of Cast Iron.—Cast iron is a metal of crystalline formation, very strong in compression and comparatively weak in tension. There are several grades of cast iron, differing chiefly in the amount of carbon contained, although distinctive properties are given to the iron by other elements, such as silicon, sulphur, phosphorus, and manganese.

9. Carbon in Cast Iron.—Cast iron usually contains from 2 to 4½ per cent. of carbon, but when there is a large percentage of manganese present there may be 6 per cent. or more of carbon. The quality of cast iron depends largely on the condition of the carbon present in the iron. Carbon exists in cast iron in two forms, namely, *combined carbon* and *graphitic carbon*. In iron containing combined carbon, the appearance of the fracture, when a piece is broken, is silvery white; while, if an iron containing graphitic carbon is broken, it shows a dark-gray fracture.

The melting point of gray iron is about 2,200° F.; and of white iron, about 2,000° F. The average value of the tensile strength of cast iron is about 20,000 pounds per square inch, and of the compressive strength about 90,000 pounds per square inch. The gray iron, which contains graphitic carbon, is weaker than the white iron, which contains combined carbon, in both tension and compression. But since the white iron is very hard and brittle, it is difficult to work, and consequently the softer gray iron is most generally used. A cubic foot of dark-gray iron weighs about 425 pounds, and a cubic foot of white iron about 475 pounds. The appearance of the fracture of the different grades of cast iron varies from a coarse semicrystalline gray to a fine close-grained white.

10. Silicon, Sulphur, Phosphorus, and Manganese in Cast Iron.—Silicon in small proportions tends to increase the strength and hardness of cast iron, and aids in preventing the formation of blowholes in castings. Sulphur tends to make the iron hard and brittle, and a high percentage causes blowholes. On the other hand, this element makes the iron more fusible. Phosphorus also makes the iron more fusible, tends to prevent blowholes, and is supposed to prevent shrinkage in cooling, so that the iron fills the mold more perfectly. On the other hand, it makes the iron brittle and liable to break under suddenly applied loads. Manganese has the property of increasing the amount of combined carbon in cast iron and so gives a harder iron, rendering it less plastic and more brittle. It also increases the shrinkage. However, it unites readily with sulphur, and thus tends to remove the latter from iron.

11. Change of Volume of Cast Iron.—It is well known that cast iron shrinks while cooling in the molds. The amount of shrinkage or contraction depends on the size and shape of the casting, the composition of the iron, and the rate of cooling. Light bars contract more than heavy ones, and slow cooling causes less contraction than rapid cooling, while a white hard iron shrinks more than a gray soft one. The usual allowance on a pattern for the shrinkage of an iron casting is $\frac{1}{8}$ inch per foot, but this does not apply in all cases. In fact, it is customary for the patternmaker to allow for shrinkage according to the methods used in the foundry and the condition of the iron used. The shrinkage usually varies from $\frac{1}{16}$ inch to $\frac{3}{16}$ inch per foot. The fact that castings of different thicknesses do not shrink at the same rate makes it necessary to use care when a casting of varying thickness is made.

It has been found that the volume of a piece of cast iron can be increased by alternately heating it to redness and allowing it to cool slowly. In some cases, the volume has been increased as much as 40 per cent. The strength, however, is reduced by this process.

12. Chilled Castings.—In many cases, when a casting is to be subjected to wear, it is desirable to have the surface quite hard. This is accomplished by pouring the molten iron into molds that have their faces lined with thick metal plates. As soon as the molten iron touches these metallic surfaces, it is chilled, the heat being conducted away quite rapidly. Sometimes the iron parts of the mold are water-jacketed, water being circulated through them to increase the chilling effect.

The result of this rapid chilling of the casting is to form a hard, close-grained surface that is admirably adapted to resist wear. Castings made in this way are called **chilled castings,** and the iron faces of the mold, by which the sudden cooling is effected, are called **chills.** The term *chill* is applied also to the hardened portion of the casting, the depth of the chill meaning the depth to which the hardened part extends.

The depth of the chill depends on the composition of the iron and the effectiveness of the chills used in the mold. Silicon, manganese, and sulphur have considerable influence on the depth of the chill. A casting made of iron poured while quite hot will chill deeper than one poured from cooler iron, and a heavy casting will chill to a less depth than a light one, the thickness of the chills in the mold being the same in each case. The fracture of the chilled portion of a casting is white, since the greater part of the carbon is in the combined form.

13. Malleable Castings.—The ordinary iron casting cannot be bent to any extent without breaking, as it is quite brittle. In order to render cast iron tough and pliable, it may be annealed, after which process it is known as a **malleable casting.** A malleable casting is made of a special grade of cast iron chilled in a sand mold and then annealed. The annealing process changes the form of the carbon contained in the iron, and a portion of it near the surface may be removed, thus changing the properties of the iron.

The castings before they are annealed are white, the carbon being in the combined form, and they are very hard and brittle. These castings are annealed by being packed in cast-iron boxes, surrounded by oxide of iron, and heated to a redness for a period of from 4 to 6 days. This process can also be performed in sand or fireclay, but the skin of the casting will not be decarbonized to the same extent as when packed in oxide of iron.

The elements found in malleable castings are the same as those in ordinary cast iron, known as gray iron, but they vary in quantity. The composition of good malleable castings should be within the following limits:

ELEMENT	PER CENT.
Silicon	.35 to 1.25
Manganese	.15 to .30
Phosphorus	.08 to .25
Sulphur	.02 to .07
Total carbon	1.50 to 4.20

14. In malleable iron there are three forms of carbon; namely, combined carbon, graphitic carbon, and temper carbon. The carbon of the white hard casting should be, as far as possible, in the combined form, for, if there is sufficient graphitic carbon to make the fracture appear light gray, the malleable casting made from it will be weaker than an ordinary gray casting.

In the process of annealing, the greater part of the combined carbon is changed to an uncombined form known as **temper carbon,** differing somewhat from the graphitic form. In forming the temper carbon, the casting expands an amount equal to about one-half of its original contraction in the mold. The malleable casting becomes a network of soft steel, formed about the particles of temper carbon, with the result that the iron becomes more ductile and may be readily bent. The casting really is converted into a piece of soft steel having about $\frac{1}{2}$ per cent. of combined carbon, the remainder of the combined carbon being changed to temper carbon, which is enclosed by the crystals of the iron. The effect is to make a casting weaker than solid steel, but still about twice as strong as ordinary cast iron.

15. A good malleable casting has a velvety-black interior, surrounded by a dark-gray band enclosed in a thin white band. A malleable casting showing a dull-gray fracture with a banded structure is generally weak. If the iron has received excessive annealing, the fracture will be almost entirely white, and will have a black spot at the center. If the interior has white spots, resembling flakes, radiating from the center, the casting is probably insufficiently annealed.

Malleable iron can be bent and worked to some extent like wrought iron, but it cannot be welded. It should not be used where it is liable to be subjected to high temperatures, as its strength will be greatly injured. The tensile strength of a good piece of malleable iron should be from 37,000 to 45,000 pounds per square inch, although iron having a tensile strength of only 35,000 pounds may be used. The tensile strength of malleable iron may run as high as 52,000 pounds per square inch, but this usually occurs in a casting that is weak in resisting shocks. A high tensile strength is obtained by adding from 2 to 5 per cent. of steel scrap to the mixture from which the casting is made.

16. Case-Hardening Malleable Castings.—The annealing process usually affects the white-iron casting to a depth of about $\frac{3}{16}$ inch, although thin castings are rendered malleable throughout. As the skin of the casting is practically soft steel, it is very low in carbon, but it may be enriched in carbon by the case-hardening process. After this process, the casting may be hardened and tempered. Castings thus treated are used largely in some grades of hardware, being much cheaper than when forged from steel. The carbonizing may be done either by dipping the casting into melted high-carbon steel, or by applying yellow prussiate of potash to the red-hot casting. Castings whose surfaces already contain sufficient carbon may be hardened by being heated and then plunged into water. If hardened malleable castings are heated and then cooled slowly, they again become soft.

WROUGHT IRON

17. Purity of Wrought Iron.—Of all the forms of iron obtained from the ores by processes of manufacture, **wrought iron** is the purest. It not only contains very little or no carbon, but the best grades are also free from the other impurities so common to cast iron and steel. Wrought iron can be produced either from the ore directly or by the conversion of pig iron in a reverberatory furnace. In the latter process, called the *puddling* process, white pig iron is melted and subjected to an oxidizing flame until the carbon is burned out or becomes less than .25 per cent.

18. Puddling Furnace.—The most common type of reverberatory, or **puddling, furnace** is shown in Fig. 3. The hearth *a* is made large enough to hold from 1,000 to 1,500 pounds of molten metal. A door *b* is built into the side wall of the furnace. Through this door the iron is charged into the furnace, and through it the workman, or

puddler, watches the iron and works it as required. The fire is built on the grate *c*, the fuel being supplied through the opening *d*. The air supply enters through the ash-pit *e* and passes up through the bed of fuel on the grate, where the combustion is only partial. The gases formed above the fire are carried back over the bridge wall, burning as they go, so that there is a long flame passing over the iron on the hearth into the flue *f*. The heat is reflected downwards by the arched top of the furnace, making the material on the hearth extremely hot, the iron being worked at temperatures ranging from 2,500° F. to 3,000° F. Pig iron is generally used for the charge; it contains from 3 to 10 per cent. of impurities, while the wrought iron produced contains less than 1 per cent. The loss of iron in the process is comparatively small. Scrap iron, machine-shop borings and turnings, etc. are used as a charge when they are available; and, as they are in a finely divided state, a heat may be finished in 20 minutes, while with a charge of pig iron it requires from 1½ to 2 hours.

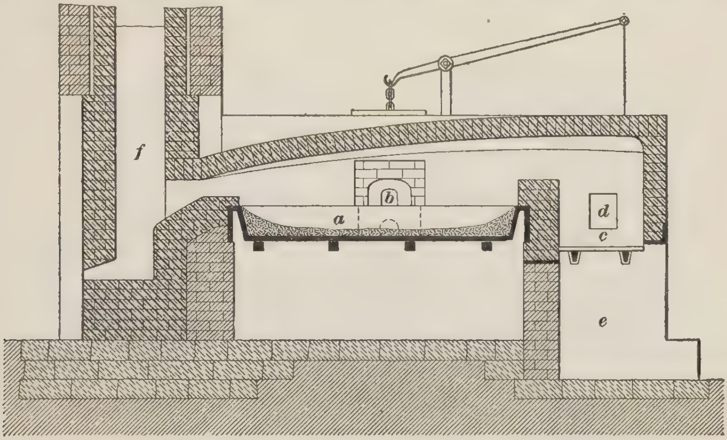

FIG. 3

19. Siemens Regenerative Furnace.—Another type of furnace used in the manufacture of wrought iron, known as the **Siemens regenerative furnace**, is shown in Fig. 4.

The furnace is called regenerative because a portion of the heat in the waste gases is returned to the furnace with the incoming air and gas. This is accomplished by the arrangement shown in section in Fig. 4, the furnace illustrated being built to use gas as a fuel. The hearth is at *a*; the crown, or roof, at *b*; the openings *c*, *c*, *c* are the charging doors; *d*, *d′* are air ports; *e*, *e′* are gas ports; *f*, *f′* and *g*, *g′* are chambers, commonly called regenerative chambers, filled with checkerwork, which consists of special checker brick about 2½ in. × 2½ in. × 9 in. in size, laid up loosely in alternate layers about 1½ inches apart; *h*, *h′* are air-inlet flues;

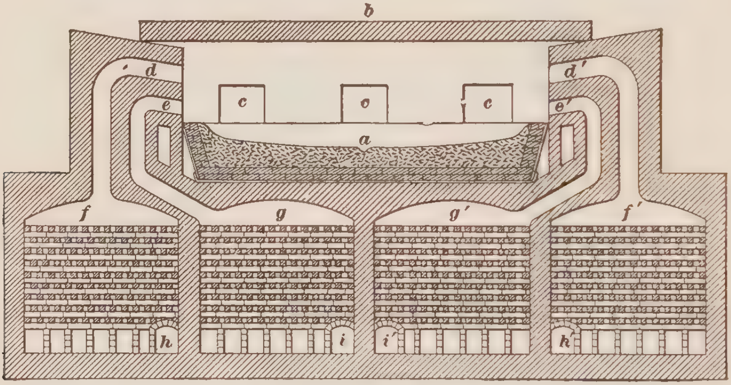

FIG. 4

and *i*, *i′* are gas-inlet flues. The flues *h*, *h′* and *i*, *i′* are connected with pipes and valves so arranged as to cause the incoming air and gas to enter either side of the furnace, as desired by the operator, the burned gases escaping at the opposite side. When the air and gas enter at *d* and *e*, the burned gases go out at *d′* and *e′*, and in circulating through the checkerwork in the chambers *f′* and *g′* heat the brick to a high temperature. The direction of the gases through the furnace can be reversed, as in the blast-furnace heating stoves shown in Fig. 1. The operation of the furnace in other respects is very much like that of the furnace shown in Fig. 3.

20. Puddling and Rolling.—In either of the furnaces illustrated in Figs. 3 and 4, the iron is heated until it melts into a thick fluid mass. While in this condition, it is thoroughly stirred and worked by means of a long iron bar, to insure all parts of the iron being treated. This working is called **rabbling.** The puddling process, carried on at a high temperature with the iron in a fluid state, causes most of the impurities to be burned out, or else separated as slag. When the process is nearly completed, the iron becomes thicker and is known as a **mat.** The workman divides this mat into masses of about 160 pounds each, and with his bar rolls them into balls on the hearth of the furnace. A small amount of slag will adhere to the balls and be rolled up in them. Consequently, as fast as they are formed, these balls are removed from the furnace and passed through a **squeezer,** which is a form of press. This forces out most of the slag remaining in the ball, and welds the iron into a solid mass, after which it is passed through rolls. The rolling process works out more slag and reduces the iron to the form of bars. These bars are then reheated and rerolled to improve their quality.

21. Properties of Wrought Iron.—At a temperature of 1,500° F. or 1,600° F., wrought iron softens; and if the surfaces of two pieces thus heated are brought together, with a flux to remove the oxide formed on the surfaces, the separate pieces can be welded or made to unite into one piece by hammering or pressing.

Good wrought iron can easily be forged and welded, but only with great difficulty can it be melted and poured into molds, like cast iron and steel, since its melting point is about 3,000° F. When broken by a tensile force, its fibrous structure is plainly apparent. When subjected to repeated shocks or loads that exceed the elastic limit, the structure changes and becomes more crystalline. Wrought iron has a greater tensile strength than cast iron, and can withstand shocks much better. The tensile strength of good wrought iron is about 55,000 pounds per square inch. When cold rolled under great pressure, as is done in the case of the best wrought-iron shafting, the strength of the material is greatly increased. Wrought iron cannot be hardened and tempered like steel, but it can be case-hardened by rubbing the surface, when at a red heat, with potassium cyanide or potassium ferrocyanide, and quenching in water. The same result may be obtained by packing the wrought iron in powdered charcoal and heating it for several hours, and then quenching it in water. This process of case-hardening converts the iron into steel to a slight depth below the surface.

Wrought-iron bars, when heated and quenched, become permanently shorter than before, in which respect wrought iron differs from cast iron, since the latter increases in volume under similar treatment.

22. Wrought iron may contain as much carbon as mild steel, but it is far more fibrous and less crystalline than steel. This is due to the manner in which it is made, the successive squeezing and rolling having a tendency to cause the fibers of the iron to lie parallel to the direction in which the bar is rolled. A small amount of slag remains in the finished product, rolled out into fibers that lie between the fibers of the iron.

The different grades of wrought iron are termed **common bar iron, best iron, double best,** and **triple best,** according to the amount of working each receives. The quality of the pig iron and the methods of manufacture also influence the quality of the wrought iron. Thus, Swedish iron is generally considered the best wrought iron, because high-grade stock is used, and great care is exercised in its manufacture. It is, however, too expensive for most classes of work.

23. Defects in Wrought Iron.—Wrought iron produced from poor ore and having an excess of phosphorus is said to be **cold short.** That is, it is very brittle when cold, and is liable to crack when bent. It can, however, be worked very well at high temperatures. If the iron contains sulphur, it is said to be **hot short,** or brittle and liable to crack when hot, although fairly good when cold. Hot-short iron, sometimes called **red short,** is useless for welding, but is tough when cold and is much used for making tin plate. In order to test wrought iron for hot-shortness, a sample may be raised to a white heat and an attempt made to forge and weld it.

The slag that remains after rolling is a troublesome factor in the use of wrought iron for finished machine parts, for when a file is applied the particles of slag work loose and, being caught between the file and the work, score the latter. This scoring is also liable to occur in a bearing having a wrought-iron journal, which would result in heating and excessive wear.

STEEL

24. Definition of Steel.—It is a difficult matter to give a concise definition of steel that will include all the grades produced, and will at the same time exclude cast iron and wrought iron. However, steel is essentially an alloy of iron and small percentages of carbon, the latter being present in greater quantities than in wrought iron and in smaller quantities than in cast iron. Small percentages of other elements are often added in order to give special properties to the product. The question of the proper classification of steels has been given much attention, but thus far no classification proposed has been generally adopted.

25. Ordinary carbon steel is made in several grades. For machine construction, a grade containing small percentages of carbon, known as **low-carbon steel, mild steel,** or **machinery steel,** is largely used. It is stronger than wrought iron, although it may contain the same percentage of carbon, and it can easily be forged. It will not, however, harden to any extent, like steels containing larger percentages of carbon.

Tool steel has the greatest percentage of carbon, is much stronger than mild steel, and will become very hard if heated and quenched. It is very difficult, if not altogether impossible, to weld it, though it can be softened or annealed, after hardening, by first heating it to redness and then allowing it to cool slowly. In order to temper it—that is, give it a certain desired degree of hardness—it is first hardened and then heated slowly. As this heating proceeds, the surface of the piece shows changing colors, each color corresponding to a certain temperature. Consequently, as soon as the desired color appears, the piece is at once quenched, when it will possess the hardness or temper corresponding to that color and temperature.

There are several substances, such as nickel, tungsten, chromium, manganese, molybdenum, aluminum, etc., that have great influence on the quality of the steel containing them. Some of these are added to the molten metal during the process of manufacture, for the purpose of modifying its quality to meet certain requirements. Others of these substances may exist in the steel as objectionable impurities, derived either from the ore or the fuel, or from both.

MANUFACTURE OF STEEL

26. The Open-Hearth Process.—Steel used for manufacturing purposes may be made by any one of three processes, known as the *open-hearth process*, the *Bessemer process*, and the *crucible process*, respectively.

Open-hearth steel is made by melting a charge of pig iron with wrought-iron or steel scrap, or by melting pig iron and iron ore in an oxidizing flame to remove the excess of carbon. The furnace is termed open-hearth because it is open at both ends. It resembles the one shown in Fig. 4, consisting of a rectangular hearth about twice as long as wide, made of firebrick, silica brick, and other refractory material. The roof is arched, so as to deflect the flame on to the charge. In the open-hearth process, the excess carbon in the charge is burned out until only the desired percentage remains, at which point the process is stopped. Gaseous fuel is used, and (except in the case of natural gas) both the gas and the air are highly heated by the waste gases in regenerative furnaces.

Open-hearth steel is used for the better grades of steel plate, forgings, machine shafts, car axles, structural steel, etc. In fact, the steel made by this process is superior for all work to that made by the Bessemer process. However, Bessemer steel is so much lower in cost than open-hearth steel that it is desirable to use the former wherever possible.

27. The **Bessemer process** consists in decarburizing, or taking out the carbon from, a charge of pig iron by forcing a blast of air through it while in a molten condition. The oxygen of the air unites with the carbon, carrying off the latter as CO_2. A quantity of pig iron rich in carbon and free from objectionable impurities is then added, so as to give just the required percentage of carbon to the steel, this operation being known as recarburizing. The molten metal is then poured into ingot molds, and the cold blocks of metal, when taken from the molds, are known as **ingots.** These are afterwards heated and rolled into commercial shapes. Bessemer steel is used for rails, nails, structural shapes, etc., wherever its cheapness makes it desirable and wherever it will be just as satisfactory as the higher grade and more expensive open-hearth steel.

28. The decarburizing and recarburizing processes used in making Bessemer steel are carried on in a vessel known as a **Bessemer converter,** a sectional view of which is shown in Fig. 5. The converter consists of a shell of heavy steel plate riveted together and lined with refractory material. It is hung on trunnions d and d', the latter being hollow and serving as a passage through which the air blast may enter the elbow e, whence it is conveyed to the bottom of the vessel and then through the tuyères to the metal. The vessel is rotated by hydraulic power applied through a rack and pinion. The construction is such that it can be made to revolve completely and empty out any slag after pouring the steel.

Referring to Fig. 5, it will be seen that the vessel consists of three principal sections keyed together to form the complete converter. The middle, or main section, b, around which the trunnion ring a extends, holds the body of metal while it is being blown. The bottom *m* is detachable, and is held to the body of the vessel by keys and links *l*. This construction facilitates repairs and speed of working. Beneath the bottom proper is the tuyère box *f*; its cover is keyed on at *k* and is air-tight; the nose of the converter *c* is also bolted to the main part, permitting its removal for repairs, etc. Converters are made in sizes of from 1 to 20 tons capacity, but are made to take about 5 tons in small plants and from 10 to 20 tons in the large plants; converters having a capacity of less than 5 tons are generally used in steel-casting plants where the output is small. The metal fills only a small part of the space, as the reaction is so violent that abundant room must be allowed for it.

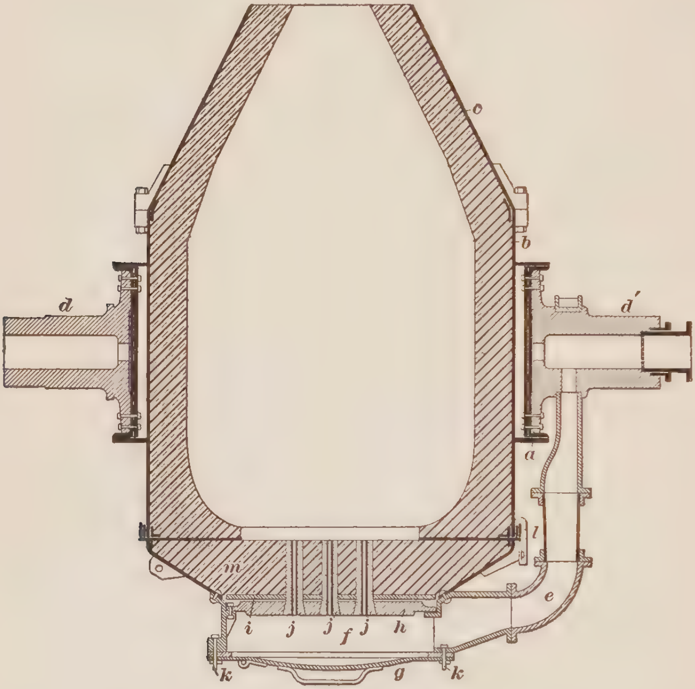

FIG. 5

CRUCIBLE STEEL

29. The Crucible Process.—The oldest and simplest process of steel manufacture is the **crucible process.** The stock is melted in a crucible, which is heated by a fire of coke, hard coal, or gas. The air supply is heated in regenerative chambers. The iron that is melted to form the steel may be either high in carbon, requiring no addition of carbon, or low in carbon, requiring recarburizing. The stock used is chiefly wrought iron and steel scrap, with sufficient charcoal to give the required percentage of carbon.

It is maintained that the highest grade of crucible steel can be manufactured only from blister steel made from the purest Swedish iron. As no sulphur or phosphorus is removed from the charge during the melting process, the stock must be free from these impurities.

In American practice, the crucible is filled with the stock while cold and before inserting it in the heating furnace. The time required for melting varies from 2¼ to 3 hours. Soft steel, or steel low in carbon, requires a longer time to melt than high-carbon steel. The presence of manganese, however, shortens the period of melting. For making tool steel, the molten contents of the crucibles are poured into ingot molds about 3 or 4½ inches square and deep enough to hold the steel from one or more crucibles. These molds open lengthwise, so that the ingot may be easily removed. The cost of making steel by the crucible process is higher than by either the open-hearth or the Bessemer process, for which reason crucible steel is used only in the manufacture of tools and in other cases where its high cost is compensated by the better quality of the product.

30. Superiority of Crucible Steel.—The superiority of **crucible steel** is believed to be due to the purity of the stock used and to the care exercised during the melting, the crucible being kept covered so that no injurious substances are absorbed by the steel from the fuel in the furnace. The materials used for making crucible steel are chiefly puddled iron and wrought iron and steel scrap; blister steel and shear steel are also sometimes used. A wide range in the composition of crucible steel is possible. Since it is largely used for tools, it generally contains from .5 to 1.5 per cent. of carbon. The percentage of manganese varies from .1 to .75 per cent., and the silicon generally ranges from a few hundredths of 1 per cent. to .2 per cent., although there is a considerable amount of steel made that contains higher percentages than these. Sulphur and phosphorus are usually kept below .02 per cent., but for less exacting purposes there may be as much as .05 per cent. of each. In the highest grades of Swedish stock, there is not more than .01 per cent. of sulphur or phosphorus. The effects of these impurities are more marked in high-carbon than in low-carbon steel, since in the former the metal is more sensitive, and small amounts of sulphur, phosphorus, and silicon exert greater influences. In determining the carbon in crucible steel, the ingots are topped; that is, the upper end is broken off and the fracture examined. An experienced eye can detect differences in the percentage of carbon as small as .05 per cent.

31. Tool Steel.—Crucible steel is used to a large extent for making machine-shop tools, and when manufactured for this purpose is called **tool steel.** The various uses to which the different grades of tool steel are put may be stated, in a general way, as follows:

Crucible steel containing from .5 to .75 per cent. of carbon is used for battering tools, hot work, dull-edge cutting tools, etc.; steel containing .75 to 1 per cent. of carbon is used for dies, axes, knives, large-sized and medium-sized drills, etc.; steel containing from 1 to 1.5 per cent. of carbon is used for razors, lathe tools, gravers' tools, small drills, etc. The best tool steel for general purposes contains from .9 to 1.1 per cent. of carbon, and is adapted to a wider range of uses than any other grade.

Table I shows the composition of several grades of crucible steel, with some of the purposes for which each is used, although by giving it suitable treatment and tempering it properly the field of usefulness of any grade may be considerably increased.

BLISTER STEEL AND SHEAR STEEL

32. Blister steel consists of wrought-iron bars that have been treated by the cementation process. This treatment consists in heating the bars to a high temperature for several hours in an air-tight compartment of a furnace and in contact with carbon. The carbon enters the iron, converting it into steel to a greater or less depth, depending on the length of time the process is continued. The surfaces of the bars become rough and spotted with blisters, and the product thus becomes known as blister steel. It is made in several grades, depending on the percentage of carbon absorbed, which usually varies from .5 to 1.5 per cent.

TABLE I

CRUCIBLE STEELS

Use	Carbon Per Cent.	Manganese Per Cent.	Silicon Per Cent.	Sulphur Per Cent.	Phosphorus Per Cent.	Tungsten Per Cent.	Nickel Per Cent.	Chromium Per Cent.
Sledges, battering tools, etc.	.50	.21	.210	.022	.020			
Hot-work shear knives, etc. . . .	.65	.20	.180	.020	.015			
Drills, reamers, dies, etc.	.85	.18	.210	.015	.014			
Chisels, knives, lathe tools, etc.	1.00	.26	.200	.010	.010			
Razor steel	1.30	.22	.200	.006	.009			
Dies, graving tools, etc.	1.30	.16	.140	.014	.012			
Cutting tools, etc. (self-hardening) .	.94	1.50	.160	.015	.012	3.40		
Krupp armor	.28	.32	.055	.016	.015		3.60	1.75

Shear steel is made from blister steel. A number of bars of blister steel are welded together so as to form a single large bar, which is then hammered or rolled down to the desired dimensions. Shear steel is made in different grades, as single shear, double shear, etc., each successive and higher grade being produced by cutting the bars of the next lower grade, welding them together, and working to size. Both blister steel and shear steel are frequently used in making crucible steel.

ALLOY STEELS

33. Tungsten Steel.—The properties of the ordinary tool steels are due almost wholly to the carbon they contain. The carbon also affects the qualities of the **alloy steels,** made by adding other elements to give them special properties. One of the most important of these elements is tungsten, and steels in which the principal properties are due to this element are known as **tungsten steels.** The amount of tungsten may vary from .1 to 10 per cent., the usual amount being from 3 to 5 per cent. The tungsten is introduced into the crucible in the form of ferrotungsten, which is simply an alloy of iron and tungsten. The amount of manganese usually runs from 1.5 to 2.5 or 3 per cent., and the percentages of silicon, sulphur, and phosphorus are the same as in carbon steel.

Air-hardening, or **self-hardening, steel**—that is, steel that will harden in air without being quenched—usually contains tungsten. The older air-hardening steels were called *mushet steel;* they contained small percentages of tungsten and from 1.5 to 2.3 per cent. of carbon. The later varieties of these steels are much harder, and contain more tungsten and less carbon owing to the danger of burning when the percentage of carbon is high. They are called high-speed steels, because of the fact that tools made from them can be used at much higher cutting speeds than ordinary steel tools. In forging air-hardening steel, great care must be taken to avoid overheating it, for it should not be heated beyond a temperature of 1,600° F. Its hardness is increased by cooling it in a blast of air, while still greater hardness is secured by quenching in oil or water. However, the steel is liable to be cracked by quenching, or else rendered so brittle as to crumble when used for cutting tools.

When air-cooled, it cannot be machined by the hardest carbon steel. It can be annealed, however, so that it will machine readily, by heating it to a bright red for from 24 to 36 hours, and then cooling it slowly, while covered with sand or ashes, in the furnace. Cutting tools made of tungsten steel will retain their hardness at much higher temperatures than will tools made of ordinary carbon steel; hence, the speed of cutting can be greatly increased. This steel will also retain magnetism better than the ordinary carbon steel.

34. Manganese Steel.—Among all the varieties of steel, the hardest and toughest is **manganese steel.** The best results are obtained with from 7 to 14 per cent. of manganese. The maximum strength is obtained with about 13 or 14 per cent. of manganese, and the greater amount manufactured contains from 12 to 14 per cent. This steel is high in carbon, because the ferromanganese used in its manufacture is high in carbon. The extreme hardness and toughness of manganese steel are secured by quenching it in water. This is one of the most noticeable peculiarities of the steel, since the other alloy steels increase in hardness, but decrease in toughness, by quenching.

Manganese steel is practically non-magnetic. Its uses are restricted to work that does not require machining, such as castings and forgings for various purposes. It works readily at a red heat. It is used principally for the jaws and plates of rock crushers and grinding machinery, car wheels, safes and vaults, etc. It may be made in crucibles, but the open-hearth process is more suitable for large quantities. Owing to the large amount of manganese that it contains, the metal is extremely fluid, and solid castings, both heavy and light, are readily made. The shrinkage is excessive, being about $\frac{5}{8}$ inch to the foot, which increases the difficulties of casting.

35. Nickel Steel.—The addition of nickel to steel will greatly increase its strength, ductility, and elasticity. The amount of nickel added usually varies from 3 to 5 per cent., although alloys containing as high 30 per cent. are made. The nickel is added to the steel either as metallic nickel or as ferronickel, which is charged with the rest of the stock into the furnace. Nickel steel is made almost entirely by the open-hearth process, though it can be made by either the Bessemer or the crucible process. It is readily worked either hot or cold, and is easily forged, but is harder to machine than ordinary carbon steel. It is largely used for armor plate, gun barrels, engine and propeller shafts, tubes for boilers and condensers, bicycle frames, and a great variety of purposes in which great strength and lightness are required, and in which high cost is not prohibitive. Nickel steel is also especially valuable because it offers a much greater resistance to corrosive influences than does carbon steel.

36. Chrome Steel.—When a small percentage of chromium is added to steel, in either the crucible or the open-hearth process, the product is a steel having great hardness and toughness. It will weld readily to itself or to wrought iron, and is not injured by excessive heating, as is the case with ordinary tool steel. When heated to a moderate heat and quenched in water, it becomes hard enough to resist the action of tools. The chromium is introduced into the steel as ferrochrome, the amount varying from .25 to 2 per cent. Chrome steel is used in making high-grade tools, liners for rock and roll crushers, stamp-mill shoes, safes, armor plate, etc. The best grades are made by the crucible process.

37. High-Speed Steel.—As already stated, **high-speed steel** is a form of air-hardening tool steel with larger percentages of tungsten than were formerly used. It takes its name from the fact that tools made of it may be used at considerable higher cutting speeds than tools made of other varieties of steel. The cutting speed has been increased in some cases from 30 to 400 feet per minute by the substitution of high-speed steel for the ordinary kinds.

High-speed steel is made by the crucible process in the same manner as carbon tool steel, but with the addition of certain other elements to give it exceptional hardness and strength. The additional elements usually employed are tungsten, chromium, molybdenum, and vanadium, with less than 1 per cent. of carbon, and less than .02 per cent. of phosphorus. To produce the highest grade of steel, great care should be taken in its manufacture. After melting in the crucible, the steel is cast into ingots, which are then rolled into bars of the desired dimensions to form the stock from which tools are made.

Aside from the special preparation of alloy steels with the design of securing the best results, it has been found necessary to forge each steel in a special way, and a tool produced may be practically worthless if there is any great variation from the required method of treating the steel; even a slight deviation from the correct temperature will lower the efficiency of the tool. If heated to a certain degree, it may be useless for cutting purposes, while a little higher temperature may bring it to a good condition, so that with a given steel the tool may be either good or bad, depending entirely on the methods of hardening and tempering and the temperatures used. High-speed steel must be heated to a white heat, in some cases as high as 2,300° F., during the process of hardening.

For grinding air-hardening steel, the wet sandstone gives the best results. If an emery wheel is used, it should be dry, and the grinding had better be done before the tool is hardened. In dry grinding with an emery wheel, the steel becomes very hot, and care must be taken not to allow cold water to flow on it while in this condition, as this would probably cause the steel to crack. The brands first made were more suitable for roughing cuts, and the finishing was done with ordinary carbon steel, but the later products are quite well suited to all classes of machine-shop tools.

STEEL CASTINGS

38. Production of Steel Castings.—The steel for castings may be made by any of the three steel-making processes already described. For the best results, however, either the crucible or the open-hearth process is used, the latter being most largely employed because it is less expensive and because large charges can be easily handled. The mixture is generally made up of steel scrap, pig iron, and iron ore. The percentage of scrap is variable, and may range from nothing up to 80 per cent. of the charge. The steel is cast in dry-sand molds, in about the same manner as cast iron; but the behavior of the steel is different, and it is more difficult to produce good castings. The shrinkage is greater, being about $\frac{1}{4}$ inch per foot. This often seriously affects the castings, because the contraction of the metal while cooling is hindered by the hard and unyielding molds. Unless the casting is of such a form that it can contract readily as the shrinkage occurs, it will be distorted, and it may even become ruptured in the mold or be so weakened as to break when in service. Since steel melts at a much higher temperature than cast iron, the facings of the molds should be less fusible, and great care must be taken to prevent shrink holes and cavities in the castings.

In order to make castings of the best quality, it is necessary to anneal them. This is done by subjecting them for several hours to a temperature of from 1,200° to 1,600° F., and then cooling them slowly without exposure to the air.

39. The difficulties in making steel castings are so great that many of the attempts to develop this line of work have resulted in failure. Their reliability has never been considered absolute, because the interior of any casting cannot be as accurately known or judged with the same certainty as that of a forging or of a rolled shape. Many steel castings have hidden defects, which greatly reduce their strength, and which cannot be detected by any method of inspection. The surfaces of the castings are usually rough. When made without defects they are very strong, and can be subjected to considerable distortion without breaking. On account of their relative cheapness, steel castings are largely used for work that formerly was made of wrought iron or steel forgings, and also, in place of cast-iron and malleable castings, for work where greater strength is required.

SPRING STEEL

40. Crucible steel that is prepared especially for the manufacture of springs is called **spring steel.** Without special treatment, it possesses a higher elastic limit than any other steel, while by hardening and tempering the elastic limit may be considerably raised. It is for this reason that crucible steel is used for making springs. Untreated mild steel may also be used, but such springs must necessarily be much larger and heavier in order to withstand the same loads as special spring steel.

The peculiar duty for which springs are designed requires that they shall yield to the action of certain forces, but shall return to their original form directly those forces cease to act. The material used for springs must therefore have a high elastic limit, so that the springs shall not be strained beyond the elastic limit of the material when subjected to their greatest load. The elastic limit may be raised and the strength increased by cold rolling or drawing into wire.

ALLOYS

41. Brass.—**Brass** is an alloy of copper and zinc. The zinc promotes solidity, and makes the alloy cast better than would copper alone. The alloy commonly used for brass castings is composed of 66 parts, by weight, of copper and 34 parts of zinc, although from 2 to 4 per cent. of tin is often added to give strength to the casting. A small amount of lead also is sometimes added to brass to facilitate the work to be done on the castings in the machine shop. Lead assists in giving the castings a smooth surface, as it forms an oxide on the surface and prevents the molten metal taking too sharply the impressions of the sand. But the addition of lead decreases the strength of the castings, changes their color, and makes them corrode more easily. It does not combine chemically with copper, but forms a mechanical mixture.

Brass may be softened by heating it to a dark red and immersing it in cold water. It is hardened by rolling or hammering. Its maximum tensile strength is about 50,000 pounds per square inch, which is secured by making the alloy with 40 per cent. of zinc. The maximum ductility is secured with about 25 per cent. of zinc. The color of brass varies from a copper red to a gray, according to the amount of zinc used. Ordinary yellow brass contains about 30 per cent. of zinc. Malleable brass is composed of 33 parts of copper and 25 parts of zinc.

42. Bronze.—An alloy of copper and tin is called **bronze.** Tin increases the fluidity of molten copper and the tensile strength of the casting, but decreases its ductility. Bronzes are largely used for bearing metals. The quality of a bronze depends on its composition, the purity of the materials used, and the care exercised in melting and pouring. Probably the greatest defect in bronzes used for bearings is caused by granules of red oxide of copper, due to a lack of proper precautions in melting and pouring. These granules are very hard, and the bearing is very liable to heat if any of these spots occur on the surface.

The maximum tensile strength of bronze is about 35,000 pounds per square inch, and this is attained when the alloy contains about 18 per cent. of tin. About 4 per cent. of tin gives a bronze of greatest ductility. The bronze having both a high tensile strength and great ductility consists of 10 parts tin and 90 parts copper. This composition is known as *gun metal*, and is very useful in machine construction.

Bronzes generally are very useful compounds. They are harder, denser, and stronger than copper and do not oxidize so easily. Their composition may be varied to suit almost any requirements. There are several varieties of bronze into which other ingredients enter that give the bronze some distinctive property, in which case the bronze takes its name from the material added.

43. Phosphor bronze is an alloy of copper and tin with a very small percentage of phosphorus. Phosphorus increases the strength, ductility, and solidity of castings. It is superior to common bronze for many kinds of bearings.

Copper oxide forms in nearly all alloys containing much copper when these are heated in the air, and reduces the strength and ductility of the alloy. The addition of phosphorus just before pouring the metal reduces the copper oxide and makes the casting more ductile. A small amount of the phosphorus may combine with the alloy and increase its strength and ductility, but the greater part prevents the harmful effect of the copper oxide.

TABLE II

COMPOSITION OF BEARING METALS

Metals	Copper	Tin	Lead	Zinc	Antimony	Iron
Camelia metal	70.20	4.25	14.75	10.20		.55
Antifriction metal	1.60	98.13				Trace
White metal			87.92		12.08	
Car brass lining		Trace	84.87		15.10	
Salgee antifriction	4.01	9.91	1.15	85.57		
Graphite bearing metal		14.38	67.73		16.73	(?) (1)
Antimonial lead			80.69		18.83	
Carbon bronze	75.47	9.72	14.57			(2)
Cornish bronze	77.83	9.60	12.40	Trace		Trace (3)
Delta metal	92.39	2.37	5.10			.07
American antifriction metal			78.44	.98	19.60	.65
Tobin bronze	59.00	2.16	.31	38.40		.11
Graney bronze	75.80	9.20	15.06			
Damascus bronze	76.41	10.60	12.52			
Manganese bronze	83.45				(4)	1.24
Ajax metal	81.24	10.98	7.27			(7)
Antifriction metal			88.32		11.93	
Harrington bronze	55.73	.97		42.67		.68
Car-box metal			84.33	Trace	14.38	.61
Hard lead			94.40		6.03	
Phosphor bronze	79.17	10.22	9.61			(5)
Ex. B. metal	76.80	8.00	15.00			(6)

OTHER CONSTITUENTS

1. No graphite
2. Possible trace of carbon
3. Trace of phosphorus
4. Manganese, 13.48
5. Phosphorus, .94
6. Phosphorus, .20
7. Phosphorus or arsenic, .37

Silicon bronze is a combination of copper and silicon, sometimes with zinc and tin added. It is largely used for wire for trolley, telephone, and telegraph service, because of its great tensile strength, good electrical conductivity, and durability.

Manganese bronze is an alloy of copper and manganese; it often contains some iron, and may also contain tin. Manganese has a great affinity for oxygen, and hence the addition of manganese tends to make a clear alloy, free from copper oxide. Manganese bronze is usually made by fusing together copper and black oxide of manganese, or copper or bronze and ferromanganese. It has great strength, and will not corrode easily. It is largely used for marine propeller wheels. Some so-called manganese bronzes contain no manganese, but are alloys of copper and tin with traces of other metals.

Aluminum bronze is made by adding from 2 to 12 per cent. of aluminum to copper. This alloy is remarkable for its strength, malleability, and ductility.

44. The composition of several brasses and bronzes used for bearing metals is given in Table II.

45. Babbitt Metal.—**Babbitt metal** is an alloy of tin, copper, and antimony. The term is applied also to a great number of alloys on the market that are used to line journal-boxes. The metal received its name from the originator of a journal-box made so that it could be lined with an antifriction alloy. The proportions of the elements in the alloy vary considerably, according to the weight that it has to carry and to the speed of the shaft. The proportions are also varied by the manufacturers, who find out by experience what combination gives the best results for each kind of work. Proportions that have been found satisfactory are, for heavy service, 50 parts tin, 2 parts copper, and 8 parts antimony; for light service, 50 parts tin, 1 part copper, and 5 parts antimony. The antimony gives hardness to the alloy and reduces contraction, while the tin gives it the antifriction quality.

For light, high-speed machinery where the bearings are not liable to become heated, **type metal,** an alloy of about 80 per cent. of lead and 20 per cent. of antimony, is also used for lining bearings. In fact, wherever there is a demand for cheapness, the tendency is to use lead instead of the more expensive metals, tin and copper. The grades having lead as a constituent should not be used in bearings that support heavy loads. The presence of lead in Babbitt metal can be determined by rubbing the metal on paper; if lead is present, it will leave a mark on the paper similar to that made by a lead pencil. Lead and antimony combine with each other without impairing the antifriction quality of the lead.

46. Solder.—The several alloys used to make metallic joints by fusion between pieces of metal are called **solders.** As the solder must be more easily fused than the metal to be united, the composition necessarily varies a great deal. Solders are designated as hard or soft, according to the temperature necessary to melt them. The fusing temperature of **soft solder,** usually composed of lead and tin, ranges from about 200° to 480° F. The addition of bismuth lowers the melting point considerably. A standard grade of soft solder is half tin and half lead; but as lead is much the cheaper metal, the tendency is to increase its amount.

Hard solder fuses only at a red heat. It is an alloy of copper, zinc, tin, antimony, silver, gold, etc., depending on the kind of metal with which it is to be used.

Spelter is a hard solder composed of copper and zinc, which may be of equal parts, though these proportions are sometimes varied, and some tin or other metal may be added.

47. German Silver.—An alloy of copper, nickel, and zinc is called **German silver.** It is a hard, white, and ductile metal, much used for making table utensils, resistance wire for electrical apparatus, ornamental work, etc. There are many varieties of German silver, and their composition varies from 20 to 30 per cent. zinc and 18 to 25 per cent.

nickel, the rest being copper. Sometimes, tin or lead is added to cheapen the alloy. Lead makes it more fusible and tin more dense, so that it will take a better polish. The addition of iron or manganese makes an alloy of whiter color, but increases its brittleness.

An alloy of copper and nickel is used for making coins. Nickel is also largely used for plating parts of machines, instruments, plumbing fixtures, ornamental work of various kinds, fittings for steam, gas, and water, etc. It produces a hard durable surface that can be highly polished.

SELECTION OF MATERIALS

48. In selecting the material for any part of a machine, that which is best suited to the conditions of service should be chosen. The materials commonly used for machine parts are cast iron, wrought iron, machinery steel, steel castings, crucible or tool steel, malleable iron, brass or bronze, and Babbitt.

When a machine-shop tool for cutting purposes is required, or a spring is needed, a material must be selected that will resist shocks and hard usage. Consequently, the piece should be made of some grade of crucible steel. If, however, the piece is to be subjected alternately to tension and compression, through a wide range of pressures, as in the case of the piston rod of a steam engine, then it should be made of machinery steel, since this is cheaper than tool steel and combines the qualities of toughness, strength, and homogeneity.

In the case of a connecting-rod, no portion of which forms a bearing surface for a sliding or rotating part, wrought iron might be used, since it can be worked cheaply in the shop, and is strong against varying pressures in tension and compression. Wrought iron should never be used for journals, however, since it contains particles of slag, which would cause cutting and heating. Steel castings are sometimes used for connecting-rods, if little machining is required, but this practice is not to be recommended.

Pinions to run with large gears are frequently made of machinery steel or steel castings, for the teeth are liable to be undercut, and are subjected to greater wear than those of the gear, since they come into action more frequently. Hence, steel is used both for its strength and its wearing qualities.

Cast iron is most useful because it can readily be molded into various shapes, can be machined easily, and possesses considerable strength. Engine cylinders, being rather intricate in construction, are made of cast iron, because it is cheap, strong, and durable, forms a good bearing surface, and is easy to cast to the desired form. The frames of most machines are made of cast iron, because it gives the necessary weight and rigidity without undue cost.

Small machine parts, such as rocker-arms, which must resist shocks, are frequently made of malleable iron. They are cheaper than they would be if made of steel, and stronger than if made of cast iron.

It has been found desirable, in most cases, to make a journal and its bearing of dissimilar metals, since this reduces the friction. That is, metals of the same kind running in contact usually produce greater friction than unlike metals. A notable exception to this rule is cast iron, which runs very well on cast iron. Usually, also, the bearing is softer than the journal, so that it will receive the greater part of the wear. It may then be replaced at small expense, whereas the cost of a new journal is considerable.

Bronze and brass are generally used to form the wearing parts of bearings, although the former is also used for propellers, pump linings, and other parts subjected to corrosive action, and which could not be made of cast iron.

Babbitt metal is used to line cast-iron journal-boxes, since it possesses good antifriction qualities. The journal should be of machinery steel, although cast iron and cast steel have been used successfully.

STRENGTH OF MATERIALS

(PART 1)

STRESS, DEFORMATION, AND ELASTICITY

STRESS AND DEFORMATION

1. Stress.—The molecules of a solid or rigid body are held together by the force of cohesion, and this force must be overcome to a greater or less degree in order to change the form and size of the body or to break it into parts. The internal resistance that a body offers to any force tending to overcome the force of cohesion is called a **stress.** If a weight of 1,000 pounds is held in suspension by a rod, there will be a stress of 1,000 pounds in the rod. The stress induced by the acting force at any section of the rod is the same as the total stress at any other section.

The stress is equal, but opposed, to the external force producing it, and is, therefore, measured and represented by this force. Thus, a force of 1,000 pounds produces a stress of 1,000 pounds. The external force is the force applied to a fixed body; the stress is the resistance offered by the body to a change of form; and when the body ceases to change its shape, as when a rod ceases to elongate, the stress just balances the external force. In the United States and England, stresses are measured in pounds or tons; in nearly all other civilized countries, in kilograms.

2. Kinds of Stresses.—Whenever a force, no matter how small, acts on a body, it produces a stress and a

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corresponding change of form. According to the manner in which forces act on a body, stresses are divided into the following classes:

1. *Tension*, which is a tensile, or pulling, stress.
2. *Compression*, which is a compressive, or crushing, stress.
3. *Shear*, which is a shearing, or cutting, stress.
4. *Torsion*, which is a torsional, or twisting, stress.
5. *Flexure*, which is a transverse, or bending, stress.

The first three—tension, compression, and shear—are *simple stresses*. A solid body may be subjected to any one of these stresses without the presence of either of the others. Torsion and flexure, however, are *compound stresses;* that is, they are a combination of two or more of the simple stresses and never appear as simple stresses themselves. When a rod is being twisted, there is a tendency for each section to shear from the one next to it, and the fibers of the outer surface lengthen and are in tension. When a bar is bent, one side is lengthened and the other is shortened, thus combining tension and compression.

3. Unit Stress.—The **unit stress,** or the **intensity of stress,** is the stress per unit of area. In the foregoing illustration, in which the rod is in tension, if the area had been 4 square inches, the unit stress would have been $1,000 \div 4 = 250$ pounds per square inch. Had the area been $\frac{1}{2}$ square inch, the unit stress would have been $1,000 \div \frac{1}{2} = 2,000$ pounds per square inch.

Let P = total stress, in pounds;
A = area of cross-section, in square inches;
S = unit stress, in pounds per square inch.

Then,
$$S = \frac{P}{A}, \text{ or } P = A\,S$$

This formula is true also when the piece is in compression or in shear. That is, *the total stress in tension, compression, or shear equals the area of the section multiplied by the unit stress.*

4. Deformation.—When a body is stretched, shortened, or in any way deformed through the action of a force, the change of form is called **deformation** or **strain,** the former term being generally considered preferable. Thus, if the rod before mentioned had been elongated $\frac{1}{10}$ inch by the load of 1,000 pounds, the deformation would have been $\frac{1}{10}$ inch. Within certain limits, to be given hereafter, deformations are proportional to the forces producing them, and consequently to the stresses that balance the forces.

5. Unit Deformation.—The **unit deformation** is the deformation per unit of length or of area, but is usually taken per unit of length and called the *elongation* per unit of length. In this Section, the unit of length will be considered as 1 inch. The unit deformation, then, equals the total deformation divided by the length of the body, in inches.

Let l = length of body, in inches;
e = elongation, in inches;
s = unit deformation.

Then,
$$s = \frac{e}{l}, \text{ or } e = l s$$

ELASTICITY

6. The **elasticity** of a body is that property by virtue of which the body tends to return to its original size and shape when distorted by an external force; it is the quality that gives spring to a material.

7. Experimental Laws.—The following laws have been established by experiment:

Law I.—*When a body is subjected to a small force causing a small stress, a small deformation is produced; and when the force is removed, the body springs back to its original shape and the stress disappears. This leads to the conclusion that, within certain limits of stress, bodies are perfectly elastic.*

Law II.—*Within certain limits of stress, the change of shape (deformation) is directly proportional to the applied force.*

Law III.—*When the force producing a stress exceeds a certain limit, a deformation is produced that is partly permanent; that is, the body does not spring back entirely to its original form when the stress disappears. This lasting part of the deformation is* *called a* **set,** *and in such cases the deformation is not proportional to the stress.*

Law IV.—*Under a still greater force, the deformation rapidly increases, and the body is finally ruptured or broken.*

Law V.—*A shock, or suddenly applied force, does greater injury than a force gradually applied.*

8. Elastic Limit.—According to the first law, the body will resume its original form when the force is removed, provided that the stress is not too great. This property is called *elasticity*. According to the second law, the deformation is proportional to the stress within certain limits. Thus, if a pull of 1,000 pounds elongates a body .1 inch, a pull of 2,000 pounds will elongate it .2 inch. This is true up to a certain limit, beyond which the body will not resume its original form on the removal of the force, but will be permanently deformed, more or less, according to the magnitude of the force. The stress at the point where the set begins is called the **elastic limit.** All deformations accompanied by stresses within the elastic limit are directly proportional to the stresses.

9. If the stress in a body is increased beyond the elastic limit, and none of the deformation disappears when the force is removed—that is, if the deformation is all permanent—the body is said to be **plastic.** Clay and wax are examples of plastic materials.

If the larger part of the deformation remains permanent, and a small part disappears with the removal of the force, the body is said to be ductile. Wrought iron and copper are good examples of ductile materials. Such materials are adapted to the manufacture of wire. Materials that cannot be deformed without fracture are said to be brittle. Brittleness is a comparative term, no substance being perfectly brittle; cast iron and glass are examples of brittle materials.

MODULUS OF ELASTICITY

10. Among engineers, the term *elasticity* means the resistance that a body offers to a permanent change of form; and by *strength*, the resistance that a body offers to division or separation into parts. Bodies that have the highest elastic limit are the most elastic.

The **modulus of elasticity,** sometimes called the **coefficient of elasticity,** is the ratio of the unit stress to the unit deformation, provided that the elastic limit is not exceeded.

Let S = unit stress;
s = unit deformation;
E = modulus of elasticity.

Then, by definition, $E = \frac{S}{s}$

Substituting the values of S and s obtained from the formulas in Arts. **3** and **5,**

$$E = \frac{S}{s} = \frac{P}{A} \div \frac{e}{l} = \frac{P\,l}{A\,e}$$

11. If, in this formula, it is assumed that $e = l$ and $A = 1$ (1 square inch), then $E = P$. That is, *the modulus of elasticity is that force which, if stress and deformation continued proportional to each other, would produce in a bar of unit area a deformation equal to the original length of the bar.* This, however, is never the case, as the elastic limit and the ultimate strength are reached before the applied force reaches this value.

EXAMPLE.—A wrought-iron bar 2 inches square and 10 feet long is stretched .0528 inch by a force of 44,000 pounds; what is the modulus of elasticity?

SOLUTION.—Using the formula in Art. **10,**

$$E = \frac{P\,l}{A\,e} = \frac{44{,}000 \times 10 \times 12}{2^2 \times .0528} = 25{,}000{,}000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

TENSION

12. When two forces act on a body in opposite directions, away from each other, the body is said to be in **tension.** The two forces tend to elongate the body and thus produce a tensile stress and deformation; a weight supported by a rope affords a good example. The weight acts downwards, and the reaction of the support to which the upper end of the rope is fastened acts upwards; the result being that the rope is stretched more or less, and a tensile stress is produced in it. Another familiar example is the connecting-rod of a steam engine on the return stroke. The crosshead then exerts a pull on one end of the rod, which is resisted by the crankpin on the other.

EXAMPLE.—An iron rod, 2 inches in diameter, sustains a load of 90,000 pounds; what is the unit tensile stress?

SOLUTION.—Using the formula in Art. 3,

$$P = A\,S, \text{ or } S = \frac{P}{A} = \frac{90{,}000}{2^2 \times .7854} = 28{,}647.82 \text{ lb. per sq. in.} \quad \text{Ans.}$$

13. The **ultimate strength** of any material is the unit stress that exists in a body when the external force is just sufficient to break it.

The **ultimate elongation** is the total elongation produced in a unit of length of the material having a unit of area when the stress equals the ultimate strength of the material.

14. For the same size, quality, and kind of material, the ultimate strength, ultimate elongation, modulus of elasticity, and elastic limit agree quite closely for different pieces. Table I gives the average values of the modulus of elasticity (E_t), elastic limit (L_t), ultimate strength (S_t), and ultimate elongation (s_t) of different materials, the quantities given being for tension only. As brick and stone are never used in tension, their values are not given.

The values in Table I are average values, and may differ from actual values in any particular case; hence, they should not be used in designing machine parts, where any considerable degree of accuracy is required. Thus, the ultimate tensile strength of steel varies from less than 60,000 to more than 180,000 pounds per square inch, according to its purity and the amount of carbon it contains; that of cast iron, from 12,000 or 13,000 to over 40,000; wrought iron varies from 40,000 to 72,000, according to quality, the latter value being for wire. Timber varies fully as much as, if not more than, any of the three preceding materials, its properties depending on the kind of wood, its degree of dryness, the manner of drying, etc.

TABLE I

TENSION

Material	Modulus of Elasticity E_t Pounds per Square Inch	Elastic Limit L_t Pounds per Square Inch	Ultimate Tensile Strength S_t Pounds per Square Inch	Ultimate Elongation s_t Inch per Linear Inch
Timber . . .	1,500,000		10,000	0.015
Cast iron . .	15,000,000		20,000	0.005
Wrought iron	25,000,000	25,000	50,000	0.200
Steel	30,000,000	40,000	65,000	0.100

15. Hereafter problems will be solved by using the average values given in the preceding and following tables; the designer, however, should not use them, but should either test the materials himself or state in the specifications what strengths the materials must have. For example, mild steel, for boiler shells, should have a tensile strength of not less than 55,000 or 65,000 pounds per square inch; Bessemer steel, for steel rails, not less than 110,000; open-hearth steel, for locomotive tires, not less than 125,000, and crucible cast steel, for tools, cutlery, etc., not less than 150,000. It is customary, also, to specify the amount of elongation. This is necessary because, as a rule, the elongation decreases as the tensile strength increases. Having tested the material about to be used, or having specified the lowest limits, the designer can ascertain the strength and stiffness of construction by means of the formulas and rules that follow.

EXAMPLE 1.—How much will a piece of steel 1 inch in diameter and 1 foot long elongate under a steady load of 15,000 pounds?

SOLUTION.—Apply the formula in Art. **10**, $E_t = \frac{P l}{A e}$, or $e = \frac{P l}{A E_t}$. From Table I, E_t = 30,000,000 for steel; hence,

$$e = \frac{15{,}000 \times 12}{1^2 \times .7854 \times 30{,}000{,}000} = .00764 \text{ in.} \quad \text{Ans.}$$

NOTE.—All lengths given in *Strength of Materials*, Parts 1 and 2, must be reduced to inches before substituting in the formulas.

EXAMPLE 2.—A piece of timber has a cross-section 2 in. × 4 in. and is 6 feet long; a certain force produces an elongation of .144 inch; what is the value of the stress, in pounds?

SOLUTION.—Applying the formula in Art. **10**, $E_t = \frac{P l}{A e}$, or

$$P = \frac{E_t A e}{l} = \frac{1{,}500{,}000 \times 2 \times 4 \times .144}{6 \times 12} = 24{,}000 \text{ lb.} \quad \text{Ans.}$$

COMPRESSION

16. If the length of the piece is not more than five times its least transverse dimension (its diameter, when round; its shorter side, when rectangular, etc.), the laws of compression are similar to those of tension. The deformation is proportional to the stress until the elastic limit has been reached; after that, it increases more rapidly than the stress, as in the

TABLE II
COMPRESSION

Material	Modulus of Elasticity E_c Pounds per Square Inch	Elastic Limit L_c Pounds per Square Inch	Ultimate Compressive Strength S_c Pounds per Square Inch
Timber	1,500,000		8,000
Brick			2,500
Stone	6,000,000		6,000
Cast iron	15,000,000		90,000
Wrought iron	25,000,000	25,000	50,000
Steel	30,000,000	40,000	65,000

case of tension. The area of the cross-section is slightly enlarged under compression. In Table II are given the average compression values of E, L, and S for timber, brick, stone, cast iron, wrought iron, and steel. (See also Table I.) E is not given for brick nor L for timber, cast iron, brick, or stone, because these values are not known. To distinguish between compression and tension when applying a formula, E_c, L_c, and S_c will be used for compression.

17. When the length of a piece subjected to compression is greater than ten times its least transverse dimension, it is called a *column*, and the material fails by a sidewise bending or flexure, often called *buckling*. Table II is strictly true only for pieces whose length does not exceed five times the least dimension of the cross-section. This statement may, without material error, be modified in practice to include pieces whose lengths are not greater than ten times their least transverse dimensions.

EXAMPLE.—How much will a wrought-iron bar 4 inches square and 15 inches long shorten under a load of 100,000 pounds?

SOLUTION.—Applying the formula in Art. **10**, $E_c = \frac{P l}{A e}$, or $e = \frac{P l}{A E_c}$,

$$e = \frac{100{,}000 \times 15}{16 \times 25{,}000{,}000} = .00375 \text{ in. Ans.}$$

SHEAR

18. When two surfaces move in opposite directions and very near together in such a manner as to cut a piece of material, or to pull part of a piece through the remainder, the piece is said to be **sheared**. A good example of shearing is given by a punch; the two surfaces in this case are the bottom of the punch and the top of the die. Another example is a bolt with a thin head; if the pull on the bolt is great enough, it will be pulled through the head and leave a hole in it, instead of the bolt breaking by pulling apart, as would be the case with a thick head. In this case, the two surfaces are the under side of the head and the surface pressed against. Other examples are afforded by a knife cutting a piece of wood and an ordinary pair of shears cutting paper. This kind of stress takes its name from the action of a pair of shears.

19. The formula in Art. **3** applies in cases of shearing stress, but the formulas in Arts. **5** and **10** are never used for shearing. In Table III, E_s and S_s are used to represent, respectively, the modulus of elasticity in shear, and ultimate shearing strength.

TABLE III

SHEAR

Material	Modulus Elasticity E_s Pounds per Square Inch	Ultimate Shearing Strength S_s Pounds per Square Inch
Timber (across the grain) . .		3,000
Timber (with the grain) . . .	400,000	600
Cast iron	6,000,000	20,000
Wrought iron	10,000,000	50,000
Steel	11,000,000	70,000

Example 1.—What force is necessary to punch a 1-inch hole in a wrought-iron plate $\frac{3}{8}$ inch thick?

Solution.— 1 in. × 3.1416 × $\frac{3}{8}$ in. = 1.1781 sq. in. = area of punched surface = area of a cylinder 1 in. in diameter and $\frac{3}{8}$ in. high. Using the formula in Art. **3**,

$$P = A\,S_s = 1.1781 \times 50{,}000 = 58{,}905 \text{ lb.} \quad \text{Ans.}$$

Example 2.—A wooden rod 4 inches in diameter and 2 feet long is turned down to 2 inches diameter in the middle, so as to leave the enlarged ends each 6 inches long; will a steady force pull the rod apart in the middle, or shear the end?

Solution.— $P = A\,S_s = 2 \times 3.1416 \times 6 \times 600 = 22{,}620$ lb., nearly, to shear off one end. The force required to rupture by tension is $P = A\,S_t = 2^2 \times .7854 \times 10{,}000 = 31{,}416$ lb. Since the former is only about two-thirds of the latter, the piece will fail through the shearing off of the end. Ans.

Had a transverse force been used, the force necessary to shear off a section of the end would have been $4^2 \times .7854 \times 3{,}000 = 37{,}700$ pounds, nearly.

FACTORS OF SAFETY

20. It is evident that no force should ever be applied to a machine part that will deform it beyond the elastic limit. The usual practice is to divide the ultimate strength of the material by some number, depending on the kind and quality of the material and on the nature of the stress; this number is called a **factor of safety.** The factor of safety for any material is the ratio of its ultimate strength to the actual stress induced, or the stress that the body is intended to resist.

In Table III, 70,000 pounds per square inch is given as the ultimate shearing strength of steel. Now, suppose that the actual stress in a piece of steel is 10,000 pounds per square inch; the factor of safety for this piece will then be $70,000 \div 10,000 = 7$.

21. To find the proper allowable working strength of a material, divide the ultimate strength for tension, compression, or shearing, as the case may be, by the proper factor of safety.

Table IV gives factors of safety generally used in American practice. Factors of safety will always be denoted by the letter f in the formulas to follow.

TABLE IV

FACTORS OF SAFETY

Material	For Steady Stress Buildings	For Varying Stress Bridges	For Shocks Machines
Timber	8	10	15
Brick and stone . .	15	25	30
Cast iron	6	15	20
Wrought iron . .	4	6	10
Steel	5	7	15

22. Twice as much deformation is caused by a suddenly applied force as by one that is gradually applied. For this reason, a larger factor of safety is used for shocks than for steady stresses. In general, the factor of safety for a given material must be chosen according to the nature of the stress.

The designer usually chooses his own factors of safety. If the material has been tested, or the specifications call for a certain strength, the factor of safety can be chosen accordingly. The general formula then becomes

$$P = \frac{A\,S}{f}$$

Example 1.—Assuming the mortar and brick to be of the same strength, how many tons can be safely laid on a brick column 2 feet square and 8 feet high?

Solution.— $P = A\,S_c = 2 \times 2 \times 144 \times 2{,}500 = 1{,}440{,}000$ lb. = 720 tons. The factor of safety for this case is 15 (see Art. **21**, Table IV); hence, $720 \div 15 = 48$ tons. Ans.

Example 2.—What must be the diameter of the journals of a wrought-iron locomotive axle to resist shearing safely, the weight on the axle being 40,000 pounds?

Solution.—Let f be the factor of safety; then, $P = \frac{A\,S_s}{f}$, or $A = \frac{P f}{S_s}$. Since the axle has two journals, the stress in each journal is 20,000 lb. Owing to inequalities in the track, the load is not a steady one, but varies; for this reason, the factor of safety will be taken as 6. Then, $A = \frac{20{,}000 \times 6}{50{,}000} = 2.4$ sq. in. Therefore, $d = \sqrt{\frac{2.4}{.7854}} = 1\frac{3}{4}$ in., nearly. Ans.

Example 3.—Considering the piston rod of a steam engine as if its length were less than ten times its diameter, what must be the diameter of a steel rod, if the piston is 18 inches in diameter and the steam pressure is 110 pounds per square inch?

Solution.—Area of piston is $18^2 \times .7854 = 254.47$ sq. in. $254.47 \times 110 = 27{,}991.7$ lb., or say, 28,000 lb. = stress in the rod. $A = \frac{P f}{S_c} = \frac{28{,}000 \times 15}{65{,}000} = 6.46$ sq. in. Hence, diameter $= \sqrt{\frac{6.46}{.7854}} = 2.869$ in., say $2\frac{7}{8}$ in. Ans.

23. When designing a machine, care should be taken to make every part strong enough to resist any force likely to be applied to it, and to make all parts of equal strength.

The reason for the first statement is obvious, and the second should be equally clear, since no machine can be stronger than its weakest part (proportioned, of course, for the load it is to bear), and those parts of the machine that are stronger than others contain an excess of material that is wasted. In actual practice, however, this second rule is frequently modified. Some machines are intended to be massive and rigid, and need an excess of material to make them so; in others there are difficulties in casting that modify the rule, etc. In most cases the designer must rely on his own judgment.

EXAMPLES FOR PRACTICE

1. A cast-iron bar is subjected to a steady tensile force of 120,000 pounds; the cross-section is an ellipse whose axes are 6 and 4 inches. (*a*) What is the stress per square inch? (*b*) What load will the bar carry with safety?
Ans. $\left\{\begin{array}{l}(a)\ 6{,}366.18 \text{ lb. per sq. in.}\\ (b)\ 62{,}832 \text{ lb.}\end{array}\right.$

2. How much will a piece of steel 2 inches square and 10 inches long shorten under a load of 300,000 pounds? Ans. .025 in.

3. A wrought-iron tie-rod is $\frac{3}{4}$ inch in diameter; how long must it be to lengthen $\frac{3}{8}$ inch under a steady pull of 5,000 pounds?
Ans. 69 ft., nearly

4. A steel bar having a cross-section of 5 in. × 4 in. and a length of 14 feet is lengthened .036 inch by a steady pull of 120,000 pounds; what is its modulus of elasticity? Ans. 28,000,000 lb. per sq. in.

5. Which is the stronger, weight for weight, a bar of chestnut wood whose tensile strength is 12,000 pounds per square inch and specific gravity .61, or a bar of steel whose tensile strength is 125,000 pounds per square inch? Ans. The wood

6. What should be the diameter of a cast-iron pin subjected to a suddenly applied shearing force of 20,000 pounds to withstand the shocks with safety? Ans. $5\frac{1}{16}$ in., nearly

7. What steady load may safely be placed on a brick column 2 feet square and 9 feet high? Ans. 96,000 lb.

PIPES AND CYLINDERS

24. A pipe or cylinder subjected to an internal pressure of steam or water is deformed equally in all its parts, and when rupture occurs, it is in the direction of its length.

Let d = inside diameter of pipe, in inches;
l = length of pipe, in inches;
p = pressure, in pounds per square inch;
P = total pressure, in one direction.

Then,
$$P = p\,l\,d \qquad (1)$$

This formula is derived from a principle of hydrostatics that the pressure of water in any direction is equal to the pressure on a plane perpendicular to that direction. In Fig. 1, suppose the direction of pressure to be as shown by the arrows; $A\,B$ will then be the plane perpendicular to this direction, the width of the plane being equal to the diameter, and the length equal to the length of the pipe. The area of the plane will then be $l \times d$, and the total pressure $P = p \times l \times d$ as before.

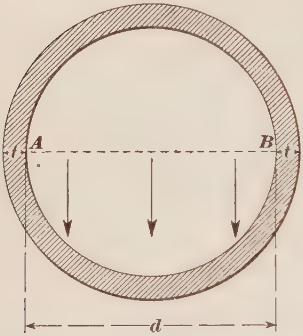


FIG. 1

Suppose the pipe to have a thickness t, and let S be the working strength of the material $= \frac{S_t}{f}$; then the resistance of the pipe on each side is $t\,l\,S$. Resistance must equal pressure; therefore, $p\,l\,d = 2\,t\,l\,S$, or

$$p\,d = 2\,t\,S \qquad (2)$$

Also,
$$p = \frac{2\,t\,S}{d} \qquad (3)$$

which is the maximum pressure a pipe of a given material and of given dimensions can safely stand.

The pressure of water per square inch may be found by the formula, $p = .434\,h$, where h is the head in feet. In pipes where shocks are likely to occur, the factor of safety should be high. The thickness of a pipe to resist a given pressure varies directly as its diameter, the pressure remaining constant. A formula for the thickness is readily obtained from formula **2**, which becomes

$$t = \frac{p\,d}{2\,S} \qquad (4)$$

EXAMPLE 1.—Find the factor of safety for a cast-iron water pipe 12 inches in diameter and $\frac{7}{8}$ inch thick, under a head of 350 feet.

SOLUTION.—Here p, pressure per square inch, equals $.434\,h = .434 \times 350 = 151.9$ lb. Using formula **3**, by transposing and substituting the values given, $p\,d = 2\,t\,S$, or $S = \frac{p\,d}{2\,t} = \frac{151.9 \times 12}{2 \times \frac{7}{8}} = 1{,}041.6$ lb. per sq. in. In Table I, Art. **14**, the ultimate tensile strength of cast iron is given as 20,000 lb. per sq. in.; then, the factor of safety is

$$f = \frac{20{,}000}{1{,}041.6} = 19.2. \quad \text{Ans.}$$

This is so near the factor for shocks that, in practice, the pipe would be considered safe against shocks.

EXAMPLE 2.—Find the proper thickness for a wrought-iron steam pipe 18 inches in diameter to resist a pressure of 140 pounds per square inch.

SOLUTION.—Using a factor of safety of 10, the working strength $S = 50{,}000 \div 10 = 5{,}000$ lb. per sq. in. From formula **4**, $t = \frac{p\,d}{2\,S} = \frac{140 \times 18}{2 \times 5{,}000} = 0.252$ in. In practice, however, the thickness is made somewhat greater than the formula requires.

CYLINDERS

25. The tendency of a cylinder subjected to internal pressure is to fail or rupture in the direction of its length, as in the case of a pipe.

Let d = inside diameter of a cylinder, in inches;
p = pressure in cylinder, in pounds per square inch;
t = thickness of cylinder wall, in inches.

Then the area of the cylinder head against which the pressure acts is $\frac{1}{4}\pi d^2$, and,

the total pressure on the cylinder head $= \frac{1}{4}\pi d^2 p$

This pressure tends to rupture the cylinder by pulling it apart on a line, around the cylinder, that has a length of

πd inches. The area of metal that resists the tendency to rupture is, therefore, $\pi d t$ square inches.

Let S, equal the working unit stress, equal $\frac{S_t}{f}$; then $\pi d t S$ equals the resistance to rupture caused by the pressure acting on the opposite cylinder heads and tending to elongate the cylinder. Since the resistance must equal the force, or pressure, $\frac{1}{4}\pi d^2 p = \pi d t S$, or

$$p d = 4 t S \qquad (1)$$

Also,
$$p = \frac{4 t S}{d} \qquad (2)$$

Since, by formula **3**, Art. **24**, for longitudinal rupture, $p = \frac{2 t S}{d}$, it is seen that a cylinder is twice as strong against transverse rupture as against longitudinal rupture. Hence, other things being equal, a cylinder will always fail by longitudinal rupture.

26. The foregoing formulas are for comparatively thin pipes and cylinders, in which the thickness is less than about one-tenth the inside radius. For pipes and cylinders whose thickness is greater than one-tenth the inside radius, the following formula is used, in which r equals the inner radius, and the other letters have the same meaning as before:

$$p = S\frac{(r+t)^2 - r^2}{(r+t)^2 + r^2} \qquad (1)$$

Using the data given in the first example of Art. **24**, and substituting them in the above formula,

$$151.9 = S\frac{(6+\frac{7}{8})^2 - 6^2}{(6+\frac{7}{8})^2 + 6^2}, \text{ or } S = \frac{151.9\left[(6+\frac{7}{8})^2 + 6^2\right]}{(6+\frac{7}{8})^2 - 6^2}$$

from which, $S = 1{,}122.7$ pounds, which is 7.7 per cent. greater than the result obtained by formula **2**, Art. **24**.

In the case of a sphere, the pressure acts outwardly, and the forces tending to rupture it along any line will amount to $\frac{1}{4}\pi d^2 p$; that is, the total force tending to rupture a sphere will be the total force tending to tear one half of the sphere from the other half. This will be resisted by the metal of thickness t around the sphere, or a total area of $\pi d t$ square inches; and if the working unit stress is S, as before, the total stress will be $\pi d t S$. The internal pressure must equal the resistance to rupture, hence, $\frac{1}{4}\pi d^2 p = \pi d t S$. The formula for spheres is, therefore, the same as that for transverse rupture of cylinders, or

$$p d = 4 t S \qquad (2)$$

27. Collapsing Pressure.—A cylinder under external pressure is, theoretically, in a condition similar to one under internal pressure, so long as its cross-section remains a true circle. A uniform internal pressure tends to preserve the true circular form, but an external pressure tends to increase the slightest variation from the circle and to render the cross-section elliptical. The distortion, when once begun increases rapidly, and failure occurs by the collapsing of the tube rather than by the crushing of the material. The flues and tubes of a steam boiler are the most common instances of cylinders subjected to external pressure.

The letters having the same meaning as before, the following formula is frequently used to find the collapsing pressure in pounds per square inch for wrought-iron pipe:

$$p = 9{,}600{,}000 \frac{t^{2.18}}{l d} \qquad (1)$$

Formula 1 is known as **Fairbairn's formula,** and was deduced from experiments on flues from 4 to 12 inches in diameter and from 20 to 60 inches long. In using the formula, a proper factor of safety should be chosen; factors of from 4 to 6 may be used for small flues under steady and varying stresses, and a factor of 10 for shocks.

The following formulas are based on recent experiments by Prof. R. T. Stewart with commercial lap-welded steel tubes from 3 to 10 inches in diameter, and of varying lengths up to 20 feet:

$$p = 86{,}670 \frac{t}{d} - 1{,}386 \qquad (2)$$

and

$$p = 1{,}000 \left(1 - \sqrt{1 - 1{,}600 \frac{t^2}{d^2}}\right) \qquad (3)$$

in which p = collapsing pressure, in pounds per square inch;

t = thickness of tube, in inches;

d = outside diameter of tube, in inches.

Formula **2** applies to those cases in which $\frac{t}{d}$ is greater than .023 and p is greater than 581 pounds, and formula **3** to cases in which $\frac{t}{d}$ and p are less than these values.

It will be seen that the length of the tube is not considered in formulas **2** and **3**. Prof. Stewart's experiments indicate that the length of the tube has practically no influence on the collapsing pressure of an ordinary lap-welded steel tube, so long as the length is not less than about six times the diameter of the tube.

Fairbairn's formula may be considered as reasonably accurate for conditions similar to those on which it is based. For cases differing from these, the results obtained are not reliable.

The thickness usually given to boiler tubes is such as to provide for a good weld and to allow the tubes to be expanded into the tube-sheets, and this thickness is in excess of that necessary to prevent collapsing.

EXAMPLE.—What must be the thickness of a wrought-iron boiler flue 8 inches in diameter and 5 feet long, if the steam pressure is to be not over 160 pounds per square inch?

SOLUTION.—Using formula **1**, with a factor of safety of 4, and solving for t,

$$t = \sqrt[2.18]{\frac{4\,p\,l\,d}{9{,}600{,}000}} = \sqrt[2.18]{\frac{4 \times 160 \times 5 \times 12 \times 8}{9{,}600{,}000}} = \sqrt[2.18]{\frac{4}{125}}$$

Hence, $\log t = \frac{\log 4 - \log 125}{2.18} = \bar{1}.31429$, or $t = .2062$, say $\frac{7}{32}$ in. Ans.

EXAMPLES FOR PRACTICE

1. What must be the thickness of a 16-inch cast-iron stand pipe that is subjected to a head of water of 250 feet? Assume that the stress is steady. Ans. .26 in.

2. What pressure per square inch can be safely sustained by a cast-iron cylinder 12 inches in diameter and 3 inches thick? Ans. 1,282 lb. per sq. in., nearly

3. What is the collapsing pressure of a lap-welded steel tube 4 inches outside diameter and .11 inch thick? Ans. 997 lb. per sq. in.

4. A cast-iron cylinder 14 inches in diameter sustains an internal pressure of 1,500 pounds per square inch; what is the necessary thickness, assuming that the pressure is gradually applied and that the cylinder is not subjected to shocks? Ans. $4\frac{3}{8}$ in.

5. A cylindrical boiler shell 3 feet in diameter is subjected to a steady hydrostatic pressure of 180 pounds per square inch; what should its thickness be if made of steel having a tensile strength of 60,000 pounds per square inch? Ans. .27 in.

ELEMENTARY GRAPHIC STATICS

FORCE DIAGRAM AND EQUILIBRIUM POLYGON

28. Before taking up the subject of flexure, some fundamental principles of graphic statics not heretofore considered will be explained and applied to the case of beams. The polygon of forces was used to find the resultant of several forces having a common point of application, or whose lines of action passed through a common point. A method of finding the resultant will now be given when the forces lie, or may be considered as lying, in the same plane, though their lines of action do not pass through a common point.

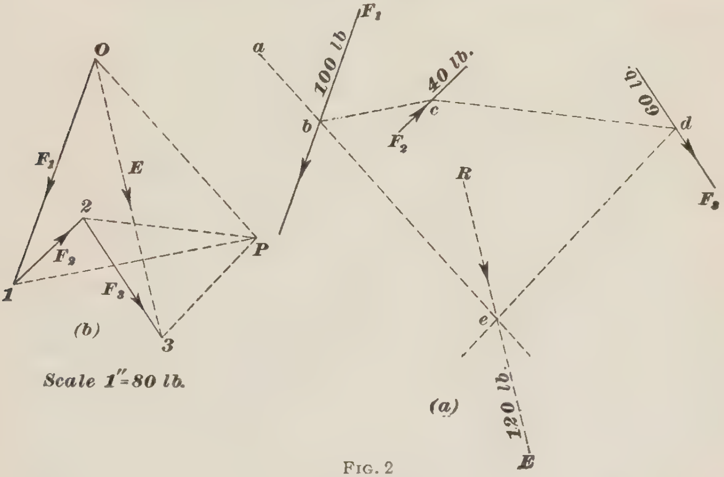


FIG. 2

In Fig. 2 (*a*), let F_1, F_2, and F_3 be three forces whose magnitudes are represented by the lengths of their respective lines, and their directions by the positions of the lines and by the arrowheads. Construct the polygon of forces *O 1 2 3 O* as shown in Fig. 2 (*b*), *O 3* representing the direction and magnitude of the resultant. Everything is now known except the line on which the point of application of the resultant must lie. To find this, proceed as follows:

Choose any point *P*, and draw *P O*, *P 1*, *P 2*, and *P 3*. Choose any point *b* on the line of direction of one of the forces, as F_1, and draw lines through *b* parallel to *P O* and *P 1*, the latter intersecting F_2, or F_2 prolonged, in *c*. Draw *c d* parallel to *P 2* and intersecting F_3, or F_3 prolonged, in *d*. Draw *d e* parallel to *P 3* and intersecting the line *a b e* parallel to *P O* in *e*. The point *e* is a point on the line of direction of the resultant of the three forces. Hence, through *e*, draw *R* parallel and equal to *O 3* and acting in the same direction; it will be the resultant. This method is applicable to any number of forces considered as acting in the same plane. The resultant can also be found when the forces act in different planes, but the method of finding it will not be described here.

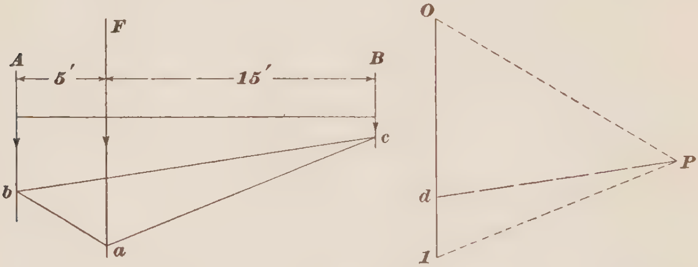


FIG. 3

The point *P* is called the **pole**; the lines *P O*, *P 1*, *P 2*, *P 3* joining the pole with the vertexes of the force polygon are called the **strings** or **rays**; the **force diagram** is the figure composed of the force polygon *O 1 2 3 O*, the pole, and the strings. The polygon *b e d c b* is called the **equilibrium polygon**. Since the pole *P* may be taken anywhere, any number of force diagrams and equilibrium polygons may be drawn, all of which will give the same value for the resultant, and whose lines de and ae will intersect on the resultant R. To test the accuracy of the work, take a new pole and proceed as before. If the work has been done correctly, de and ae will intersect on R.

The equilibrium polygon gives an easy method of resolving a force into two components.

Example.—In Fig. 3, let $F = 16$ pounds be the force, and let it be required to resolve it into two parallel components, A and B, at distances respectively of 5 feet and 15 feet from F. What will be the magnitudes of A and B?

Solution.—Draw $O\,1$ to represent $F = 16$ lb. Choose any convenient pole P and draw the rays PO and $P1$. Take any point a on F and draw ab parallel to PO, intersecting A in b, and ac parallel to $P1$, intersecting B in c. Join b and c by the line bc. Through the pole P draw Pd parallel to bc, intersecting $O\,1$ in d. Then Od is the magnitude of A, measured to the scale to which $O\,1$ was drawn, and $d\,1$ is the magnitude of B to the same scale.

29. If the components are not parallel to the given force, they must intersect its line of direction in a common point. In Fig. 4, let $F = 16$ pounds be the force; it is required to resolve it into two components A and B, intersecting at a, as shown. Draw $O\,1$ to some convenient scale equal to 16 pounds; then draw OP and $1\,P$ parallel to A and B, and OP and $P1$ are the values of the components A and B, respectively, both in magnitude and direction.

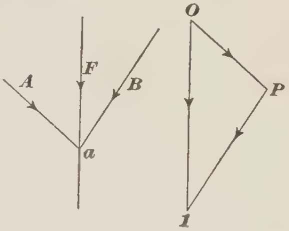


Fig. 4

Example.—Let F_1, F_2, F_3, F_4, and F_5, Fig. 5, be five forces whose magnitudes are 7, 10, 5, 12, and 15 pounds, respectively; it is required to find their resultant and to resolve this resultant into two components parallel to it and passing through the points a and b.

Solution.—Choose any point O, Fig. 5, and draw $O\,1$ parallel and equal to F_1; 1–2 parallel and equal to F_2, etc.; $O\,5$ will be the value of the resultant, and its direction will be from O to 5, opposed to the other forces acting around the polygon. Choose a pole P, and complete the force diagram. Choose a point c on F_1, and draw the equilibrium polygon *c d e f g h c*; the intersection of *c h*, parallel to *P O*, and *g h*, parallel to *P 5*, gives a point *h*. Through *h*, draw *R* parallel to *O 5*, and it will be the position of the line of action of the resultant of the five forces. The components must pass through the points *a* and *b*, according to the conditions; hence, through *a* and *b*, draw V_1 and V_2 parallel to *R*. Since *O 5* represents the magnitude of *R*, draw *h k* and *h l* parallel to *P O* and *P 5*, respectively, as in Fig. 3 (they, of course, coincide with *c h* and *g h*, since the same pole *P* is used), intersecting V_1 and V_2 in *k* and *l*. Join *k* and *l*, and draw *P Q* parallel to *k l*. Then, $OQ = V_1$, and $Q5 = V_2$.

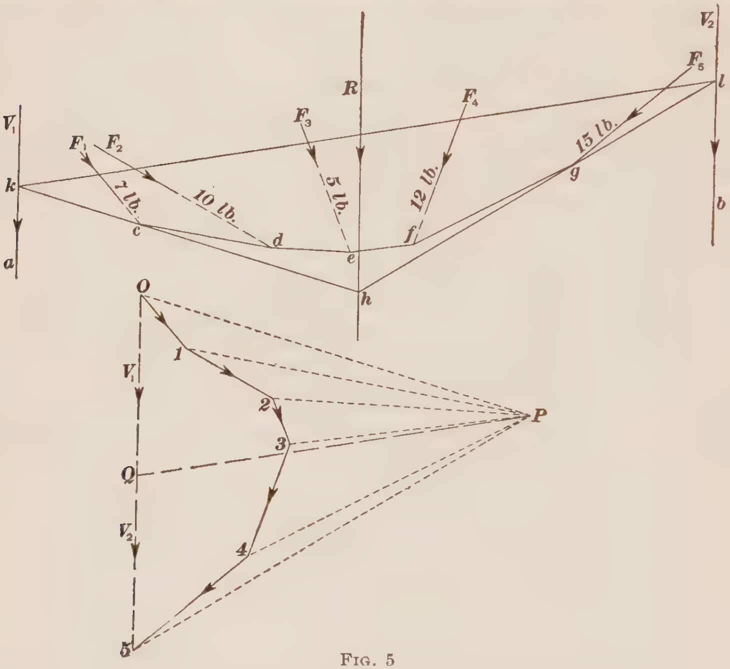


FIG. 5

COMPOSITION OF MOMENTS

30. According to the principles of mechanics, the **moment** of a force about a point is the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force. A force can act in two ways on a body: it can either produce a motion of translation—that is, cause all the points of the body to move in straight parallel lines—or it can produce a motion of rotation—that is, make the body turn. A moment measures the capacity of a force to produce rotation about a given point. For example, suppose, in Fig. 6, that AC is a lever 30 inches long, having a fulcrum at B 10 inches from A. If a weight is suspended from C, it will cause the bar to rotate about B in the direction of the arrow at that end. A weight suspended from A will cause it to revolve in the opposite direction, as indicated by the arrow. Suppose, for simplicity, that the bar itself has no weight. If two weights of 12 pounds each are hung at A and C, it is evident that the bar will revolve in the direction of the arrow at C, because the arm BC is longer than the arm AB. Let the weight at A be increased until it equals 24 pounds. The bar will then balance exactly, and any additional weight at A will cause the bar to rotate in the opposite direction, as shown by the arrow at that point. When the lever is balanced, it will be found that $24 \times 10 = 12 \times 20$, or considering B as the center of moments, $24 \times$ perpendicular distance $AB = 12 \times$ perpendicular distance BC. In other words, the moment of W about B must equal the moment of P about B—that is, the two moments must be equal. Further, P tends to cause rotation in the direction in which the hands of a watch move and for convenience will be considered positive, or $+$; W tends to cause rotation in a direction opposite to the hands of a watch and will be considered negative, or $-$. Adding the two algebraically, $P \times BC + (-W \times AB) = P \times BC - W \times AB = 0$, since the two moments are equal. Hence the following general rule:

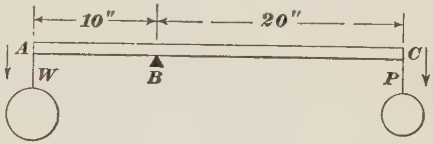


FIG. 6

Rule.—*One of the necessary conditions of equilibrium is that the algebraic sum of the moments of all the forces about a given point shall equal zero.*

Applications of this rule will occur further on.

GRAPHIC EXPRESSIONS FOR MOMENTS

31. The moment of a single force may be expressed graphically in the following manner: In Fig. 7, let $F = 10$ pounds be the given force and c, at a distance from $F = fc = 7\frac{1}{2}$ feet, be the center of rotation or center of moments. Draw $O1$ parallel to F and equal to 10 pounds. Choose any point P as a pole, and draw the rays PO and $P1$; also draw $P2$ perpendicular to $O1$. Through any point b on F, draw the sides ab and gb of the equilibrium polygon; they correspond to be and de, Fig. 2, through the intersection of which the resultant must pass, the resultant F being given in the present case. Prolong ab, and draw ed through c parallel to F, intersecting bg and ab in d and e, respectively.

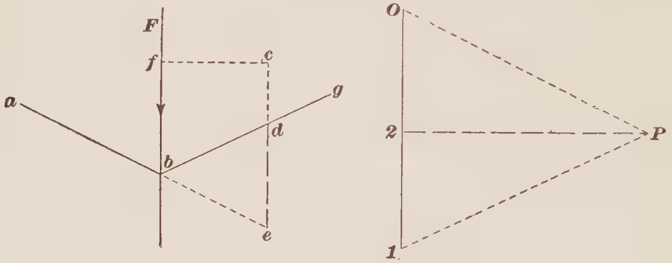


FIG. 7

By geometry, it can be proved that the triangles bde and $O1P$ are similar and that the sides of the one are proportional to the corresponding sides of the other. Hence $P1 : O1 = bd : de$, and since $P2$ and fc are perpendiculars from the vertexes to the bases of similar triangles, $P2 : O1 = fc : de$, or $\frac{P2}{O1} = \frac{fc}{de}$. Then $P2 \times de = O1 \times fc$. But $O1 = F$ and fc = the length of the moment arm. Then $O1 \times fc$ = the moment of F about c, and therefore $P2 \times de$, which equals $O1 \times fc$, is also equal to the moment of F about c, when $P2$ is measured to the same scale as $O1$, and ed is measured to the same scale as cf.

The line $P2$ is called the **pole distance** and will hereafter always be denoted by the letter H. The line de is called the

intercept. Hence, the pole distance multiplied by the intercept equals the moment, or, denoting the intercept by y, the moment $= Hy$.

The statement just made is one of the most important facts in graphical statics, and should be thoroughly understood. In the triangle $PO1$, the lines PO and $P1$ represent the components of the force F in the directions ab and gb, respectively, while the lines of action of those components are ab and gb, meeting at b. As de is limited by gb and ab (produced), the following definition is given: The *intercept* of a force whose moment about a point is to be found is the segment (or portion) that the two components (produced, if necessary) cut off from a line drawn through the center of moments parallel to the direction of the force.

32. The pole distance and intercept for the moment of several forces about a given point may be determined in a similar manner by first finding the magnitude and position of the resultant of all the forces; the moment of this resultant about the given point will equal the value of the resultant moment of all the forces that tend to produce rotation about that point.

Example.—Let $F_1 = 20$ pounds, $F_2 = 25$ pounds, and $F_3 = 18$ pounds be three forces acting as shown in Fig. 8; find their resultant moment about the point C.

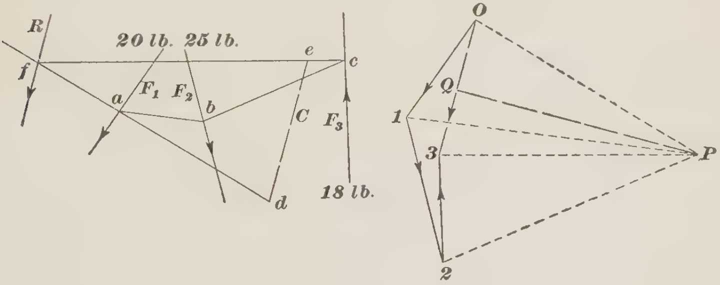


Fig. 8

Solution.—Draw the force diagram, equilibrium polygon, and resultant R as previously described. Draw dCe parallel to R. The intercept de, multiplied by the pole distance PQ = the resultant moment.

33. If all the forces are parallel, the force polygon is a straight line; this is evident, since, if a line of the force polygon be drawn parallel to one of the forces and from one end of this line a second line be drawn parallel to another force, the second line will coincide with the first.

EXAMPLE.—Let $F_1 = 30$, $F_2 = 20$, and $F_3 = 20$, all in pounds, be three parallel forces acting downwards, as shown in Fig. 9. It is required to find their resultant moment (algebraic sum of the moments) and the moment of their resultant, all moments to be taken about the point C.

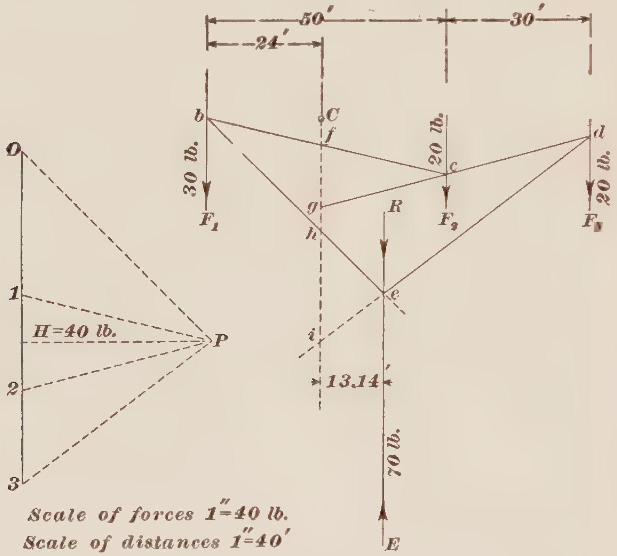


FIG. 9

SOLUTION.—Lay off $O\,1$ = 30 lb. = F_1, 1–2 = 20 lb. = F_2, and 2–3 = 20 lb. = F_3, and $O\,3$ is the value of the resultant. Choose some point P as a pole and draw the rays. Take any point, as b, on any force, as F_1, and complete the equilibrium polygon $b\,c\,d\,e\,b$; then, the line of action of the resultant must pass through e. Through C draw $C\,i$ parallel to R and prolong $d\,e$. The moment of R about C equals the pole distance H, multiplied by the intercept $h\,i$, since $h\,i$ is that part of the line drawn through C parallel to R, and included between the lines $b\,e$ and $d\,e$ of the equilibrium polygon, which meet on R. Measuring $h\,i$ to the scale of distances (1 in. = 40 ft.), it equals 23 ft. Measuring H to the scale of forces, it equals 40 lb.

Consequently, the moment of R about $C = 40 \times 23 = 920$ ft.-lb. Considering the force F_3, the intercept is $g\,i$, since F_3 is parallel to R, and, consequently, to $C\,i$; also $g\,i$ is that part of the line $C\,i$ included between the sides $c\,d$ and $d\,e$, which meet on F_3. Measuring $g\,i$, it is found to equal 28 ft. Hence, the moment of F_3 about $C = 40 \times 28 = 1{,}120$ ft.-lb. The moment of F_2 about $C = H \times fg = 40 \times 13 = 520$ ft.-lb. The moment of $F_1 = H \times f\,h = 40 \times 18 = 720$ ft.-lb. Now F_3 and F_2 have positive moments, since they tend to cause rotation in the direction of the hands of a watch, while F_1 has a negative moment, since it tends to cause rotation in the opposite direction. Consequently, adding the moments algebraically, the resultant moment $= 1{,}120 + 520 - 720 = 920$ ft.-lb., the same as the moment of the resultant.

Having described the fundamental principles of graphic statics, the consideration of the strength of materials will now be continued, and the stresses due to flexure and torsion discussed.

BEAMS

DEFINITIONS

34. Any bar resting in a horizontal position on supports is called a **beam.**

A **simple beam** is a beam resting on two supports very near its ends.

A **cantilever** is a beam resting on one support in its middle, or which has one end fixed (as in a wall) and the other end free.

An **overhung beam** is a beam resting on two supports with one or both ends projecting beyond the supports and sustaining loads.

A **restrained beam** is one that has both ends fixed (as a plate riveted to its supports at both ends).

A **continuous beam** is one that rests on more than two supports. The continuous beam will not be discussed here, as the subject requires a knowledge of higher mathematics.

A **load** on a beam is the weight that the beam supports.

A **concentrated** load is one that acts on the beam at one point.

A **uniform** load is one that is equally distributed over the beam, so that each unit of length has the same load.

In the solution of problems in flexure, two methods are used, namely, the **analytic** and the **graphic.** By the analytic method, the results are obtained by calculation and the use of formulas; by the graphic method, they are obtained by means of diagrams drawn accurately to scale, as illustrated in the preceding articles on Graphic Statics. In the following explanations and solutions, both methods will be given. In practice, the one may be used that is best suited to the work that is to be done; or one may be used as a check on the other.

SIMPLE BEAMS

REACTIONS OF SUPPORTS

35. All forces that act on beams will be considered as vertical, unless distinctly stated otherwise. According to the third law of motion, every action has an equal and opposite reaction; hence, when a beam is acted on by downward forces, the supports react upwards. In problems of construction in which beams are used, the weights, or downward forces, acting on the beams are usually known; from these, it is required to find the value of the reaction at each support. If a simple beam is uniformly loaded or has a load in the middle, it is evident that the reaction of each support is one-half the load plus one-half the weight of the beam. If the load is not uniformly distributed over the beam, or if the load or loads are not in the middle, the reactions of the two supports will be different. In order that a beam may be in equilibrium, three conditions must be fulfilled:

1. The algebraic sum of all vertical forces must equal zero; otherwise the beam will move up or down.
2. The algebraic sum of all horizontal forces must equal zero; otherwise the beam will move sidewise.
3. The algebraic sum of the moments of all forces about any point must equal zero; otherwise the beam will turn about that point.

Since the loads act downwards and the reactions upwards, the first condition states that the sum of all the loads must equal the sum of the reactions of the supports. The reactions and forces acting upwards will be considered as positive, or +, and the downward weights as negative, or −. It is plain that the first condition of equilibrium is satisfied when the sum of the positive forces and reactions is equal to the sum of the negative forces.

36. Reactions for Concentrated Loads.—The principles just stated apply directly to a simple beam with concentrated loads. Let AB, Fig. 10, be such a beam, with supports at A and B, through which the lines of the reactions R_1 and R_2 pass, and let F_1, F_2, and F_3 be concentrated loads at distances l_1, l_2, and l_3 from the left support, as shown, l being the length of the beam between supports. Let l, l_1, l_2, and l_3 be in feet, and R_1, R_2, F_1, F_2, and F_3 be in pounds. Then $R_1 + R_2 - F_1 - F_2 - F_3 = 0$, or

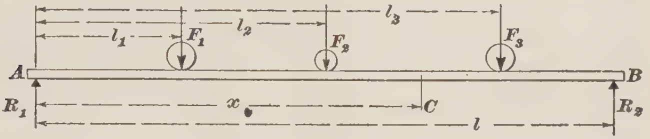


FIG. 10

$$R_1 + R_2 = F_1 + F_2 + F_3 \qquad (1)$$

Taking the moments about R_1 as a center, $R_2 l - F_1 l_1 - F_2 l_2 - F_3 l_3 = 0$, or

$$R_2 = \frac{F_1 l_1 + F_2 l_2 + F_3 l_3}{l} \qquad (2)$$

The moments may also be taken about the right support. Then, $R_1 l - F_1(l - l_1) - F_2(l - l_2) - F_3(l - l_3) = 0$, or

$$R_1 = \frac{F_1(l - l_1) + F_2(l - l_2) + F_3(l - l_3)}{l} \qquad (3)$$

No matter how many concentrated loads there may be on the beam, the same principle applies. The above formulas are the expression of the statements 1 and 3 of Art. **35.**

EXAMPLE.—Let R_1, Fig. 11, be the reaction of the left support, R_2 the reaction of the right support, and let the distance between the two supports be 14 feet; suppose that loads of 50, 80, 100, 70, and 30 pounds are placed on the beam at distances from the left support equal to 2, 5, 8, 10, and 12½ feet, respectively. What are the reactions of the supports, neglecting the weight of the beam?

ANALYTIC SOLUTION.—Writing formula **2** for five loads,

$$R_2 = \frac{F_1 l_1 + F_2 l_2 + F_3 l_3 + F_4 l_4 + F_5 l_5}{l}$$

Substituting the values given in the example,

$$R_2 = \frac{50 \times 2 + 80 \times 5 + 100 \times 8 + 70 \times 10 + 30 \times 12.5}{14} = \frac{2{,}375}{14} = 169\tfrac{9}{14} \text{ lb.}$$

Applying formula **3** in the same manner,

$$R_1 = \frac{F_1(l - l_1) + F_2(l - l_2) + F_3(l - l_3) + F_4(l - l_4) + F_5(l - l_5)}{l}$$

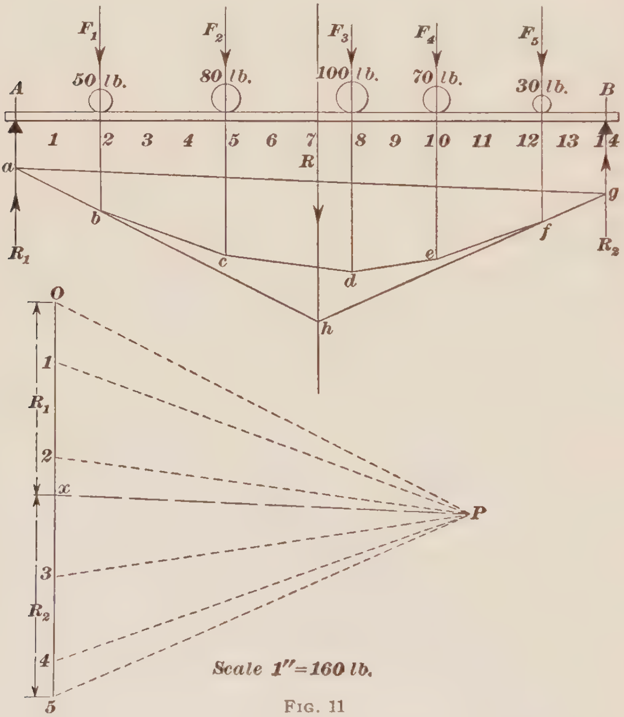


FIG. 11

Substituting the values,

$$R_1 = \frac{50 \times 12 + 80 \times 9 + 100 \times 6 + 70 \times 4 + 30 \times 1.5}{14} = \frac{2{,}245}{14} = 160\tfrac{5}{14} \text{ lb.}$$

Now, formula **1** can be applied as a check. Written for five loads, it becomes $R_1 + R_2 = F_1 + F_2 + F_3 + F_4 + F_5$. Substituting, $R_1 + R_2 = 160\tfrac{5}{14} + 169\tfrac{9}{14} = 330$; and $F_1 + F_2 + F_3 + F_4 + F_5 = 50 + 80 + 100 + 70 + 30 = 330$ lb. The results are thus shown to be correct.

Graphic Solution.—The reactions may also be found graphically by resolving the resultant of the weights (which, in this case, acts vertically downwards) into two parallel components, passing through the points of support. The sum of the reactions is equal to the sum of the components, but the two sums have different signs. Draw the beam to some convenient scale, and locate the loads as shown in Fig. 11. Draw the force diagram, making *O 1* = 50 lb., *1–2* = 80 lb., *2–3* = 100 lb., *3–4* = 70 lb., and *4–5* = 30 lb.; *O 5* represents by its length the sum of the weights or their resultant, thus representing the force polygon. Choose any point as *P* and draw *P O*, *P 1*, *P 2*, etc.

Choose a point *b* on the line of action of the force F_1, and draw the equilibrium polygon *a b c d e f g a*, as in Fig. 9; *a b* and *f g* intersect at *h*, the point through which the resultant *R* must pass. Draw *P x* parallel to *a g*, and *O x* will be the reaction (= component) R_1, and *x 5* the reaction R_2. Measuring *O x* and *x 5* to the same scale used to draw *O 5*, $R_1 = 161$ lb., and $R_2 = 169$ lb. By calculation, it was found that $R_1 = 160\frac{5}{14}$ lb., and $R_2 = 169\frac{9}{14}$ lb. This shows that the graphic method is accurate enough for all practical purposes. The larger the scale used, the more accurate will be the results.

VERTICAL SHEAR

37. In Fig. 11, imagine that part of the beam at a very short distance to the left of a vertical line passing through the point of support *A* to be acted on by the reaction $R_1 = 160$ pounds, and that part to the right of the line to be acted on by the equal downward force due to the load. The two forces acting in opposite directions tend to shear the beam.

Suppose that the line had been situated at the point *3* instead of at *A*; the reaction upwards would then be partly counterbalanced by the 50-pound weight, and the total reaction at this point would be $160 - 50 = 110$ pounds. Since the upward reaction must equal the downward load at the same point, the downward force at *3* also equals 110 pounds, and the shear at this point is 110 pounds. At the point *6*, or any point between *5* and *8*, the downward force due to the weight at the left is $50 + 80 = 130$ pounds, and the upward reaction is 160 pounds. The resultant shear is, therefore, $160 - 130 = 30$ pounds. At any point between *8* and *10*, the shear is $160 - (50 + 80 + 100) = -70$ pounds.

The negative sign means nothing more than that the weights exceed the reaction of the left-hand support.

The vertical shear equals the reaction of the left-hand support, minus all the loads on the beam to the left of the point considered. Therefore, the point of zero vertical shear comes at that section where the sum of the loads to the left of and at the section first equals the left-hand reaction.

For a simple beam, the greatest positive shear is at the left-hand support, and the greatest negative shear is at the right-hand support, and both shears are equal to the reactions at these points.

38. Graphic Representation of Shear.—The vertical shear may be represented graphically as shown in Fig. 12, which is Fig. 11 repeated. Draw the force diagram,

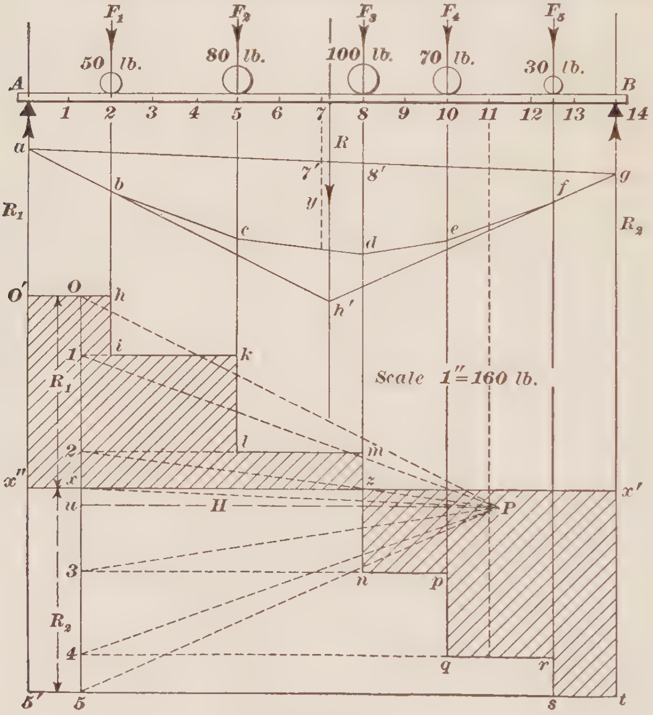


FIG. 12

continue the lines of action of R_1 and R_2 downwards, and make $O' 5' = O 5$. Through x draw the horizontal line $x'' x'$ called the **shear axis.** The vertical shear is the same for any point between A and 2 and equals $O x = O' x'' = 160$; hence, draw $O' h$ parallel to $x x'$, and any ordinate measured from $x x'$ to $O' h$ between O' and $h = 160$ pounds = the vertical shear at any point between O' and h when $O' h = A 2$. $O 1$, it will be remembered, is equal to 50 pounds, or the force F_1. Through 1 draw $1 k$ parallel to $x'' x'$, and project the points 2 and 5 on it, in i and k. Then, the length of the ordinate between $x'' x'$ and $i k$ equals the vertical shear between 2 and 5. In the same way, find the remaining points l, m, etc. The broken line $O' h i \ldots t$ is called the **shear line,** and the figure $O' h i \ldots r s t x' x''$ is called the **shear diagram.** To find the shear at any point, as *11*, project the point on the shear axis and measure the ordinate to the shear line, drawn through the projected point. If the ordinate is measured from the shear axis upwards, the vertical shear is positive; if downwards, it is negative. For the point *11*, the vertical shear is -140 pounds. The maximum negative vertical shear is -170 pounds $= x' t = x 5$. The greatest shear, whether positive or negative, is the one which the beam must be designed to withstand. It should be noticed that the shear passes from a positive to a negative value under point *8*, as shown by the line $m n$ crossing $x'' x'$.

BENDING MOMENT

39. A beam seldom fails by shearing, but generally by bending and breaking under its load; that is, it fails by flexure. In order to design a beam to resist flexure, the maximum bending moment must be known.

The **bending moment** *at any point of a beam is the algebraic sum of the moments of all the forces (the reaction included) acting on the beam, on either side of that point, the point being considered as the center of moments.*

The expression *algebraic sum* refers to the fact that, when considering the forces acting at the left of the point taken, the moments of all the forces acting upwards are considered positive, and the moments of all the forces acting downwards are considered negative. Hence, the algebraic sum is the moment of the left reaction about the given point, minus the sum of the moments, about the same point, of all the downward forces between the reaction and the given point. Should there be any force or forces acting upwards, their moments must be added, since they are positive. If the forces on the right of the point are considered, all lever arms are negative, distances to the left of the point being +, those to the right being − (see Art. **30**). Hence, the downward forces give positive moments, and the right reaction gives a negative moment. This is as it should be, for the downward forces on the right and the upward forces on the left tend to rotate the beam in the direction of the hands of a watch; while the downward forces on the left and the upward forces on the right tend to rotate the beam in the opposite direction.

40. Bending Moment by Analytic Method.—Consider the beam AB, Fig. 10, with concentrated loads F_1, F_2, and F_3, and reactions R_1 and R_2. Let C be any point in the beam at a distance of x feet from R_1. The algebraic sum of the moments of the forces on either side of the point C are equal; those to the left will be positive, having the plus sign, and those to the right will be negative, having the minus sign. Letting M represent the bending moment at C, in foot-pounds, and taking the moments of the forces to the left of C in order to get positive values for M, then with C as a center of moments,

$$M = R_1 x - F_1(x - l_1) - F_2(x - l_2) \qquad (1)$$

If C is taken between R_1 and F_1, and the moments of the forces to the left of C are considered, the formula for the bending moment is

$$M = R_1 x \qquad (2)$$

This expression gives the moment for any section of the beam between R_1 and F_1. At R_1 the value of M is zero because x is zero and the value of M, the bending moment, increases from this point to F_1; that is, the bending moment for the different sections of the beam increases as x increases from 0 to l_1.

If the point C is taken between F_1 and F_2 and the moments taken to the left of C, the formula for the bending moment is

$$M = R_1 x - F_1(x - l_1) \qquad (3)$$

This expands to $M = R_1 x - F_1 x + F_1 l$; and by combining terms containing x, it becomes $M = (R_1 - F_1)x + F_1 l_1$. From this expression, it is seen that when R_1 is greater than F_1, the bending moment M increases as x increases from l_1 to l_2; and if F_1 is greater than R_1, M decreases as x increases from l_1 to l_2; but if $R_1 = F_1$, the value of M remains constant, because then $(R_1 - F_1) = 0$ and $F_1 l_1$ does not change in value.

If the point C is taken between F_2 and F_3, formula **1** is obtained, which expands to $M = R_1 x - F_1 x + F_1 l_1 - F_2 x + F_2 l_2$. Combining the terms containing x, the expression becomes, $M = (R_1 - F_1 - F_2)x + F_1 l_1 + F_2 l_2$. As $F_1 l_1$ and $F_2 l_2$ are constant, the value of M in the formula increases as x increases from l_2 to l_3, when R_1 is greater than $(F_1 + F_2)$; it decreases when R_1 is less than $(F_1 + F_2)$, and remains constant when $R_1 = (F_1 + F_2)$.

From these formulas it will be seen that, when concentrated loads alone are considered, the change from an increasing to a decreasing bending moment always occurs at the point of application of one of the concentrated loads. It follows that the maximum bending moment occurs at the point of application of one of these loads. The maximum bending moment may, therefore, be found by calculating the bending moments at the points where the loads are applied and comparing the results.

NOTE.—The expression foot-pounds, used in stating the value of a moment, must not be confounded with foot-pounds of work. The former means simply that a force has been multiplied by a distance, while the latter means that a resistance has been overcome through a distance. In expressing the value of a moment, the force is usually measured in pounds or tons, and the distance in inches or feet; hence, the moment may be inch-pounds or inch-tons and foot-pounds or foot-tons. Unless otherwise stated, the bending moment will always be expressed in inch-pounds, the length of the beam being always measured in inches, and, consequently, the length of the intercept y also.

41. Bending Moment by Graphic Method.—To find the bending moment for any point of a beam, as 7, Fig. 12, by the graphic method, draw the vertical line 7–7′ through the point. Let y = that part of the line included between $a g$ and $a b c d f g$ of the equilibrium polygon (= vertical intercept). Then $H \times y$ = the bending moment. H, of course, equals the pole distance = $P u$. For any other point on the beam, the bending moment is found in the same manner, that is, by drawing a vertical line through the point and measuring that part of it included between the upper and lower lines of the equilibrium polygon. The scale to which the intercept y is measured is the same as that used in drawing the length $A B$ of the beam. The pole distance H is measured to the same scale as $O 5$. In the present case, $y = 2.05$ feet, and $H = 349$ pounds; hence, the bending moment for the point 7 is $H y = 349 \times 2.05 = 715.45$ foot-pounds.

If expressed in inch-pounds, the value of the moment just found is $715.45 \times 12 = 8{,}585.4$ inch-pounds. It will be noticed that after the force diagram and equilibrium polygon have been drawn, the value of the bending moment depends solely on the value of y, since the length $P u = H$ is fixed. At the points a and g, directly under the points of support of the beam, $y = 0$; hence, for these two points, bending moment $= H y = H \times 0 = 0$; that is, for any simple beam, the bending moment at either support is zero. The greatest value for the bending moment will evidently be at the point 8, since $d 8'$ is the longest vertical line which can be included between $a g$ and $a c f g$.

The figure $a c e g a$ is called the **diagram of bending moments.**

SIMPLE BEAM WITH UNIFORM LOAD

42. Analytic Method.—The vertical shear, in the case of a simple beam carrying a uniform load, decreases uniformly from the maximum positive shear at the left support to the maximum negative shear at the right support. Hence, at the middle point of the beam, the vertical shear will be zero. Let AB, Fig. 13, be a simple beam with a uniform load of w pounds per square foot and a length of l feet. The reactions R_1 and R_2 are equal to each other, and each is equal to $\frac{wl}{2}$; that is, each reaction is equal to half the load. This may be expressed as a formula thus:

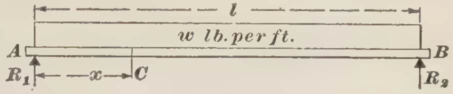


FIG. 13

$$R_1 = R_2 = \frac{wl}{2} \qquad (1)$$

The bending moment at any point, as C, is then

$$M = \frac{wl}{2}x - wx\frac{x}{2} \qquad (2)$$

The load for any part of the beam x feet long is wx pounds; and being a uniform load, it may be regarded as acting at the middle point of the portion considered.

When $x = 0$ or l, the value of M in formula **2** becomes 0, and when $x = \frac{l}{2}$ it is a maximum, and the formula becomes

$$M = \frac{wl^2}{8} \qquad (3)$$

But $x = \frac{l}{2}$ indicates the middle point of the beam; hence, in this case, the maximum bending moment comes at the point of zero vertical shear.

43. Graphic Method.—The graphic method will now be applied to the case of a simple beam uniformly loaded. Let the distance between the supports in Fig. 14 be 12 feet, and let the total load uniformly distributed over the beam be 216 pounds. Divide the load into a convenient number of equal parts, the more the better—12 in this case. The load that each part represents is 216 ÷ 12 = 18 pounds. For convenience, lay off OC on the vertical line through the left-hand support, equal to 216 pounds to the scale chosen, and divide it into twelve equal parts, Oa, ab, etc.; each part will represent 18 pounds to the same scale. Choose a pole P and draw the rays PO, Pa, Pb, etc. Through the points d, e, etc., the centers of gravity of the equal subdivisions of the load, draw the verticals *d 1*, *e 3*, *f 5*, etc., intersecting the horizontals through *O*, *a*, *b*, etc., in *1*, *3*, *5*, etc. Draw *O 1*, *1–2*, *2–3*, *3–4*, *4–5*, etc., and the broken line thus found will be the shear line. In drawing the shear line for a uniform load in this manner, it is assumed that each part of the total load is concentrated at its center of gravity, or, in other words, that a force equal to each small load (18 pounds) acts on the beam at each of the points *d*, *e*, *f*, etc.

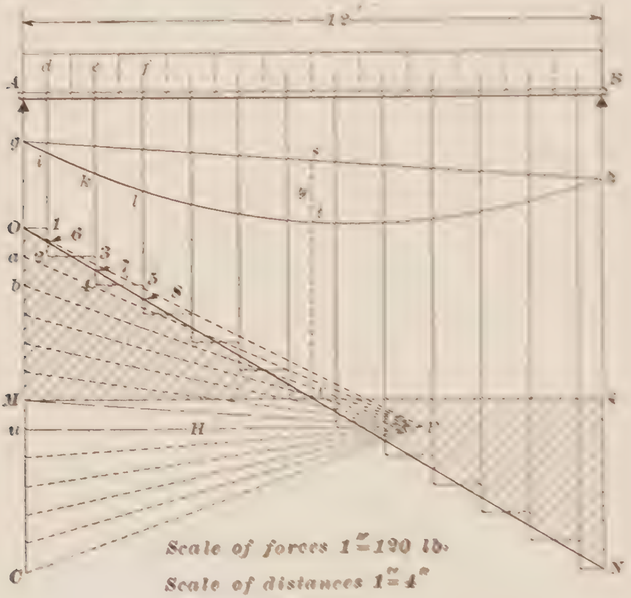


FIG. 14

Construct the diagram of bending moments in the ordinary manner by drawing *g i* parallel to *P O*, *i k* parallel to *P a*, etc. Draw *P M* parallel to *g h*, and *M g* horizontal; *M g* is the shear axis. When the load is uniform and the work has been done correctly, *O M* should equal *M C*, that is, the reactions of the two supports are equal.

44. The shear line is not in reality a broken line, as shown, since the load is distributed evenly over the entire beam, and not divided into small loads concentrated at *d*, *e*, *f*, etc., as was assumed. The points *1*, *3*, etc. are evidently too high, and the points *2*, *4*, etc. too low. To find the real shear line, bisect the lines *1–2*, *3–4*, etc., locating the points *6*, *7*, *8*, etc. Trace a line through *O*, *6*, *7*, *8* . . . *N*, and it will be the real shear line. For all cases of a uniform load, the shear line will be straight and may be drawn from *O* to *N* directly.

The diagram of bending moments is also not quite exact, but may be corrected by tracing a curve through the points *g* and *h*, which will be tangent to *g i*, *i k*, *k l*, etc. at their middle points, as shown in the figure.

45. To find the maximum bending moment for any beam having two supports, draw a vertical line through *r*, where the shear line cuts the shear axis; the intercept, $s\,t = y$, on the diagram of bending moments will be the greatest value of *y*, and consequently the greatest bending moment $= H \times y$. In the present example, $H = P u = 247.5$ pounds, and maximum $y = s\,t = 15.72$ inches; hence, the maximum bending moment $= H y = 247.5 \times 15.72 = 3{,}890.7$ inch-pounds. No matter how the beam may be loaded, the foregoing is still true. In Fig. 12, the shear line cuts the shear axis at *z*, and *d 8′*, on the vertical through *z*, was shown previously to be the maximum intercept.

SIMPLE BEAMS WITH MIXED LOADS

46. Analytic Method.—Let *A B*, Fig. 15, represent a simple beam *l* feet long, with a uniform load of *w* pounds per foot of length, and with loads of F_1 and F_2 pounds at a distance of l_1 and l_2 feet from R_1. The algebraic sum of the vertical forces being equal to zero, $R_1 + R_2 - F_1 - F_2 - w\,l = 0$, or,

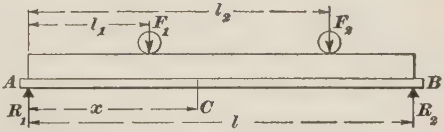


FIG. 15

$$R_1 + R_2 = F_1 + F_2 + w\,l \qquad (1)$$

F_1, F_2, and $w\,l$ being known from the conditions of the problem, it becomes necessary to find R_1 or R_2 before the other can be determined by formula **1**. In order to determine R_2, take the moments of all the vertical forces about R_1 as a center. The moment of R_1 will then be zero, and the moment of R_2, the only other force acting upwards, will equal the moments of all the downward forces, or $R_2 l = F_1 l_1 + F_2 l_2 + \frac{1}{2} w l^2$. Dividing both sides of the equation by l gives

$$R_2 = \frac{F_1 l_1 + F_2 l_2 + \frac{1}{2} w l^2}{l} \qquad (2)$$

This value for R_2 may be substituted in formula **1** to determine R_1, or R_1 may be determined in a manner similar to that by which R_2 was found, that is, by taking moments about R_2 as a center. Thus,

$$R_1 = \frac{F_1(l - l_1) + F_2(l - l_2) + \frac{1}{2} w l^2}{l} \qquad (3)$$

Any two of the above formulas may be used to find R_1 and R_2, and the third may be used to check the results.

It has been shown that the shear passes from a positive to a negative value, that is, it has a zero value, at the section at which the maximum bending moment occurs, for beams uniformly loaded or loaded with concentrated loads only. It can be shown, by an intricate process, that the point of zero shear for a simple beam loaded with a mixed load is also at the point of the maximum bending moment. It is very convenient to find the maximum bending moment for a simple beam with a mixed load by first finding the point of zero shear and then finding the bending moment for this point. This method will, therefore, be used here.

The maximum positive shear occurs at the point of application of the left reaction and is equal to the reaction. In order to find the point of zero shear, it is necessary to locate the point where the sum of the loads, between this point and either reaction, passes from a value less than to a value greater than the reaction. To do this, it may be necessary to make trials at the points of application of several different concentrated loads; and where a uniform load is applied, to find how much of the uniform load must be considered in addition.

This method will be applied to the simple beam carrying a mixed load, as shown in Fig. 15. If the uniform load is w pounds per foot, the load between R_1 and F_1 is $w\,l_1$. If $w\,l_1$ is greater than R_1, the point of zero shear will lie at a distance of x feet to the right of R_1, so that $R_1 = w\,x$; and x may be found by the formula

$$x = \frac{R_1}{w} \qquad (4)$$

If $w\,l_1$ is less than R_1 but $(w\,l_1 + F_1)$ is greater than R_1, the point of zero shear will be at the point of application of F_1. If $(w\,l_1 + F_1)$ is less than R_1 but $(w\,l_2 + F_1)$ is greater than R_1, the point of zero shear will be at some point, as C, between F_1 and F_2, at a distance of x feet from R_1, such that

$$x = \frac{R_1 - F_1}{w} \qquad (5)$$

If $(w\,l_2 + F_1)$ is less than R_1 but $(w\,l_2 + F_1 + F_2)$ is greater than R_1, the point of zero shear is at the point of application of F_2. If, however, $(w\,l_2 + F_1 + F_2)$ is less than R_1, the point of zero shear will be at the distance of x feet from R_1, and x may be found by the formula

$$x = \frac{R_1 - F_1 - F_2}{w} \qquad (6)$$

In case there are more concentrated loads, this method can be applied until the point of zero shear is found. The same method can be applied when considering the right reaction first; but as the left reaction is considered positive, it is customary to start from that point.

Having located the point of zero shear at some point, as C, at a distance of x feet from R_1 and between F_1 and F_2, the maximum bending moment is

$$M = R_1 x - \tfrac{1}{2} w\,x^2 - F_1(x - l_1) \qquad (7)$$

47. Graphic Method.—If there is a uniform load on the beam and one or more concentrated loads, as in Fig. 16, the method of finding the moment diagram and shear line is similar to that used in Arts. **43** to **45**. In Fig. 16, let the length $A\,B$ of the beam be 15 feet, the uniformly distributed load 180 pounds, with two concentrated loads, one of 24 pounds, 5 feet from *A*, and the other of 30 pounds, 11 feet from *A*. Draw the beam and loads as shown, the length of the beam and the distances of the weights from *A* being drawn to scale. Divide the uniform load into a convenient number of equal parts—say 10 in this case; each part will then represent $\frac{180}{10} = 18$ pounds. Draw *A O C*, as usual, and lay off three of the 18-pound subdivisions from *O* downwards; then lay off 24 pounds to represent the first weight. Lay off four more of the equal subdivisions, and then the 30-pound weight. Finally, lay off the remaining three equal subdivisions, the point *C* being the end of the last 18-pound subdivision. *O C* should then equal 180 + 24 + 30 = 234 pounds to the scale to which the weights were laid off. It will be noticed that the equal subdivisions of the load were laid off on *O C* until that one was reached on

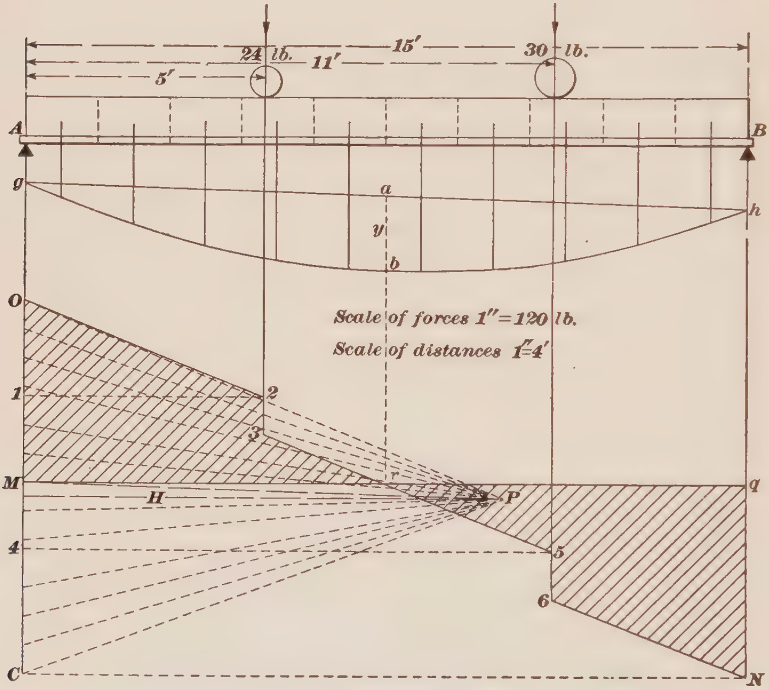


FIG. 16

which the concentrated loads rested, and that the concentrated loads were laid off before the equal subdivision on which the concentrated load rested was laid off. Had one of the concentrated loads been to the right of the center of gravity of the subdivision on which it rests, the weight of the subdivision would have been laid off first. Locate the centers of gravity of the equal subdivisions and draw the verticals through them as in the previous case. Choose a pole *P*, draw the rays, the diagram of bending moments, and the shear axis $M q$, as previously described. To find the maximum bending moment, the shear line must be drawn and its intersection with the shear axis determined.

The weight of the uniform load per foot of length is $\frac{180}{15} = 12$ pounds. The weight of that part between *A* and the center of the 24-pound weight is $12 \times 5 = 60$ pounds. Lay off $O 1 = 60$ pounds and draw *1–2* horizontal, cutting the vertical through the center of the 24-pound weight in *2*. Draw $O 2$ and it will be a part of the shear line. Lay off *2–3* vertically downwards, equal to 24 pounds, locating the point *3*. Lay off $O 4$ equal to $12 \times 11 + 24 = 156$ pounds, and draw the horizontal *4–5*, cutting the vertical through the center of the 30-pound weight in *5*. Join *3* and *5* by the straight line *3–5*. Lay off *5–6* vertically downwards equal to 30 pounds. Draw the horizontal $C N$, intersecting the vertical $B q N$ in *N*, and join *6* and *N* by the straight line *6 N*. The broken line *O–2–3–5–6–N* is the shear line and cuts the shear axis in the point *r*. Drawing the vertical $r b a$, through *r*, it intersects the moment diagram in *a* and *b*; hence, $a b$ is the maximum *y*. For this case, the maximum bending moment is $H \times y = 300 \times 18.36 = 5{,}508$ inch-pounds.

It is better, in ordinary practice, to choose the pole *P* on a line perpendicular to $O C$ and at some distance from $O C$ easily measured with the scale used to lay off $O C$. Thus, suppose $O C$ to be laid off to a scale of 1 inch = 60 pounds. At some convenient point, as *M*, on $O C$, draw a perpendicular line and choose a point on this line whose distance from $O C$ shall be easily measurable, say $3\frac{1}{2}$ inches. Then, *H* is known to be exactly $60 \times 3\frac{1}{2} = 210$ pounds, and will not have to be measured when finding the bending moment Hy.

48. If the method of constructing the shear and moment diagrams for concentrated loads is clearly understood, no difficulty will be encountered in the preceding operations, which may be expressed by the following:

Rule.—*Divide the beam into an even number of parts (the greater the better), and the uniform load into half as many. Consider the divisions of the load as concentrated loads applied, alternately, at the various points of division of the beam (the ends included); that is, the first point of division (the support) carries no load, the next one does, the following one does not, etc. Then proceed as in the ordinary case of concentrated loads.*

OVERHUNG BEAMS AND CANTILEVERS

49. Overhung Beams.—A beam overhung at one end and loaded with concentrated loads is shown in Fig. 17. The principles that have been applied to simple beams can be applied to overhung beams, but the directions of forces must be kept carefully in mind, so that the positive and negative shears and moments may be distinguished. If a beam overhangs the left support, the vertical shear will be zero at its left end and will have increasing negative values up to the left support, where it will change to a large positive value that decreases to zero and then increases to a large negative value at the right reaction. If the beam overhangs the right support, the large negative shear at the right support will pass to a large positive shear that will decrease to zero to the right of the overhung load.

As the vertical shear passes from a positive to a negative value, that is, has a zero value at more than one point, it is necessary to calculate the bending moments for all such points, and to compare these results in order to determine the maximum bending moment.

The application of the graphic method does not involve any new principles in cases of overhung beams, yet there are points in the construction that should be noted carefully. The reactions are very similar to those of a simple beam, and the force diagram is practically the same. The shear diagram and the bending moment diagram will show the differences in shear and bending forces. The distinction between positive and negative forces and moments should be carefully kept in mind. Forces acting downwards are negative, and moments tending to produce rotation in the direction of the hands of a clock are positive; the others are negative.

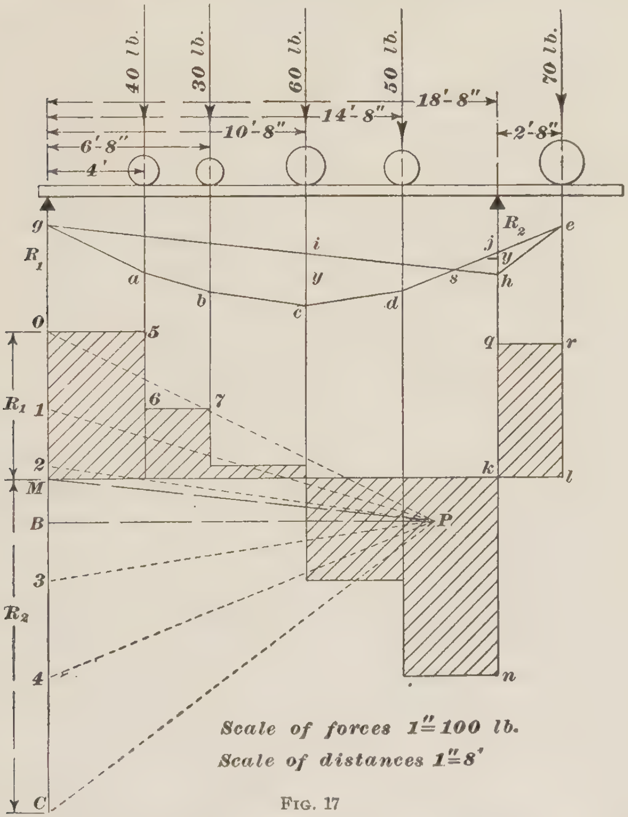


FIG. 17

EXAMPLE 1.—Find the reactions of the supports, the maximum bending moments, and the maximum vertical shear of the beam shown in Fig. 17, which has one overhanging end.

Analytic Solution.—The sum of the reactions is equal to the sum of the loads, that is, $R_1 + R_2 = 40 + 30 + 60 + 50 + 70 = 250$ lb. Taking the moments about R_1, by formula **2**, Art. **36**,

$$R_2 = \frac{40 \times 4 + 30 \times 6\frac{2}{3} + 60 \times 10\frac{2}{3} + 50 \times 14\frac{2}{3} + 70 \times 21\frac{1}{3}}{18\frac{2}{3}} = 172\tfrac{6}{7} \text{ lb.}$$

From formula **3**, Art. **36**,

$$R_1 = \frac{40 \times 14\frac{2}{3} + 30 \times 12 + 60 \times 8 + 50 \times 4 - 70 \times 2\frac{2}{3}}{18\frac{2}{3}} = 77\tfrac{1}{7} \text{ lb.}$$

Hence, $R_1 + R_2 = 172\frac{6}{7} + 77\frac{1}{7} = 250$ lb., which checks with the above result of the sum of the loads.

The vertical shear is $77\frac{1}{7}$ lb. from R_1 to the point of application of the 40-lb. load, and $77\frac{1}{7} - 40 = 37\frac{1}{7}$ from this point to the point of application of the 30-lb. load. The shear then reduces to $37\frac{1}{7} - 30 = 7\frac{1}{7}$ lb., and continues the same to the point of application of the 60-lb. load, when it becomes $7\frac{1}{7} - 60 = -52\frac{6}{7}$ lb. Hence, at 10 ft. 8 in. from R_1, the shear passes from a positive to a negative value; the same is also true at the right support, where R_2 acts. The bending moments at R_2 and the point of application of the 60-lb. load must now be calculated and the results compared. The bending moment for the beam at the 60-lb. load is $M_1 = (77\frac{1}{7} \times 10\frac{2}{3} - 40 \times 6\frac{2}{3} - 30 \times 4)\,12 = 5{,}234\frac{2}{7}$ in.-lb. The bending moment about R_2 should now be calculated and compared with M_1. Since the bending moment is the same at R_2, on whichever side the forces are considered, it is simpler to take the one force to the right of R_2. The bending moment to the right of R_2 is positive, while that to the left is negative, and, since the moments to the left are considered, the bending moment in this case is taken as negative, and equal to $M_2 = -70 \times 2\frac{2}{3} \times 12 = -2{,}240$ in.-lb. M_1 having a larger numerical value than M_2, it is the maximum bending moment for the beam.

Graphic Solution.—Draw $O\,C$ and the force diagram in the usual manner. Construct the bottom curve of the moment diagram in the same manner as in the preceding cases. The side $d\,e$ is parallel to $P\,4$; $e\,h$ is parallel to $P\,C$ and cuts the vertical through the right reaction in h. Join h and g by the straight line $g\,h$, and draw $P\,M$ parallel to $g\,h$. Then, $O\,M = 77$ lb. = left reaction, and $M\,C = 173$ lb. = right reaction. The shear line is drawn until the point n, on the vertical $h\,n$, is reached; $k\,n$ here denotes the vertical shear for any point between the 50-lb. weight and the right support, and this shear is negative. The point k denotes the intersection of the shear axis and the vertical through the right support. For any point to the right of k, between k and l, the vertical shear is positive, and is equal to 70 lb.; hence, lay off $k\,q$ upwards equal to 70 lb., and draw $q\,r$ horizontal. The line, O–5–6–7 . . . $n\,q\,r$ is the shear line. Measuring $O\,M$, $k\,n$, and $k\,q$, it is found that $O\,M = 77$ lb., $k\,n = -103$ lb., and $k\,q = 70$ lb.; therefore, $k\,n = -103$ lb. = maximum vertical shear; ci is evidently the maximum y; hence, the maximum bending moment $= Hy = PB \times ci = 200 \times 26$ in. $= 5{,}200$ in.-lb. Any value of y measured in the polygon $gabcdsg$ is positive, and any value measured in the triangle ehs is negative. Consequently, the bending moment for any point between s and the vertical, through the center of the 70-lb. weight, is negative, since $H \times (-y) = -Hy$. In all cases, when designing beams with overhanging ends, the maximum bending moment, whether positive or negative, should be used. In the present case, the maximum negative y, or hj, is less than the

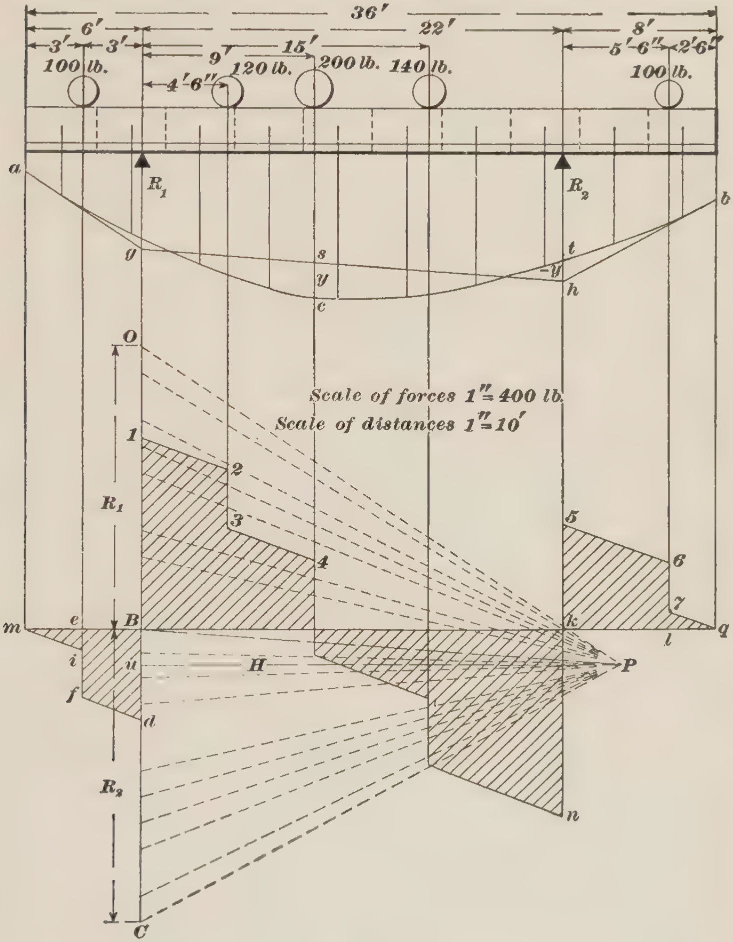


Fig. 18

maximum positive y, or $c\,i$; therefore, the maximum negative bending moment is also less than the maximum positive moment.

EXAMPLE 2.—Fig. 18 shows a beam overhanging both supports, which carries a uniform load of 15 pounds per foot of length and has five concentrated loads at distances from the supports as marked in the figure. Find the reactions of the supports, the maximum positive and negative bending moments, and the maximum vertical shear.

ANALYTIC SOLUTION.—The beam here has a uniform load and several concentrated loads; hence, a modification of formula **1**, Art. **46**, applies. This gives

$$R_1 + R_2 = 100 + 120 + 200 + 140 + 100 + 15 \times 36 = 1{,}200 \text{ lb.}$$

Formula **2**, Art. **46**, also applies. $R_2 = (-100 \times 3 - 6 \times 15 \times 3 + 120 \times 4\frac{1}{2} + 200 \times 9 + 140 \times 15 + 100 \times 27\frac{1}{2} + 15 \times 30 \times 15) \div 22$

$$= \frac{13{,}370}{22} = 607\tfrac{8}{11} \text{ lb.}$$

Using formula **3**, Art. **46**, $R_1 = (-100 \times 5\frac{1}{2} - 15 \times 8 \times 4 + 140 \times 7 + 200 \times 13 + 120 \times 17\frac{1}{2} + 100 \times 25 + 28 \times 15 \times 14) \div 22$

$$= \frac{13{,}030}{22} = 592\tfrac{3}{11} \text{ lb.}$$

Hence, $R_1 + R_2 = 592\frac{3}{11} + 607\frac{8}{11} = 1{,}200$ lb., which agrees with the results obtained above. The vertical shear has a zero value at both supports and at some point between; this last point should be determined and the bending moments for the three points calculated. The one with the largest numerical value will be the maximum bending moment. At the left support, the shear is R_1 minus the load at the left of this support, or $592\frac{3}{11} - 100 - 6 \times 15 = 402\frac{3}{11}$ lb. The shear should next be calculated under the 120-lb. load. This is $402\frac{3}{11} - 120 - 4\frac{1}{2} \times 15 = 214\frac{17}{22}$. Next, calculate the vertical shear under the 200-lb. load. Without the 200-lb. load it is $214\frac{17}{22} - 4\frac{1}{2} \times 15 = 147\frac{3}{11}$ lb. With the 200-lb. load, it is $147\frac{3}{11} - 200 = -52\frac{8}{11}$ lb. The bending moment at the point of application of the 200-lb. load should now be calculated; it is

$$M_1 = (592\tfrac{3}{11} \times 9 - 100 \times 12 - 120 \times 4\tfrac{1}{2} - 15 \times 15 \times 7\tfrac{1}{2})\,12$$
$$= 22{,}835\tfrac{5}{11} \text{ in.-lb.}$$

As both the moments to the right of R_2 are greater than the two at the left of R_1, it will not be necessary to calculate R_1. The bending moment at R_2 is $M_2 = (100 \times 5\frac{1}{2} + 8 \times 15 \times 4)\,12 = 12{,}360$ in.-lb. Comparing M_1 with M_2, it is seen that M_1 is the larger; hence, the maximum bending moment is at the point of application of the 200-lb. load.

GRAPHIC SOLUTION.—Construct the force diagram and equilibrium polygon in the ordinary manner, continuing the latter to b and a, points on the verticals passing through the ends of the beam. Draw $b\,h$ and $a\,g$ parallel to $P\,C$ and $P\,O$, respectively, intersecting the verticals through the points of support in h and g. Join g and h and draw PB parallel to gh. Then, $OB = 588$ lb. $= R_1$, and $BC = 612$ lb. $= R_2$. Through B, draw the shear axis mq. To draw the shear line, proceed as follows: The shear for any point to the left of the left support is negative, and for any point to the right of the right support is positive; between the two supports, it is positive or negative, according to the manner of loading and the point considered. The negative shear at the left support $= 15 \times 6 + 100 = 190$ lb.; hence, lay off Bd downwards equal to 190 lb. For a point at a minute distance to the right of e, the shear is $15 \times 3 + 100 = 145$ lb. $= ef$, and for a minute distance to the left, it is $15 \times 3 = 45$ lb. $= ei$; at m it is 0. Consequently, $mifd$ is the shear line between the end of the beam and the left support. Lay off $O1 = Bd = 190$ lb. and draw the shear line $1\text{–}2\text{–}3\text{–}4 \ldots n$ in the usual manner. Draw the shear line $5\text{–}6\text{–}7\,q$, laying off $k5 = 15 \times 8 + 100 = 220$ lb.; $l6 = 100 + 15 \times 2\tfrac{1}{2} = 137\tfrac{1}{2}$ lb., and $6\text{–}7 = 100$ lb. At q, the vertical shear is again 0. The broken line $mifd\ 1\text{–}2\text{–}3 \ldots n\ 5\text{–}6\text{–}7\,q$ is the shear line. The maximum positive bending moment is $H \times y = Pu \times sc = 1{,}000 \times 22.5 = 22{,}500$ in.-lb. The greatest negative maximum moment is $H \times (-y) = Pu \times -th = 1{,}000 \times -11.6 = -11{,}600$ in.-lb. It will be noticed that there are two negative and one positive maximum bending moments.

It should now be easy to find the bending moment for any beam having but two supports, whatever the character of the loading. In all cases of loading heretofore discussed, no other forces than the loads themselves have been considered. Should forces that are not vertical act on the beam, the force polygon will be no longer a straight line, but a broken line, somewhat similar in character to *0–1–2–3–4–5* in Fig. 5.

50. Cantilevers.—A horizontal beam with one end built into a wall, the wall being its only support, is a **cantilever**. Such a beam is shown in Fig. 19. In this case, the beam AB is loaded with a mixed load—a uniform load of w pounds per foot and a concentrated load of F_1 pounds at a distance of l_1 feet from the wall. There being only one support, the beam will have only one reaction R, which will equal the sum of the loads, or

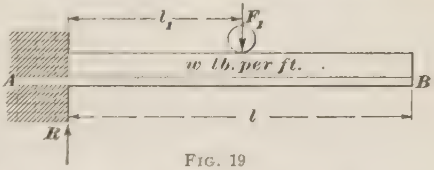


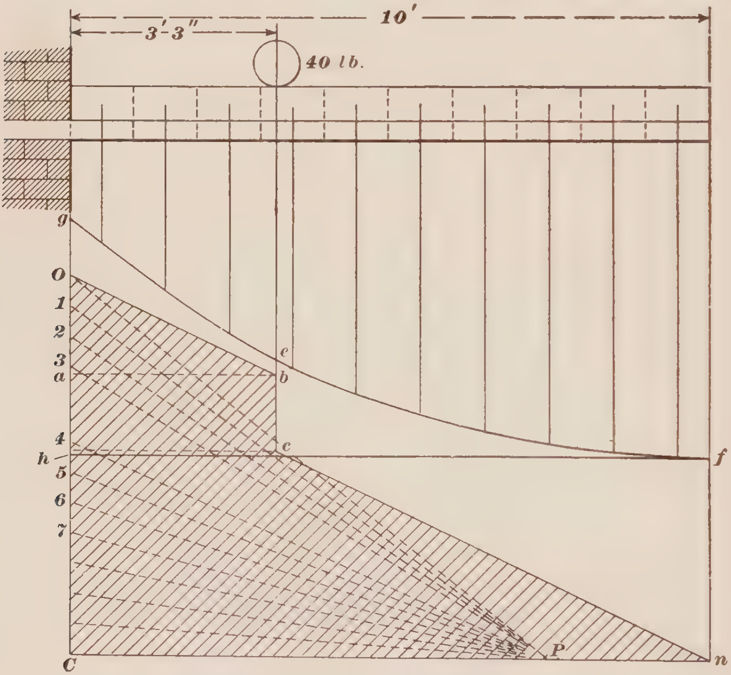
FIG. 19

$$R = F_1 + wl \qquad (1)$$

The wall above the beam at the fixed end holds the beam in position; hence it exerts a downward force and produces a negative shear at the point of support of the beam. The reaction R acting upwards, however, gives a positive shear. so that at the point of support of the beam the vertical shear passes from negative to positive; that is, it has a zero value. This point will, therefore, be the point of maximum bending moment, and the formula for M is

$$M = F_1 l_1 + \tfrac{1}{2} w l^2 \qquad (2)$$

51. Graphic Method for Cantilever.—In Fig. 20 is shown a cantilever projecting 10 feet from the wall. It carries a uniform load of 16 pounds per foot of length, and a concentrated load of 40 pounds at a distance of $3\frac{1}{4}$ feet



Scale of forces 1"=100 lb.

Scale of distances 1"=3'

FIG. 20

from the wall. The maximum bending moment is required. The method is similar to that shown in Fig. 18, except that, as there is but one support, there can be but one reaction. Since the beam is 10 feet long, the total weight of the uniform load is $16 \times 10 = 160$ pounds. Hence, the reaction $= 160 + 40 = 200$ pounds.

Draw OC equal to 200 pounds to some convenient scale. Draw PC perpendicular to OC at C and choose the pole P at a convenient distance from OC. For convenience, divide the uniform load into ten equal parts, as shown; then, each part will represent 16 pounds. Lay off $C1$, 1–2, 2–3, each equal to 16 pounds, and 3–4 equal to 40 pounds. Also, 4–5, 5–6, 6–7, etc., equal to 16 pounds each. 3 feet 3 inches $= 3\frac{1}{4}$ feet, and $16 \times 3\frac{1}{4} = 52$ pounds. Lay off Oa $= 52$ pounds, and draw ab, meeting the vertical through the center of the weight in b. Draw Ob; lay off bc equal to 40 pounds and draw Cn. $Obcn$ is the shear line. The perpendicular through the point C coincides with PC; hence, nPC is the shear axis. Draw the line gef of the moment diagram as in the previous cases. In Fig. 16, and in the preceding figures, the line gh was drawn connecting the extreme ends of the bottom line; in other words, it joined the points where the equilibrium polygon cut the lines of direction of the reactions of the supports. This cannot be done in this case, because there is no right reaction; therefore, gh must be drawn by means of some other property of the polygon. In the previous cases, the shear axis was drawn perpendicular to OC at the point where a line through the pole P parallel to gh cut OC, or, in other words, the shear axis was drawn through the point which marked the end M of the left reaction OM. In the present case, Fig. 20, the point C is the end of the left reaction; hence, fh is drawn parallel to PC, hg is the maximum y, and the bending moment $= Hy = PC \times hg = 90 \times 124 = 11{,}160$ inch-pounds.

It will also be noticed that the distance $y = hg$ is measured from the line hf upwards, while, for all points between the supports in the previous examples, this distance was measured downwards. The same observation is true for any point between *h* and *f*; hence, for a cantilever, all bending moments are negative.

EXAMPLES FOR PRACTICE

1. A simple beam 24 feet long carries four concentrated loads of 160, 180, 240, and 120 pounds at distances from the left support of 4, 10, 16, and 21 feet, respectively. (*a*) What are the values of the reactions? (*b*) What is the maximum bending moment in inch-pounds?

Ans. $\begin{cases} (a) \ R_1 = 333\frac{1}{3} \text{ lb.};\ R_2 = 366\frac{2}{3} \text{ lb.} \\ (b) \ 28{,}480 \text{ in.-lb.} \end{cases}$

2. A simple beam carries a uniform load of 40 pounds per foot, and supports two concentrated loads of 500 and 400 pounds at distances from the left support of 5 and 12 feet, respectively; the length of the beam is 18 feet. What are: (*a*) the reactions? (*b*) the maximum bending moment in inch-pounds?

Ans. $\begin{cases} (a) \ R_1 = 854\frac{4}{9} \text{ lb.};\ R_2 = 765\frac{5}{9} \text{ lb.} \\ (b) \ 48{,}844\frac{17}{27} \text{ in.-lb.} \end{cases}$

3. A cantilever projects 10 feet from a wall and carries a uniform load of 60 pounds per foot; it also supports three concentrated loads of 100, 300, and 500 pounds at distances from the wall of 2, 5, and 9 feet, respectively. Required: (*a*) the maximum vertical shear; (*b*) the maximum bending moment in inch-pounds.

Ans. $\begin{cases} (a) \ -1{,}500 \text{ lb.} \\ (b) \ 110{,}400 \text{ in.-lb.} \end{cases}$

4. A beam that overhangs one support sustains six concentrated loads of 160 pounds each at distances from the left support of 4 feet 9 inches, 7 feet, 9 feet 6 inches, 12 feet, 15 feet, and 18 feet 3 inches, respectively, the distance between the supports being 16 feet. What are: (*a*) the reactions? (*b*) the maximum bending moment?

Ans. $\begin{cases} (a) \ R_1 = 295 \text{ lb.};\ R_2 = 665 \text{ lb.} \\ (b) \ 20{,}460 \text{ in.-lb.} \end{cases}$

5. A beam that overhangs both supports equally carries a uniform load of 80 pounds per foot, and has a load of 1,000 pounds in the middle, the length of the beam being 15 feet, and the distance between the supports 8 feet. What is: (*a*) the vertical shear? (*b*) the maximum bending moment?

Ans. $\begin{cases} (a) \ 820 \text{ lb.} \\ (b) \ 25{,}800 \text{ in.-lb.} \end{cases}$

STRENGTH OF MATERIALS

(PART 2)

BEAMS

STRENGTH OF BEAMS

1. Beams of various kinds were defined in *Strength of Materials*, Part 1, and the external forces acting on them were considered. When a beam is acted on by external forces it is deformed, thus bringing into action internal forces or stresses that resist the external forces.

The strength of the beam depends on the stresses that the material of which it is composed is capable of exerting. These stresses are different for different parts of the cross-section of any beam. Formulas for the strength of the beam, therefore, take into consideration the maximum stresses.

2. The Neutral Axis.—In Fig. 1, let $ABCD$ represent a cantilever. Suppose that a force F acts on it at its extremity A. The beam will then be bent into the shape shown by $A'BCD'$. It is evident, from the illustration, that the upper part $A'B$ is now longer than it was before the force was applied; that is, $A'B$ is longer than AB. It is also evident that $D'C$ is shorter than DC. Hence, the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten the lower ones. In other words, when a cantilever is bent through the action of

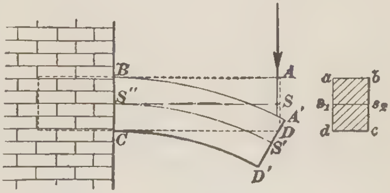


FIG. 1

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a load, the upper fibers are in tension and the lower fibers in compression. The reverse is the case in a simple beam in which the upper fibers are in compression and the lower fibers in tension.

Further consideration will show that there must be a fiber $S\,S''$ that is neither lengthened nor shortened when the beam is bent, that is, $S'\,S''$ equals $S\,S''$. When the beam is straight, the fiber $S\,S''$, which is neither lengthened nor shortened when the beam is bent, is called the **neutral line.** There may be any number of neutral lines dependent only on the width of the beam. For, let $b\,a\,d\,c$, Fig. 1, be a cross-section of the beam. Project S on it in s_1. Make $b\,s_2$ equal $a\,s_1$ and draw $s_1\,s_2$; then, any line in the beam that touches $s_1\,s_2$ and is parallel to $S\,S''$ is a neutral line. Thus, in Fig. 2, $S_1\,S'$, $S\,S$, $S_2\,S''$, etc., are all neutral lines. The line $S_1\,S_2$ is called the **neutral axis,** and the surface $S_1\,S'\,S''\,S_2$ is called the **neutral surface.** The neutral axis, then, is the line of intersection of a cross-section with the neutral surface. It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of the beam.*

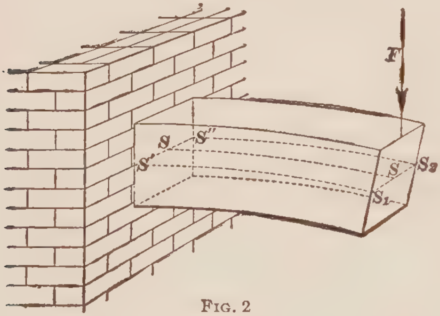

FIG. 2

3. Experimental Law.—*When a beam is bent, the horizontal elongation or compression of any fiber is directly proportional to its distance from the neutral surface, and, since the deformations are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided that the elastic limit is not exceeded.*

4. External and Internal Forces.—The **external forces** acting on a beam are the loads that the beam supports and the reactions of the supports. For nearly all practical cases these forces act vertically; and when they do not, they can be resolved into components parallel and perpendicular to the beam. These forces induce or cause stresses of two kinds. One is a shearing stress that is equal to the vertical shear on any section of the beam; the other is a bending stress that resists the bending action of the external forces. The bending stress is composed of tension stresses on one side, and of compression stresses on the other side of the neutral axis of the beam.

As these **internal stresses**, for any section of a beam, are opposite in the upper and lower parts, they produce a turning moment about the neutral axis of the section. This internal moment, or the moment of the stresses in any section, is called the *moment of resistance* of the section. From the fact that the beam remains in one position, the moment of the external forces at any section must equal the moment of resistance of the same section. The moment of resistance depends directly on the maximum stress of the material.

5. Theory of Beams.—The generally accepted **theory of beams** may be stated thus: *The bending moment at any section of a beam is equal to the maximum fiber stress in that section multiplied by the modulus of the section.* The modulus of the section is dependent entirely on the size and shape of the section and not on the material of the beam. This modulus will be considered more fully later.

The theory of beams is based on three important assumptions, which are: (*a*) that the fibers of a beam do not slide, lengthwise of the beam, on one another; (*b*) that the fiber stress is proportional to the distance of the fiber from the neutral axis; (*c*) that the modulus of elasticity is the same for both tension and compression. While these assumptions may not be strictly true for all cases, yet they are so nearly true that formulas based on this theory are sufficiently reliable for practical use. The value of the theory is that it furnishes a satisfactory basis for formulas for beams. It also has the advantange of making an otherwise complicated subject comparatively simple.

RESISTANCE TO BENDING

6. Equation of Moments in Bending.—Suppose the beam to be a rectangular prism; then every cross-section will be a rectangle, and the neutral axis will pass through the center *o*, Fig. 3. Let the perpendicular distance from the neutral axis MN to the outermost fiber be denoted by c, and let the horizontal unit stress (stress per square inch) for bending, at the distance c from the axis, be denoted by S_b. If a is the area of a fiber, the stress on the outermost fibers will be $a\,S_b$. The stress on a fiber at the distance unity (1 inch) from MN is $\frac{a\,S_b}{c}$; and the stress on a fiber at the distance r_1 is $\frac{a\,S_b}{c} \times r_1 = \frac{a\,S_b\,r_1}{c}$. The moment of this stress about the axis MN is $\frac{a\,S_b\,r_1}{c} \times r_1 = \frac{a\,S_b\,r_1^2}{c} = \frac{S_b}{c} a\,r_1^2$. The moment of the stress on any other fiber at a distance r_2 from MN is evidently $\frac{S_b}{c} a\,r_2^2$, and for a distance r_3, $\frac{S_b}{c} a\,r_3^2$, etc. If n is the number of fibers, the sum M of the moments of the horizontal stresses on all the fibers is

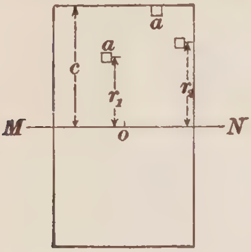

Fig. 3

$$M = \frac{S_b}{c} a\,r_1^2 + \frac{S_b}{c} a\,r_2^2 + \frac{S_b}{c} a\,r_3^2 +, \text{ etc.}$$

$$= \frac{S_b}{c}(a\,r_1^2 + a\,r_2^2 + a\,r_3^2 + \ldots a\,r_n^2)$$

$$= \frac{S_b}{c} a(r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2)$$

Now, let r be a quantity whose square equals the mean of the squares of $r_1, r_2, r_3, \ldots r_n$. Then,

$$r^2 = \frac{r_1^2 + r_2^2 + \ldots + r_n^2}{n}$$

and, therefore, $r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2 = n\,r^2$. Substituting, we get $M = \frac{S_b}{c} n\,a\,r^2$. But, since a is the area of one fiber, na is the area of all the fibers, that is, the area A of the cross-section; hence, the sum of the moments of all the horizontal stresses is $\frac{S_b}{c} A r^2$.

7. The expression $A r^2$ that is found by dividing a section into a large number of minute areas (a, a, etc.), multiplying each area by the square of its distance from an axis (r_1^2, r_2^2, r_3^2, etc.), and then adding the products thus obtained, is called the **moment of inertia** of the section with respect to that axis, and is usually denoted by the letter I. Hence,

$$I = A r^2$$

The quantity r whose square is the mean of the squares of all the distances of the minute areas from the axis is called the **radius of gyration.**

8. The sum of the moments of all the horizontal stresses may then be written as $\frac{S_b}{c} A r^2 = \frac{S_b}{c} I$, or $S_b \frac{I}{c}$; this expression is called the **moment of resistance,** since it is the measure of the resistance of the beam to bending (and consequently to breaking) when loaded. The resisting moment must equal the bending moment when the beam is in equilibrium; hence, denoting the bending moment by M,

$$M = S_b \frac{I}{c}$$

The term $\frac{I}{c}$ in this formula is called the **modulus of the section.**

9. Table of Ultimate Strength.—The values of I and c depend wholly on the size and form of the cross-section of the beam, and S_b is the ultimate bending strength of the material. In Table I, the average ultimate strengths in bending, S_b, are given for a number of materials. These values represent the average results of a large number of experiments, in which beams of the several materials were so loaded that the loads could be accurately found and the bending moments M calculated. The values of I and c were then determined from the measurements of the cross-sections of the beams, and the values of S_b were calculated by the formula in Art. 8.

TABLE I

ULTIMATE STRENGTH IN BENDING

Material	Pounds per Square Inch S_b
Cast iron	30,000
Wrought iron	45,000
Steel	65,000
Ash	8,000
Concrete	700
Stone	1,200
Hemlock	3,500
Oak, white	6,000
Pine, white	4,000
Pine, yellow	7,000
Spruce	3,000
Chestnut	4,500

10. Table of Moments of Inertia.—Exact values of the moment of inertia I for most cross-sections can only be determined by the aid of higher mathematics. The least value of I occurs when the axis passes through the center of gravity of the cross-section, that is, when I is found with reference to the neutral axis. The least moments of inertia I for a number of sections are given in the table of Moments of Inertia at the end of this Section; also, the area A of the sections and the values of c. The dotted line indicates the position of the neutral axis, about which the moment of inertia is taken.

In the table of Moments of Inertia, A is the area of the section, and π is the ratio of the circumference of a circle to its diameter = 3.1416. It will be noticed that d is always taken vertically, except in section 3.

11. Moment of Inertia of Combined Shapes.—When the moment of inertia of simple forms of cross-sections, like those shown in the table of Moments of Inertia, is known, the moments of inertia of any shape composed of these shapes may be found.

Let I_1 = moment of inertia about an axis through center of gravity of section;
I = moment of inertia about any parallel axis;
A = area of cross-section;
r = distance between axes of I_1 and I.

Then,
$$I = I_1 + A r^2$$

This formula may be expressed in words thus: The moment of inertia of any area about any axis is equal to its moment of inertia about a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes. The sum of the moments of inertia of the parts of any figure about a given axis is equal to the moment of inertia of the entire figure about the same axis.

Example.—What is the moment of inertia of the section of a beam shown in Fig. 4, about the horizontal axis $a\,b$ through the center of gravity?

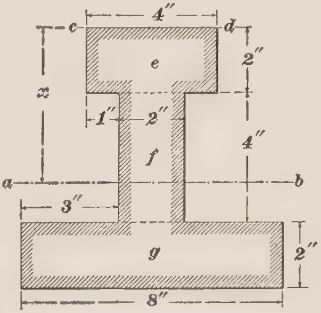

Fig. 4

Solution.—It is first necessary to find the center of gravity of the section. Let the axis $a\,b$ pass through the center of gravity at a distance of x inches from the top line $c\,d$. The figure can be readily divided into rectangles, and the center of gravity of each rectangle can be found. Then, by the principle of moments, the moment of the entire section about $c\,d$ is equal to the sum of the moments of the parts about the same axis. Let e, f, and g be the rectangles into which the section is divided. The area of e is $2 \times 4 = 8$ sq. in., and its center of gravity is 1 in. from $c\,d$. The area of f is $2 \times 4 = 8$ sq. in., and its center of gravity is $2 + 2 = 4$ in. from $c\,d$. The area of g is $2 \times 8 = 16$ sq. in., and its center of gravity is $1 + 4 + 2 = 7$ in. from $c\,d$. The area of the entire section is $e + f + g = 8 + 8 + 16 = 32$ sq. in., and its center of gravity is x inches from $c\,d$. Then by the principle of moments stated above, $32\,x = 8 \times 1 + 8 \times 4 + 16 \times 7$, or $x = \frac{8 \times 1 + 8 \times 4 + 16 \times 7}{32} = 4\frac{3}{4}$ in. After locating the center of gravity of the entire figure, find the distance between the axis $a\,b$, through the center of gravity of the figure, and the center of gravity of each part. The center of gravity of e is $4\frac{3}{4} - 1 = 3\frac{3}{4}$ in. from $a\,b$; the center of gravity of f is $4\frac{3}{4} - 4 = \frac{3}{4}$ in. from $a\,b$; and the center of gravity of g is $7 - 4\frac{3}{4} = 2\frac{1}{4}$ in. from $a\,b$. The moment of inertia of a rectangle about an axis through its center of gravity is, from the table, $I = \frac{b\,d^3}{12}$. Letting the moments of inertia of e, f, and g be I_a, I_b, and I_c, respectively, then $I_a = \frac{4 \times 2 \times 2 \times 2}{12} = 2\frac{2}{3}$, $I_b = \frac{2 \times 4 \times 4 \times 4}{12} = 10\frac{2}{3}$, and $I_c = \frac{8 \times 2 \times 2 \times 2}{12} = 5\frac{1}{3}$. Let r_a, r_b, and r_c represent the distances of their centers of gravity from $a\,b$, and let their moments of inertia about $a\,b$ be I_a', I_b', and I_c'. Then $I_a' = I_a + e\,r_a^2 = 2\frac{2}{3} + 8 \times 3\frac{3}{4} \times 3\frac{3}{4} = 115\frac{1}{6}$. $I_b' = I_b + f\,r_b^2 = 10\frac{2}{3} + 8 \times \frac{3}{4} \times \frac{3}{4} = 15\frac{1}{6}$. $I_c' = I_c + g\,r_c^2 = 5\frac{1}{3} + 16 \times 2\frac{1}{4} \times 2\frac{1}{4} = 86\frac{1}{3}$. The moment of inertia of the entire figure, about $a\,b$, is the sum of the moments of inertia of the parts, or

$$I = I_a' + I_b' + I_c' = 115\tfrac{1}{6} + 15\tfrac{1}{6} + 86\tfrac{1}{3} = 216\tfrac{2}{3}. \quad \text{Ans.}$$

12. Design of Beams.—Taking the formula in Art. 8, the bending moment, in inch-pounds, may be found by either of the methods described in *Strength of Materials*, Part 1, or calculated by means of the table of Bending Moments given at the end of this Section. If it is desired to find the size of a beam that will safely resist a given bending moment, take S_b from Table I, and divide it by the proper factor of safety taken from *Strength of Materials*, Part 1. With the factor of safety f introduced into the formula of Art. 8, it becomes

$$M = \frac{S_b I}{f c} \qquad (1)$$

From this

$$\frac{I}{c} = \frac{M f}{S_b} \qquad (2)$$

Substituting the values of M, f, and S_b, the value of $\frac{I}{c}$ is found.

The material of the beam and its shape will usually be determined by the conditions under which it is to be used, and the cross-section will probably have the form of some one of the sections shown in the table of Moments of Inertia. The kind and shape of beam having been decided on, the size can be so proportioned that $\frac{I}{c}$ for the section shall not be less than the value calculated above. An example will make this clear.

EXAMPLE.—What should be the size of an ash girder to resist safely a bending moment of 16,000 inch-pounds, the cross-section to be rectangular and the load steady?

SOLUTION.—From formula **2**, $\frac{I}{c} = \frac{Mf}{S_b}$. $M = 16{,}000$; from Table I, $S_b = 8{,}000$; from *Strength of Materials*, Part 1, $f = 8$; then $\frac{I}{c} = \frac{16{,}000 \times 8}{8{,}000} = 16$. From the table of Moments of Inertia, $I = \frac{b\,d^3}{12}$ and $c = \frac{d}{2}$ for a rectangle; hence, $\frac{I}{c} = \frac{b\,d^3}{12} \times \frac{2}{d} = \frac{b\,d^2}{6} = 16$, or $b\,d^2 = 96$. Any number of values of b and d can be found that will satisfy this equation. If b is taken as 6 in., $d^2 = \frac{96}{6} = 16$ and $d = \sqrt{16} = 4$. Hence, the beam may be 6 in. × 4 in., with the short side vertical. When possible, it is always better to have the longer side vertical. If b is taken as 2 in., $d^2 = 48$ and $d = \sqrt{48} = 7$ in., nearly; hence, a 2″ × 7″ beam will also answer the purpose. The advantage of using a 2″ × 7″ beam instead of a 6″ × 4″ beam is evident, since the 6″ × 4″ beam contains nearly twice as much material as the 2″ × 7″ beam. Thus, the area of the cross-section of a 6″ × 4″ beam is 24 sq. in., and of a 2″ × 7″ beam, 14 sq. in. Moreover, the 2″ × 7″ beam, with its long side vertical, is slightly stronger than the 6″ × 4″ beam, with its short side vertical, since $\frac{I}{c} = \frac{2 \times 7^2}{6} = 16\frac{1}{3}$ for the former, and $\frac{6 \times 4^2}{6} = 16$ for the latter. If the 6″ × 4″ beam had its longer side vertical, thus making it a 4″ × 6″ beam, $\frac{I}{c}$ would then equal $\frac{4 \times 6^2}{6} = 24$, and the safe bending moment could be increased to $M = \frac{S_b I}{fc} = \frac{8{,}000 \times 24}{8} = 24{,}000$ in.-lb. Ans.

13. Ultimate Strength.—If the breaking bending moment, the form, and the size of cross-section of the beam are known, the ultimate strength in bending S_b can be readily found from the formula in Art. **8** by substituting the values of M, I, and c, and solving for S_b.

EXAMPLE.—A cast-iron bar 2 inches square breaks when the maximum bending moment is 63,360 inch-pounds; what is its ultimate strength in bending?

SOLUTION.— $M = S_b \frac{I}{c}$, or $S_b = \frac{Mc}{I}$. $c = \frac{d}{2} = 1$ in.; $I = \frac{d^4}{12} = \frac{2^4}{12}$; therefore, $S_b = \frac{63{,}360 \times 1}{\frac{1}{12} \times 2^4} = 47{,}520$ lb. per sq. in. Ans.

14. Table of Bending Moments.—In order to save time in calculating, the bending moments for cases of simple loading are given in the table of Bending Moments at the end of this Section. W denotes a concentrated load, and w the uniform load per inch of length. All dimensions are to be taken in inches when using the formulas.

For any other manner of loading than is described in the table of Bending Moments, the maximum bending moment may be obtained by finding the point of zero shear and calculating the bending moment for that point.

EXAMPLE 1.—A wrought-iron cantilever, 6 feet long, carries a uniform load of 50 pounds per inch; the cross-section of the beam is an equilateral triangle, with the vertex downwards; what should be the length of a side?

SOLUTION.—From the table of Bending Moments, $M = \frac{w l^2}{2} = \frac{50 \times (6 \times 12)^2}{2} = 129{,}600$ in.-lb. $I = \frac{bd^3}{36}$ and $c = \frac{2}{3}d$, from the table of Moments of Inertia; hence, $\frac{I}{c} = \frac{b\,d^2}{24}$. $S_b = 45{,}000$, from Table I, and $f = 4$, from *Strength of Materials*, Part 1. Therefore, $129{,}600 = \frac{45{,}000}{4} \times \frac{b\,d^2}{24}$, or $b\,d^2 = \frac{129{,}600 \times 4 \times 24}{45{,}000} = 276.48$. Since an equilateral triangle has been specified, b cannot be given any convenient value in order to find d. For an equilateral triangle, $d = b \sin 60° = .866\,b$. Hence, $b\,d^2 = b\,(.866\,b)^2 = .75\,b^3$. Therefore, $b\,d^2 = .75\,b^3 = 276.48$, $b = \sqrt[3]{\frac{276.48}{.75}}$, $\log b = \frac{1}{3}(\log 276.48 - \log .75) = .85553$, or $b = 7.17$ in., nearly. Ans.

EXAMPLE 2.—What weight will be required to break a round steel bar 4 inches in diameter, 16 feet long, fixed at both ends and loaded in the middle?

Solution.—Use the formula in Art. 8, $M = \frac{S_b I}{c}$. Here, from the table of Bending Moments, $M = \frac{W l}{8}$, and from Table I, $S_b = 65{,}000$; $\frac{I}{c} = \frac{\frac{1}{64}\pi d^4}{\frac{1}{2} d} = \frac{\pi d^3}{32}$. Hence, $\frac{W l}{8} = \frac{W \times (16 \times 12)}{8} = \frac{65{,}000 \times 3.1416 \times 4^3}{32}$, or $W = \frac{65{,}000 \times 3.1416 \times 64 \times 8}{16 \times 12 \times 32} = 17{,}017$ lb. Ans.

DEFLECTION OF BEAMS

15. Formula for Deflection.—The **deflection,** or amount of bending, produced in a beam by one or more loads is given by certain general formulas, whose derivations are too complicated to be given here. The formulas, however, will be given and their uses illustrated by examples.

In the third column of the table of Bending Moments are given expressions for the value of the greatest deflection of a beam when loaded as shown in the first column. From this, it is seen that the deflection s equals a constant (depending on the manner of loading the beam and on the condition of the ends—whether fixed or free), multiplied by $\frac{W l^3}{E I}$.

Let a = a constant, the value of which varies from $\frac{1}{384}$ to $\frac{1}{3}$;
s = deflection;
E = modulus of elasticity taken from *Strength of Materials*, Part 1;
l = length, in inches;
W = concentrated load, in pounds;
W' = total uniform load, in pounds;
I = moment of inertia about neutral axis.

Then,
$$s = a \frac{W l^3}{E I}$$

It will be noticed that the deflection is given for only nine cases; for any other manner of loading a beam than those here given, it is necessary to use higher mathematics to obtain the deflection.

Example.—What will be the maximum deflection of a simple wooden beam 9 feet long, whose cross-section is an ellipse having axes of 6 inches and 4 inches (short axis vertical), under a concentrated load of 1,000 pounds applied at the middle?

Solution.—Use the formula $s = a\,\frac{W l^3}{E I}$. From the table of Bending Moments, $a = \frac{1}{48}$; $W = 1{,}000$ lb.; $l = 9 \times 12 = 108$ in.; $E = 1{,}500{,}000$, and $I = \frac{\pi b d^3}{64} = \frac{\pi \times 6 \times 4^3}{64}$. Hence,

$$s = \frac{1{,}000 \times 108^3 \times 64}{48 \times 1{,}500{,}000 \times \pi \times 6 \times 64} = .9282 \text{ in. Ans.}$$

16. Stiffness of Beams.—The principal use of the formula for deflection is to determine the stiffness of a beam or shaft. In designing machinery, it frequently occurs that a piece may be strong enough to sustain the load with perfect safety, but the deflection may be more than circumstances will permit; in this case, the piece must be made larger than is really necessary for mere strength. An example of this occurs in the case of locomotive guides, and the upper guides of a steam engine when the engine runs under. It is obvious that they must be very stiff. In such cases it is usual to allow a certain deflection, and then proportion the piece so that the deflection shall not exceed the amount decided on.

Example.—The guides of a certain locomotive are to be made of cast iron, 38 inches long between the points of support, and of rectangular cross-section; the breadth must not exceed $2\frac{1}{4}$ inches, nor the deflection $\frac{1}{400}$ inch. Regarding the guides as fixed at both ends: (*a*) what must be their depth to resist a load of 10,000 pounds at the middle? (*b*) What weight will these guides be able to support with safety?

Solution.—(*a*) Since the load comes on two guides, each piece must support $10{,}000 \div 2 = 5{,}000$ lb. In the formula, $s = \frac{1}{400} = a\,\frac{W l^3}{E I}$. $a = \frac{1}{192}$ for this case, $W = 5{,}000$, $l = 38$, $E = 15{,}000{,}000$, and $I = \frac{b d^3}{12} = \frac{2\frac{1}{4} d^3}{12} = \frac{3}{16} d^3$. Substituting,

$$s = \frac{5{,}000 \times 38^3 \times 16}{192 \times 15{,}000{,}000 \times 3 d^3} = \frac{1}{400}$$

or $d = \sqrt[3]{\frac{5{,}000 \times 38^3 \times 16 \times 400}{192 \times 15{,}000{,}000 \times 3}} = 5.88$, nearly, say $5\frac{7}{8}$ in. **Ans.**

(*b*) To find the weight that these guides can support with safety, use formula **1**, Art. **12**, $M = \frac{S_b I}{f c}$, in which $M = \frac{W l}{8}$, $S_b = 30{,}000$.

$f = 15$, $\frac{I}{c} = \frac{b\,d^2}{6} = \frac{2\frac{1}{4} \times (5\frac{7}{8})^2}{6} = \frac{6,627}{512}$. Substituting,

$$W = \frac{30,000 \times 6,627 \times 8}{15 \times 512 \times 38} = 5,326 \text{ lb. Ans.}$$

Hence, the beam is over 6 per cent. stronger than necessary, the extra depth being required for stiffness.

EXAMPLES FOR PRACTICE

1. How much will a simple wooden beam 16 feet long, 2 inches wide, and 4 inches deep deflect under a load in the middle of 120 pounds? Ans. 1.106 in.

2. What should be the size of a rectangular yellow-pine girder 20 feet long to sustain a uniformly distributed load of 1,800 pounds? Assume a factor of safety for a varying stress, and make $b = \frac{2}{3} d$.
Ans. 6 in. × 9 in.

3. A hollow cylindrical beam, fixed at both ends, has diameters of 8 inches and 10 inches. The beam is 30 feet long and is made of cast iron. (*a*) What steady load will it safely support at 15 feet from one of the supports? (*b*) What force will be required to rupture the beam if applied at this point?
Ans. (*a*) 6,440.3 lb.
(*b*) 38,641.7 lb.

4. A simple cylindrical wrought-iron beam, resting on supports 24 feet apart, sustains three concentrated loads of 350 pounds each, at distances from one of the supports of 5, 12, and 19 feet; what should be the diameter of the beam to withstand shocks safely?
Ans. 4.71 in., say $4\frac{3}{4}$ in.

5. Find the value of $\frac{I}{c}$ for a hollow rectangle whose outside dimensions are 10 inches and 13 inches, and inside dimensions are 8 inches and 10 inches: (*a*) when the long side is vertical; (*b*) when the short side is vertical.
Ans. (*a*) 179.103
(*b*) $131\frac{1}{3}$

6. What is the deflection of a steel bar 1 inch square and 6 feet long, which supports a load of 100 pounds at the center? Ans. .311 in.

7. Which will be the stronger, a beam whose cross-section is an equilateral triangle, one side measuring 15 inches, or one whose cross-section is a square, one side measuring 9 inches? Both beams are of the same length. Ans. The one having the square cross-section

8. A wooden beam of rectangular cross-section sustains a uniform load of 50 pounds per foot. If the beam is 8 in. × 14 in. × 16 ft., how much more will it deflect when the short side is vertical than when the long side is vertical? Ans. .0554 in.

COMPARISON OF STRENGTH AND STIFFNESS OF BEAMS

17. Comparative Strength of Beams.—Consider two rectangular beams, loaded in the same manner, having the same lengths and bending moments but different breadths and depths. Then,

$$M = S_b \frac{I}{c} = S_b \frac{b\,d^2}{6} \qquad (1)$$

and

$$M = S_b \frac{b_1\,d_1^2}{6} \qquad (2)$$

Dividing (1) by (2) $\frac{M}{M} = \frac{6\,S_b\,b\,d^2}{6\,S_b\,b_1\,d_1^2} = \frac{b\,d^2}{b_1\,d_1^2} = 1$, or

$$b\,d^2 = b_1\,d_1^2 \qquad (3)$$

Equation (3) shows that, if both beams have the same depth, their strengths will vary directly as their breadths; that is, if the breadths are increased 2, 3, 4, etc. times, their strengths will also be increased 2, 3, 4, etc. times. It also shows that if the breadths are the same and the depths are increased, the strengths will vary as the square of the depth; that is, if the depths are increased 2, 3, 4, etc. times, the strengths will be increased 4, 9, 16, etc. times. Hence, it is always best, when possible, to have the long side of a beam vertical.

If the bending moments are the same, but the weights and lengths are different,

$$M = g\,W\,l \qquad (4)$$

and

$$M = g\,W_1\,l_1 \qquad (5)$$

when g denotes the fraction $\frac{1}{2}$, $\frac{1}{4}$, etc., according to the manner in which the ends are secured, and the manner of loading. Dividing (4) by (5), $\frac{M}{M} = \frac{g\,W\,l}{g\,W_1\,l_1}$, or

$$W\,l = W_1\,l_1 \qquad (6)$$

Equation (6) shows that if the load W or W_1 be increased, the length l or l_1 must be decreased; consequently, the strength of a beam loaded with a given weight varies inversely as its length; that is, if the load be increased 2, 3, 4, etc. times, the length must be shortened 2, 3, 4, etc. times, the breadth and depth remaining the same.

EXAMPLE 1.—If a simple beam, loaded in the middle, has its breadth and depth reduced one-half, what proportion of the original load can it carry?

SOLUTION.—It has been shown that the strength varied as the product of the breadth and the square of the depth, or $b_1 d_1^2 = \frac{1}{2} \times (\frac{1}{2})^2 = \frac{1}{8}$. Consequently, the beam can support only one-eighth of the original load. Had the breadth remained the same, $(\frac{1}{2})^2 = \frac{1}{4}$ of the original load could have been supported. Had the depth remained the same, one-half of the original load could have been supported. Ans.

EXAMPLE 2.—A beam 10 feet long, loaded in the middle, has a breadth of 4 inches and a depth of 6 inches. The length is increased to 12 feet, the breadth to 6 inches, and the depth to 8 inches; how many times the original load can it now support?

SOLUTION.—The strength varies directly as the product of the breadth and the square of the depth, and inversely as the length, or as $\frac{b\,d^2}{l}$. If b, d, and l denote the original sizes, the strength of the two beams will be to each other as $\frac{b_1\,d_1^2}{l} : \frac{b\,d^2}{l}$, or as $\frac{6 \times 8^2}{12} : \frac{4 \times 6^2}{10}$. $\frac{6 \times 8^2}{12} = 32$ and $\frac{4 \times 6^2}{10} = 14.4$. $\frac{32}{14.4} = 2\frac{2}{9}$. Consequently, the beam will support a load $2\frac{2}{9}$ times as great as the original beam. Ans.

18. Comparative Stiffness of Beams.—By a process of reasoning similar to that just employed, it can be shown that the maximum deflection of a beam varies inversely as the cube of the depth and directly as the cube of the length. In other words, if the depth be increased 2, 3, 4, etc. times, the deflection will be decreased 8, 27, 64, etc. times; and if the length be increased 2, 3, 4, etc. times, the deflection will be increased 8, 27, 64, etc. times. Hence, if a beam is required to be very stiff, the length should be made as short and the depth as great as circumstances will permit.

COLUMNS

19. Compression in Columns.—When a piece ten or more times as long as its least diameter or side (in general, its least transverse dimension) is subjected to compression, it is called a **column** or **pillar.** The ordinary rules for compression do not apply to columns, for the reason that when a long piece is loaded beyond a certain amount, it buckles and tends to fail by bending. This combination of bending and compression causes the column to break under a load considerably less than that required to crush the material. It is likewise evident that the strength of a column is principally dependent on its diameter, since that part having the least thickness is the part that buckles or bends. A column free to turn in any direction, having a cross-section of 3 in. × 8 in. is not nearly so strong as one whose cross-section is 4 in. × 6 in. The strength of a very long column varies, practically, inversely as the square of the length, the other dimensions remaining the same; that is, if column *b* is twice as long as column *a*, the strength of *b* is $(\frac{1}{2})^2 = \frac{1}{4}$ the strength of *a*, the cross-sections being equal.

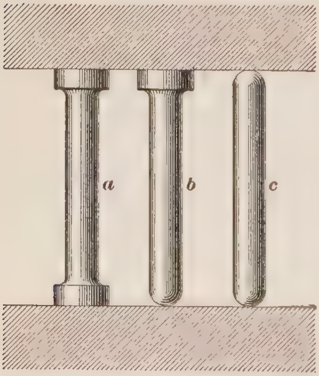

FIG. 5

20. Ends of Columns. The condition of the ends of a column plays a very important part in determining the strength of the column, and should always be taken into consideration. In Fig. 5 are shown three classes of columns. The column *a* is used in architecture, while columns similar to *b* and *c* are used in bridge and machine construction.

According to theory, which is confirmed by experiment, a column having one end flat and the other rounded, like *b*, is

$2\frac{1}{4}$ times as strong as a column having both ends rounded, like *c*. One having both ends flat, like *a*, is 4 times as strong as *c*, which has both ends rounded, the three columns being of the same length. If the length of *c* be taken as 1, the length of *b* may be $1\frac{1}{2}$, and that of *a* may be 2 for equal strength, the cross-sections all being the same; for, since the strengths vary inversely as the squares of the lengths, the strength of *c* is to that of *b* as $1 : \frac{1}{(1\frac{1}{2})^2}$, or as $1 : \frac{4}{9}$. But, since *b* is $2\frac{1}{4} = \frac{9}{4}$ times as strong as *c*, $\frac{4}{9} \times \frac{9}{4} = 1$; or, the length of *b* being $1\frac{1}{2}$ times that of *c*, its strength is the same. Similarly, when *a* is twice as long as *c*, its strength is the same.

Columns like *b* and *c* do not actually occur in practice, an eye being formed at the end of the column and a pin inserted, forming what may be termed a *hinged end*. A steam-engine connecting-rod is a good example of a column having two hinged ends, and a piston rod of a column having one end hinged and one end flat.

21. Rankine's Formula for Columns.—There are numerous formulas for calculating the strengths of columns, but the one that gives the most satisfactory results for columns of all lengths is the following, which is known as **Rankine's formula:**

$$W = \frac{S_c A}{f\left(1 + \frac{A l^2}{g I}\right)}$$

In this formula,

W = load;

S_c = ultimate strength for compression, taken from *Strength of Materials*, Part 1;

A = area of section of column, in square inches;

f = factor of safety;

l = length of column, in inches;

g = a constant, to be taken from Table II;

I = least moment of inertia of the cross-section, that is, the moment of inertia about an axis passing through the center of gravity of the cross-section and parallel to the longest side.

If the column has a rectangular cross-section, whose longer side is b and shorter side d, the least moment of inertia is $\frac{b\,d^3}{12}$, the axis in this case being parallel to the long side b. The values of g are given in Table II.

TABLE II

VALUES OF CONSTANT g FOR COLUMNS

Material	Both Ends Fixed	One End Hinged	Both Ends Hinged
Timber	3,000	1,690	750
Cast iron	5,000	2,810	1,250
Wrought iron	36,000	20,250	9,000
Steel	25,000	14,060	6,250

Example.—The section of a hollow, rectangular, cast-iron column has the following dimensions: $d = 8$ inches, $d_1 = 6$ inches, $b = 6$ inches, and $b_1 = 3\frac{1}{2}$ inches. If the length is 10 feet and the ends are fixed, what steady load will the column sustain with safety?

Solution.—From the table of Moments of Inertia, least I = $\frac{1}{12}(d\,b^3 - d_1\,b_1^3) = \frac{8 \times 6^3 - 6 \times 3.5^3}{12} = 122.5625$. $A = b\,d - b_1\,d_1 = 6 \times 8 - 3.5 \times 6 = 27$ sq. in. From *Strength of Materials*, Part 1, $S_c = 90{,}000$ and $f = 6$, $l = 10 \times 12 = 120$ in., and $g = 5{,}000$. Therefore,

$$W = \frac{90{,}000 \times 27}{6\left(1 + \frac{27 \times 120^2}{5{,}000 \times 122.5625}\right)} = \frac{2{,}430{,}000}{9.806} = 247{,}800 \text{ lb., nearly. Ans.}$$

Had the column been less than $10 \times 6 = 60$ in. = 5 ft. long, the safe load would have been $\frac{90{,}000 \times 27}{6} = 405{,}000$ lb. Had it been twice as long, it would have supported a safe load of only

$$\frac{90{,}000 \times 27}{6\left(1 + \frac{27 \times 240^2}{5{,}000 \times 122.5625}\right)} = 114{,}500 \text{ lb., nearly.}$$

22. Euler's Formula for Columns.—Another formula, known as **Euler's formula,** gives very satisfactory results for long columns, but, generally, it should not be used where the length is not at least thirty times the diameter or width of the column. It is, however, often used for calculating the strength of connecting-rods for steam engines. The formula is

$$W = C\frac{\pi^2 E I}{f l^2}$$

in which W = load in pounds;

C = a constant, 1 for both ends round or pinned, $2\frac{1}{4}$ for one end fixed and one round, and 4 for both ends fixed;

π^2 = 9.8696 (for approximate value use 10);

E = modulus of elasticity;

I = least moment of inertia;

l = length of column, in inches;

f = factor of safety.

It has been found by extensive investigation that Euler's formula gives results for connecting-rods that agree very closely with the best American practice. This formula will therefore be used for calculating the dimensions of connecting-rods in the examples given later. Since both ends of a connecting-rod are hinged, the value of C in the formula should be taken as 1 for such cases. For all other columns in which the length is less than thirty times the least width, the formula in Art. **21** will be used.

Example.—What thrust can be transmitted by a wrought-iron connecting-rod 6 feet long, and of rectangular cross-section $2\frac{1}{4}$ in. × 4 in.?

Solution.—Apply the formula $W = C\frac{\pi^2 E I}{f l^2}$. $C = 1$; $\pi^2 = 10$; from *Strength of Materials*, Part 1, $E = 25{,}000{,}000$; from the table of Moments of Inertia, $I = \frac{b d^3}{12} = \frac{2\frac{1}{4} \times 4 \times 4 \times 4}{12} = 12$; from *Strength of Materials*, Part 1, $f = 10$; $l^2 = (6 \times 12)^2 = 5{,}184$. Therefore,

$$W = \frac{10 \times 25{,}000{,}000 \times 12}{10 \times 5{,}184} = 57{,}870 \text{ lb., nearly. Ans.}$$

23. Designing Columns.—In the actual designing of a column, the size of the cross-section is not known, but the form (square, round, etc.) is known, also the length, material, condition of ends, and load it is to carry. To find the size of the cross-section, take the formula

$$P = A S$$

from *Strength of Materials*, Part 1,

where P = total load in pounds;

A = area of cross-section, in square inches;

S = unit stress, in pounds per square inch.

Then substitute $\frac{S_c}{f}$ in the formula for S, and solve for A, obtaining $A = \frac{P f}{S_c}$. Substituting, in this equation, the values of $P\ (= W)$, f, and S_c, this gives the value of A for a short piece less than ten times the length of the shortest side, or diameter. Assume a value of A somewhat larger than that just found, and dimension a cross-section of the form chosen so that its area shall equal that assumed. Calculate the moment of inertia and substitute the values of W, A, I, l, and g in the formula in Art. **21**, and solve for $\frac{S_c}{f}$. If the result last found equals the value of $\frac{S_c}{f}$ taken from *Strength of Materials*, Part 1, the assumed dimensions are correct; if larger, the assumed dimensions must be increased; if smaller, they should be diminished; in both cases, the value of $\frac{S_c}{f}$ should be recalculated. An example will serve to illustrate the process.

EXAMPLE.—What should be the diameter of a steel piston rod 5 feet long, the diameter of the piston being 18 inches and the greatest pressure 130 pounds per square inch?

SOLUTION.— S_c for this case = 65,000 lb. Since the piston rod is liable to shocks, a factor of safety of 15 should be used; hence, $\frac{S_c}{f} = \frac{65{,}000}{15} = 4{,}333\frac{1}{3}$ lb. The load $W = 18^2 \times .7854 \times 130 = 33{,}081$ lb. $A = \frac{P f}{S_c} = \frac{33{,}081}{4{,}333\frac{1}{3}} = 7.63$ sq. in., nearly. Assume that 8 sq. in. is needed. The diameter of a circle corresponding to an area of 8 sq. in. is $\sqrt{\frac{8}{.7854}} = 3.19$ in. Assume the diameter to be $3\frac{1}{4}$ in.; the area will be $(3\frac{1}{4})^2 \times .7854 = 8.3$ sq. in.

$$I = \frac{\pi d^4}{64} = \frac{3.1416 \times (3\frac{1}{4})^4}{64} = 5.48. \quad W = \frac{S_c A}{f\left(1 + \frac{A l^2}{g I}\right)}$$

Consequently,

$$\frac{S_c}{f} = \frac{W}{A}\left(1 + \frac{A\,l^2}{g\,I}\right) = \frac{33{,}081}{8.3}\left(1 + \frac{8.3 \times (5 \times 12)^2}{14{,}060 \times 5.48}\right) = 5{,}532 \text{ lb., nearly.}$$

As this value exceeds 4,333⅓ lb., the diameter of the rod must be increased. Trying 3½ in. as the diameter, the area = 9.62 sq. in. $I = 7.366$ and $\frac{S_c}{f} = \frac{33{,}081}{9.62}\left(1 + \frac{9.62 \times 60^2}{14{,}060 \times 7.366}\right) = 4{,}588$ lb., which is still greater than 4,333⅓ lb. Trying 3⅝ in. as the diameter, the area is 10.32 sq. in. and $I = 8.476$. Substituting these values, as before,

$$\frac{S_c}{f} = \frac{33{,}081}{10.32}\left(1 + \frac{10.32 \times 60^2}{14{,}060 \times 8.476}\right) = 4{,}205 \text{ lb.}$$

Hence, a rod 3⅝ in. in diameter is sufficient. Ans.

24. The method given in Art. **23** for determining the dimensions of the cross-section, when the load and length are given, is perfectly general, and may therefore be used in every case. It is, however, somewhat long and cumbersome. For the special cases of square, circular, and rectangular columns, the following formulas may be applied, if preferred. They seem complicated, but, when substitutions are made for the quantities given, the formulas will be found of relatively easy application.

For square columns, the side c of the square is given by the formula

$$c = \sqrt{\frac{W f}{2\,S_c} + \sqrt{\frac{W f}{S_c}\left(\frac{W f}{4\,S_c} + \frac{12\,l^2}{g}\right)}} \qquad (1)$$

For circular columns, the diameter d of the circle is given by the formula

$$d = 1.4142\sqrt{\frac{.3183\;W f}{S_c} + \sqrt{\frac{.3183\;W f}{S_c}\left(\frac{.3183\;W f}{S_c} + \frac{16\,l^2}{g}\right)}} \qquad (2)$$

For rectangular columns, assume the shorter dimension (depth = d). Then the longer dimension (breadth = b) is given by the formula

$$b = \frac{W f\left(1 + \frac{12\,l^2}{d^2 g}\right)}{d\,S_c} \qquad (3)$$

Should the dimensions given by the last formula be too much out of proportion, a new value may be assumed for d, and a new value found for b.

Example 1.—Required, the section of a square timber pillar to stand a steady load of 20 tons, the length of the column being 30 feet, and its ends both flat.

Solution.—Here S_c = 8,000 lb., f = 8, g = 3,000, W = 40,000 lb., $l = 30 \times 12 = 360$ in. These values, substituted in formula **1**, give

$$c = \sqrt{\frac{40{,}000 \times 8}{2 \times 8{,}000} + \sqrt{\frac{40{,}000 \times 8}{8{,}000}\left(\frac{40{,}000 \times 8}{4 \times 8{,}000} + \frac{12 \times 360^2}{3{,}000}\right)}}$$

$$= \sqrt{20 + \sqrt{40\left(10 + \frac{4 \times 36^2}{10}\right)}}$$

$$= \sqrt{20 + \sqrt{21{,}136}} = \sqrt{20 + 145.38}$$

$$= \sqrt{165.38} = 12.90 = 12\tfrac{7}{8} \text{ in., nearly, or say 13 in.} \quad \text{Ans.}$$

Example 2.—Let it be required to solve the problem worked out by the general method in Art. **23.**

Solution.—Here S_c = 65,000 lb., f = 15, g = 14,060, W = 33,000 lb., nearly, and $l = 5 \times 12 = 60$ in. From these data,

$$\frac{.3183\ Wf}{S_c} = \frac{.3183 \times 33{,}000 \times 15}{65{,}000} = 2.424$$

$$\frac{16\ l^2}{g} = \frac{16 \times 60^2}{14{,}060} = \frac{8 \times 360}{703} = 4.0967$$

Then, by formula **2**,

$$d = 1.4142\sqrt{2.424 + \sqrt{2.424 \times 6.5207}}$$

$$= 1.4142\sqrt{2.424 + 3.976} = 1.4142\sqrt{6.400}$$

$$= 1.4142 \times 2.53 = 3.58 \text{ in., or about } 3\tfrac{5}{8} \text{ in.}$$

as found by the general or trial method. Ans.

EXAMPLES FOR PRACTICE

1. What safe steady load will a hollow, cylindrical, cast-iron column support, which is 14 feet long, outside diameter 10 inches, inside diameter 8 inches and has flat ends? Ans. 273,500 lb., nearly

2. Suppose a wrought-iron connecting-rod to be of rectangular cross-section twice as long as wide and of uniform size throughout its length. If the diameter of the steam cylinder is 40 inches, steam pressure 110 pounds per square inch, and the length of the rod is 12½ feet, what should be the dimensions of the cross-section of the rod?
Ans. 3¾ in. × 7½ in., nearly

TORSION AND SHAFTS

25. Effect of Twisting.—When a force is applied to a beam in such a manner that it tends to twist it, the stress thus produced is termed **torsion.** In Fig. 6, *b c* represents a beam fixed at one end; a load *W* applied at the end of a lever arm *o n* twists the beam. If a straight line *c b* is drawn parallel to the axis before the load is applied, it will be found, after the weight *W* has been hung from *n*, that the line *c b* will take a position *c a*, forming a helix. If the load does not deform the material beyond its elastic limit, *c a* will return to its original position *c b* when *W* is removed. It will also be found that the angles *a c b* and *a o b* are directly proportional to the loads.

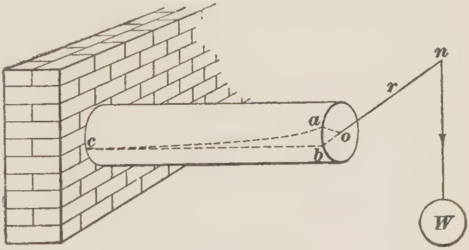

FIG. 6

Torsion manifests itself in the case of rotating shafts. Instead of one end being fixed, as in the previous case, the resistance that the shaft has to overcome takes the place of the force that before was necessary for fixing one end. Should the shaft be too small, the resistance will overcome the strength of the material and rupture it.

The angle *a o b*, which may be called the angle of twist, plays an important part in the designing of shafts.

26. Twisting Moment and Moment of Resistance. If the line of action of a force passes through the axis of a shaft, the shaft is subjected to bending; but if the line of action passes to one side of the axis the shaft is subjected to twisting. The product of the turning force and the perpendicular distance from the axis of the shaft to the line of action of the force is called the **twisting moment.** The material of the shaft resists the twisting, and this resisting force has a moment about the axis of the shaft, known as the moment of resistance. When the positions of the fibers of a shaft subjected to twisting do not change in relation to one another, the moment of resistance is equal to the twisting moment. For, if they are not equal, the twisting moment will continue to twist the shaft, changing the relative positions of the fibers, and will finally cause rupture of the shaft.

In Fig. 7, let a represent a pin that prevents the two adjacent parts b, d of a shaft from turning independently of each other. Let the distance of the pin from the axis $o\,o$ of the shaft be c, and let the force P, at the distance r from $o\,o$, tend to turn the shaft about its axis. It is evident that when the end b of the shaft is acted on by the force P, and the end d is prevented from turning, that the pin a is in shear.

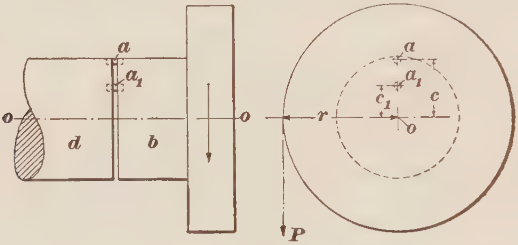

FIG. 7

The unit shearing stress is S_s, and $S_s\,a$ is the total shearing stress of the pin if a represents its area. The twisting moment T of the force P about the axis is $T = P\,r$, and the moment of the shearing resistance is $S_s\,a\,c$ about the same center. These two moments must be equal, or $T = P\,r = S_s\,a\,c$. If there is a second pin with area a_1 at a distance c_1 from $o\,o$, and with a shearing stress S_s', the external moment must equal the sum of the moments of shear in both pins, or

$$T = S_s\,a\,c + S_s'\,a_1\,c_1 \qquad (1)$$

But the unit stresses in the pins are proportional to their distances from o, provided that the stress is not greater than the elastic limit; hence, $\frac{S_s'}{S_s} = \frac{c_1}{c}$, or $S_s' = \frac{S_s c_1}{c}$. Substituting this value of S_s' in formula **1**, it becomes $T = S_s a c + \frac{S_s a_1 c_1^2}{c}$, or

$$T = \frac{S_s a c^2}{c} + \frac{S_s a_1 c_1^2}{c} \qquad (2)$$

Now, if c is the distance from o of a pin at the circumference of the shaft, and if the number of pins be increased until they cover the entire area of the shaft, or if the area of any cross-section of the shaft be considered as divided into very small areas, the resistance of the entire section is the sum of the resistances of all these very small areas, and it may be expressed thus: $T = \frac{S_s a c^2}{c} + \frac{S_s a_1 c_1^2}{c} + \frac{S_s a_2 c_2^2}{c} + \frac{S_s a_3 c_3^2}{c} + \text{etc.}$, or

$$T = \frac{S_s}{c}(a c^2 + a_1 c_1^2 + a_2 c_2^2 + a_3 c_3^2 + \text{etc.}) \qquad (3)$$

Now, the sum of the products of all these very small areas, multiplied by the squares of their distances from the axis, is called the **polar moment of inertia**, which is represented by J.

Then formula **3** becomes

$$T - \frac{S_s J}{c} \qquad (4)$$

and if divided by f, the factor of safety, it becomes

$$T = \frac{S_s J}{f c} \qquad (5)$$

It will be noticed that this formula resembles closely the formula $M = \frac{S_b I}{f c}$ for bending, given in Art. **12**, the only difference being that S_s is used instead of S_b, and the polar moment of inertia J instead of the rectangular moment of inertia I. While formula **5** is strictly true for shafts of circular sections only, it is approximately true for other shafts, and may be applied to square, rectangular, and elliptical shafts when the factor of safety is so large that exact results are not extremely important.

27. Polar Moment of Inertia.—Let Fig. 8 represent a section of a body and a the area of any very small part of the section at a distance y from the axis XX and a distance x from the axis YY. The axes XX and YY pass through the center of gravity of the section at right angles to each other. The moment of inertia of a about XX is $a y^2$, and that about the axis YY is $a x^2$. The polar moment of inertia about the center of gravity o is $a r^2$. But, $x^2 + y^2 = r^2$. Multiplying both sides of this equation by a gives $a x^2 + a y^2 = a r^2$. This is true for all points, hence it is true for the whole section. Calling I_x and I_y the moments of inertia of the section about axes at right angles to each other,

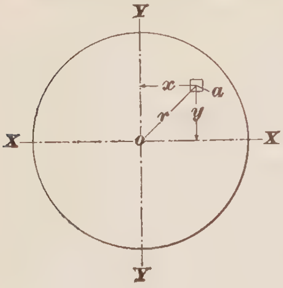

FIG. 8

$$J = I_x + I_y$$

That is, the polar moment of inertia is equal to the sum of the rectangular moments of inertia about two axes at right angles to each other.

The method of determining the polar moment of inertia may be illustrated by the use of Fig. 9, which is a rectangle with axes XX and YY passing through its center of gravity, at right angles to each other. The moment of inertia of the figure about XX is, from the table of Moments of Inertia, $\frac{b d^3}{12}$, and about YY it is $\frac{d b^3}{12}$. The sum of these two is

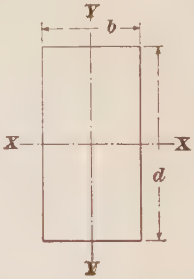

FIG. 9

$$J = \frac{b d^3}{12} + \frac{d b^3}{12} = \frac{b d (d^2 + b^2)}{12}$$

For a solid square section, $J = \frac{d^4}{6}$

For a hollow square section, $J = \frac{d^4 - d_1^4}{6}$

For a solid circular section, $J = \frac{\pi d^4}{32}$

For a hollow circular section, $J = \frac{\pi(d^4 - d_1^4)}{32}$

EXAMPLE.—What belt pull on a pulley 8 feet in diameter will a steel shaft 4 inches in diameter stand with safety when there is liability to shocks?

SOLUTION.—Apply formula **5**, Art. **26**, $T = \frac{S_s J}{f c}$, or, as $T = Pr$, $P = \frac{S_s J}{f c r}$. $S_s = 70{,}000$ lb., $J = \frac{\pi d^4}{32} = 25.133$, $f = 15$, $c = 2$ in., $r = 4 \times 12 = 48$ in. Then,

$$P = \frac{70{,}000 \times 25.133}{15 \times 2 \times 48} = 1{,}221.7 \text{ lb., nearly. Ans.}$$

28. Bending of Shafts.—The foregoing formulas do not take into account the bending of the shaft; hence, for long shafts carrying bodies, such as pulleys, between supports, they should not be used. Where shafts are subjected to bending only, they are treated as beams, although they may rotate. The formulas and graphic solutions for beams apply directly to such cases, but not to cases in which there is combined bending and torsion. In the latter case, an equivalent twisting moment may be found to take the place of the twisting and bending moments. This equivalent twisting moment may then be used in formula **5**, Art. **26**.

Let M = bending moment for any section;
T = twisting moment for same section;
T_1 = equivalent twisting moment.

Then, by higher mathematics, the following formula is derived:

$$T_1 = M + \sqrt{M^2 + T^2} \qquad (1)$$

The twisting moment T_1 is sometimes called the **ideal twisting moment.** The bending moment M is obtained as explained in *Strength of Materials*, Part 1, or by the use of the formulas in the table of Bending Moments; the twisting moment T is obtained by the method explained in Art. **26**.

Formula **5**, Art. **26**, may be changed to a more convenient form for circular shafts, by finding a value for the diameter.

Thus, by transforming the formula, $\frac{J}{c} = \frac{Tf}{S_s}$. But, $J = \frac{\pi d^4}{32}$ and $c = \frac{d}{2}$. Hence, substituting, $\frac{\frac{\pi d^4}{32}}{\frac{d}{2}} = \frac{Tf}{S_s}$, or $\frac{\pi d^4}{32} \times \frac{2}{d} = \frac{Tf}{S_s}$; from which $\frac{\pi d^3}{16} = \frac{Tf}{S_s}$. Solving this for d^3, $d^3 = \frac{16\,Tf}{\pi S_s}$, and

$$d = \sqrt[3]{\frac{16\,Tf}{\pi S_s}} \qquad (2)$$

Now, substituting for T the equivalent twisting moment T_1, formula **2** becomes

$$d = \sqrt[3]{\frac{16\,T_1 f}{\pi S_s}} \qquad (3)$$

EXAMPLE 1.—A wrought-iron shaft subjected to shocks is required to withstand safely a twisting moment of 25,000 inch-pounds; what should be its diameter?

SOLUTION.—Using formula **2**, and making $T = 25{,}000$, $f = 10$, and $S_s = 50{,}000$,

$$d = \sqrt[3]{\frac{16 \times 25{,}000 \times 10}{3.1416 \times 50{,}000}} = 2.94 \text{ in., or say 3 in. Ans.}$$

EXAMPLE 2.—Find the diameter of a steel shaft to withstand safely an equivalent twisting moment of 35,000 inch-pounds, using a factor of safety of 15.

SOLUTION.—Using formula **3**, and making $T_1 = 35{,}000$, $f = 15$, and $S_s = 70{,}000$,

$$d = \sqrt[3]{\frac{16 \times 35{,}000 \times 15}{3.1416 \times 70{,}000}} = 3.37 \text{ in., or say } 3\tfrac{3}{8} \text{ in. Ans.}$$

EXAMPLES FOR PRACTICE

1. If the bending moment is 24,000 inch-pounds and the twisting moment is 15,000 inch-pounds, find the equivalent twisting moment.
Ans. 52,302 in.-lb., nearly

2. A cast-iron shaft is subjected to torsion, the twisting moment being 8,000 inch-pounds; what should be the diameter, assuming a factor of safety of 15? Ans. $3\frac{1}{8}$ in., very nearly

3. Find the diameter of a steel shaft subjected to combined torsion and bending, the twisting moment being 16,000 inch-pounds and the bending moment 8,000 inch-pounds; take a factor of safety of 7.

Ans. $2\frac{3}{8}$ in.

4. A wrought-iron shaft is subjected to torsion by a force of 2,500 pounds acting at a distance of 20 inches from its axis; assuming the force to be steadily applied, find the required diameter of the shaft. Ans. $2\frac{3}{4}$ in., nearly

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ROPES AND CHAINS

ROPES

29. Hemp and Manila Ropes.—The strength of **hemp and manila ropes** varies greatly, depending not so much on the material and area of cross-section as on the method of manufacture and the amount of twisting. Hemp ropes are about 25 per cent. to 30 per cent. stronger than manila ropes or tarred hemp ropes. Ropes laid with tar wear better than those laid without tar, but their strength and flexibility are greatly reduced. For most purposes the following formula may be used for the safe working load of any of the three ropes just mentioned:

$$P = 100\, C^2$$

in which P = working load, in pounds;

C = circumference of rope, in inches.

This formula gives a factor of safety of from $7\frac{1}{2}$ for manila or tarred hemp rope to about 11 for the best three-strand hemp rope. When excessive wear is likely to occur, it is better to make the circumference of the rope considerably larger than that given by the formula.

30. Wire Ropes.—**Wire rope** is made by twisting a number of wires (usually nineteen) together into a strand and then twisting several strands (usually seven) together to form the rope. It is very much stronger than hemp rope, and may be much smaller in size to carry the same load.

For iron-wire rope of seven strands, nineteen wires to the strand, the following formula may be used, the letters having the same meaning as in the formula in Art. **29**:

$$P = 600\,C^2 \qquad \textbf{(1)}$$

Steel-wire ropes should be made of the best quality of steel wire; when so made they are superior to the best iron-wire ropes. If made from an inferior quality of steel wire, the ropes are not as good as the better class of iron-wire ropes. When substituting steel for iron ropes, the object in view should be to gain an increase of wear rather than to reduce the size. The following formula may be used in computing the size or working strength of the best steel wire rope, seven strands, nineteen wires to the strand:

$$P = 1{,}000\,C^2 \qquad \textbf{(2)}$$

Formulas **1** and **2** are based on a factor of safety of 6.

31. Long Ropes.—When using ropes for the purpose of raising loads to a considerable height, the weight of the rope itself must also be considered and added to the load. The weight of the rope per running foot, for different sizes, may be obtained from the manufacturer's catalog.

EXAMPLE 1.—What should be the allowable working load of an iron-wire rope whose circumference is $6\frac{3}{4}$ inches? Weight of rope not to be considered.

SOLUTION.—Using formula **1**, Art. **30**,

$$P = 600 \times (6\tfrac{3}{4})^2 = 27{,}337.5 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The working load, including weight, of a hemp rope is to be 900 pounds; what should be its circumference?

SOLUTION.—Using the formula in Art. **29**,

$$C = \sqrt{\frac{P}{100}} = \sqrt{\frac{900}{100}} = 3 \text{ in.} \quad \text{Ans.}$$

32. Sizes of Ropes.—In measuring ropes, the circumference is used instead of the diameter, because the ropes are not round and the circumference is not equal to 3.1416 times the diameter. For three strands, the circumference is about $2.86\,d$, and for seven strands about $3\,d$, d being the diameter.

CHAINS

33. The size of a **chain** is always specified by giving the diameter of the iron from which the link is made. The two kinds of chain most generally used are the **open-link chain** and the **stud-link chain.** The former is shown in Fig. 10 (*a*) and the latter in Fig. 10 (*b*). The stud prevents the two sides of a link from coming together when under a heavy pull, and thus strengthens the chain.

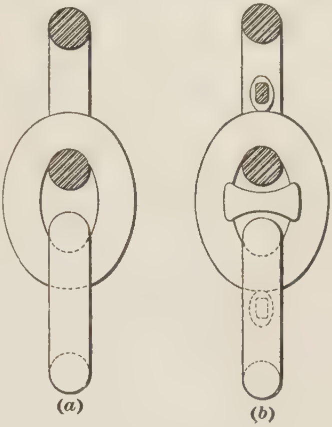


FIG. 10

It is a good practice to anneal old chains that have become brittle by overstraining. This renders them less liable to snap from sudden jerks. The annealing process reduces their tensile strength, but increases their toughness and ductility, two qualities that are sometimes more important than mere strength.

Let $P =$ safe load, in pounds;
$d =$ diameter of link, in inches.

Then, for open-link chains made from a good quality of wrought iron,

$$P = 12{,}000\, d^2 \qquad (1)$$

and, for stud-link chains,

$$P = 18{,}000\, d^2 \qquad (2)$$

EXAMPLE 1.—What load will be safely sustained by a $\frac{3}{4}$-inch open-link chain?

SOLUTION.—Using formula **1**,

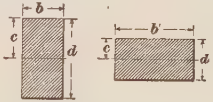
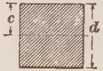
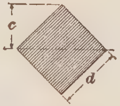
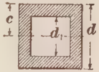
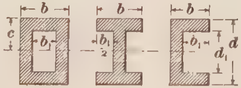
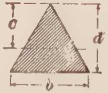
$$P = 12{,}000\, d^2 = 12{,}000 \times \left(\tfrac{3}{4}\right)^2 = 6{,}750 \text{ lb.} \quad \text{Ans.}$$

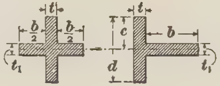
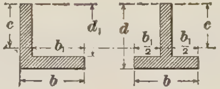
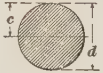
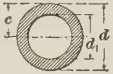
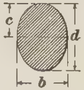
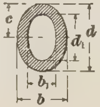
EXAMPLE 2.—What must be the diameter of a stud-link chain to carry a load of 28,125 pounds?

SOLUTION.—Use formula **2**, $P = 18{,}000\, d^2$. Hence,

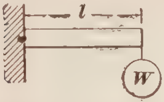
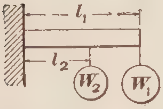
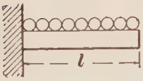
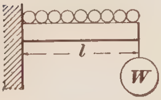
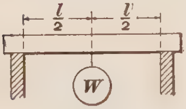
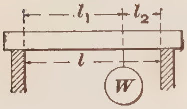
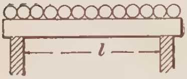
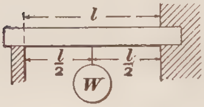
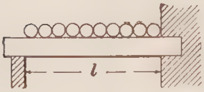
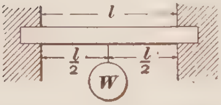
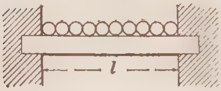
$$d = \sqrt{\frac{P}{18{,}000}} = \sqrt{\frac{28{,}125}{18{,}000}} = 1\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

MOMENTS OF INERTIA

Form of Section	Dotted Line Shows Position of Neutral Axis	A	I	c
1. Rectangle . .		bd	$\frac{1}{12}bd^3$	$\frac{1}{2}d$
2. Square		d^2	$\frac{1}{12}d^4$	$\frac{1}{2}d$
3. Axis through Diagonal . . .		d^2	$\frac{1}{12}d^4$	$.707d$
4. Hollow Square		$d^2 - d_1{}^2$	$\frac{1}{12}(d^4 - d_1{}^4)$	$\frac{1}{2}d$
5. Hollow Rectangle, **I** or Channel Iron .	 b c b_1 b $\frac{b_1}{2}$ b b_1 d d_1	$bd - b_1d_1$	$\frac{1}{12}(bd^3 - b_1d_1{}^3)$	$\frac{1}{2}d$
6. Triangle . . .		$\frac{1}{2}bd$	$\frac{1}{36}bd^3$	$\frac{2}{3}d$

Form of Section	Dotted Line Shows Position of Neutral Axis	A	I	c
7. Cross		$td + t_1 b$	$\frac{1}{12}(td^3 + bt_1{}^3)$	$\frac{1}{2}d$
8. Angle Iron . .		$bd - b_1 d_1$	$\frac{(bd^2 - b_1 d_1{}^2)^2 - 4bdb_1 d_1(d - d_1)^2}{12(bd - b_1 d_1)}$	$\frac{d}{2} + \frac{b_1 d_1}{2}\left(\frac{d - d_1}{bd - b_1 d_1}\right)$
9. Circle		$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	$\frac{1}{2}d$
10. Hollow Circle .		$\frac{\pi}{4}(d^2 - d_1{}^2)$	$\frac{\pi(d^4 - d_1{}^4)}{64}$	$\frac{1}{2}d$
11. Ellipse		$\frac{\pi}{4}bd$	$\frac{\pi bd^3}{64}$	$\frac{1}{2}d$
12. Hollow Ellipse		$\frac{\pi}{4}(bd - b_1 d_1)$	$\frac{\pi(bd^3 - b_1 d_1{}^3)}{64}$	$\frac{1}{2}d$

BENDING MOMENTS AND DEFLECTIONS

	Manner of Supporting Beams	Maximum Bending Moment, M	Maximum Deflection, s	Remarks
1.		$W\,l$	$\frac{1}{3}\frac{W\,l^3}{E\,I}$	Cantilever, load at free end
2.		$W_1\,l_1 + W_2\,l_2$		Cantilever, more than one load
3.		$\frac{w\,l^2}{2}$	$\frac{1}{8}\frac{W'\,l^3}{E\,I}$	Cantilever, uniform load w lb. per unit of length. $W' = w\,l$
4.		$\frac{w\,l^2}{2} + W\,l$	$\frac{1}{3}\frac{W'\,l^3}{E\,I} + \frac{1}{8}\frac{W''\,l^3}{E\,I}$	Cantilever, load partly uniform, partly concentrated
5.		$\frac{W\,l}{4}$	$\frac{1}{48}\frac{W\,l^3}{E\,I}$	Simple beam, load at middle
6.		$W\frac{l_1\,l_2}{l}$		Simple beam, load at some other point than the middle
7.		$\frac{w\,l^2}{8}$	$\frac{5}{384}\frac{W'\,l^3}{E\,I}$	Simple beam, uniformly loaded
8.		$\frac{3}{16}W\,l$	$.0182\,\frac{W\,l^3}{E\,I}$	One end fixed, other end supported, load in the middle
9.		$\frac{w\,l^2}{8}$	$.0054\,\frac{W'\,l^3}{E\,I}$	One end fixed, other end supported, uniformly loaded
10.		$\frac{W\,l}{8}$	$\frac{1}{192}\frac{W\,l^3}{E\,I}$	Both ends fixed, load in the middle
11.		$\frac{w\,l^2}{12}$	$\frac{1}{384}\frac{W'\,l^3}{E\,I}$	Both ends fixed, uniformly loaded

THE TESTING OF MATERIALS

METHODS AND APPLIANCES

1. Purpose of Testing Materials.—In designing machinery, it is essential that the designer shall be familiar with the physical properties of the materials used, in order that he may be able to give the various parts the proportions and strengths necessary to enable them to withstand the forces to which they will be subjected. In ordinary machine construction, the materials most generally used are cast iron, wrought iron, and steel. In order to learn what are the properties and characteristics of these, as well as other metals, when under load, the simplest way is to study their action under test. Many manufacturing establishments maintain testing departments in which are tested the various materials used. In this way, information as to the strength, elasticity, ductility, etc. of a material is readily obtained, and thus it is known just what load can safely be carried by each part of a machine.

2. Kinds of Tests.—The stresses to which the parts of a machine are most commonly subjected are those producing tension, compression, and flexure. Consequently, the greater number of tests are made to determine the strengths of materials subjected to these stresses. In some machines, the parts are subjected to torsion and shearing, but these are usually of less importance than the others, so that tests for strength in torsion and in shearing are less frequently made.

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TEST PIECES

3. Selection of Test Pieces.—In making a physical test of a material, a bar of that material is taken and subjected to the kind of stress under which its strength is desired to be known. The bar thus taken is known as a **test piece** or **test specimen.** This piece should be selected with care in order that it may represent the average quality of the material of which it is a specimen. It is not uncommon to find variations of quality in the same piece of material, as, for example, in a boiler plate. It is unwise, therefore, to depend on the results obtained from the testing of a single specimen; a number of tests should be made and the average of the results taken. This will give a more nearly correct idea of what may be expected of the material in actual service.

4. Forms of Test Pieces for Tension.—Some of the forms of specimens used in tensile tests are shown in Fig. 1, in which (*a*) is a rolled flat or square bar taken just as it comes from the rolls; (*b*) is a round rolled bar; (*c*) is a specimen cut from a plate; (*d*) is a specimen that has been turned down in the middle so as to give a uniform section for a considerable portion of its length; and (*e*) is a circular bar having a sectional area at the middle of about 1 square inch.

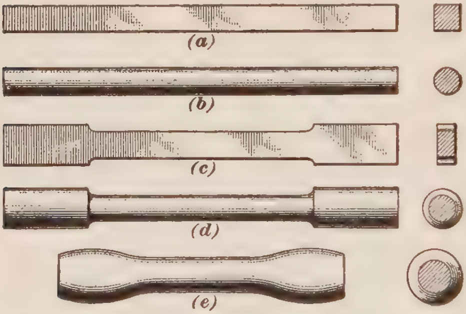


Fig. 1

For ordinary work, specimens of square, round, or flat bars are cut directly from the rolled stock and used without further preparation. In all such cases, the least sectional area of the piece must be accurately measured and recorded. If the specimen is cut from a plate, the usual form is that shown in Fig. 1 (*c*). The front and back surfaces are rolled, and are not machined in any way. The edges, however, are machined so as to make them parallel and about 1½ inches apart. For testing pieces from large bars, the middle part of the specimen is turned down, as in Fig. 1 (*d*). The large ends thus form convenient places to grip the test piece in the testing machine, while the break in the piece must occur somewhere between the larger ends, because of the reduction in cross-section.

5. A test piece with threaded ends is shown in Fig. 2. This form is adopted by the American section of the International Association for Testing Materials. For steel forgings, the dimensions of the test piece are as shown in the figure. The ends are threaded with United States standard threads, ten to the inch. The advantages of this form of specimen for steel forgings are as follows: (1) It is shorter than other forms and less material is wasted. (2) Less time and labor are required to prepare a small specimen. (3) The testing of a number of short specimens gives, at equal cost, a better idea of the quality of the material than the testing of one long specimen. (4) The shorter piece can be taken from parts of a forging that will not give a long piece. (5) The elastic limit, elongation, and reduction of area of cross-section in tension, can be determined quite as accurately by short as by long specimens.

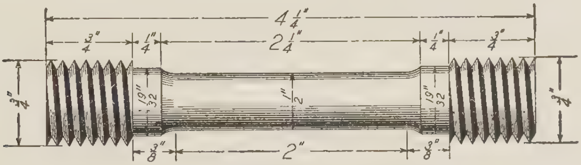


Fig. 2

There is no generally accepted standard form of test piece for tension. It has been found, however, that pieces having the same relative dimensions or of similar shapes have the same percentages of elongation and the same strengths per square inch, so that under these conditions comparisons of the materials tested may be made without error. A rule that may be safely followed is to make the length of the test piece eight times the diameter, in case of a round bar, and nine times the thickness in case of a square bar. A bar 8 inches long and 1 inch in diameter is used quite commonly in practice.

6. Test Pieces for Compression.—There are a number of forms of test pieces for compression, but when it is desired to find the compressive strength only, the length of the piece should not exceed ten times its least transverse dimension. Short square blocks are sometimes used for compression specimens, but the most common form is a short cylinder. There are two convenient sizes of this form, one being 1 inch in diameter by 2 inches in height, and the other .798 inch in diameter by 1 inch in height. The area of cross-section of the first is .7854 square inch, while that of the latter is ½ square inch, which is very convenient in calculating the stress per square inch. In preparing test pieces for compression, great care must be taken to have the ends quite flat and parallel, as well as at right angles to the sides of the piece. The pressure should be applied in a direction parallel to the sides. If the ends are not at right angles to the sides, or if they are not flat, the load will not be distributed uniformly and the results of the test will be less trustworthy.

7. Test Pieces for Flexure.—Test pieces for flexure are ordinarily made of square or of rectangular cross-section and of any convenient length. The small specimens of wrought iron and steel used in the testing machine are usually taken from bars 1 inch square, just as they come from the rolls, and are not machined or finished. Cast-iron specimens are also 1 inch square in section and are used just as they come from the mold, except that the sand is brushed off them. Larger test pieces may be made of rolled shapes, and full-sized beams may be tested, provided that a machine of ample size is available.

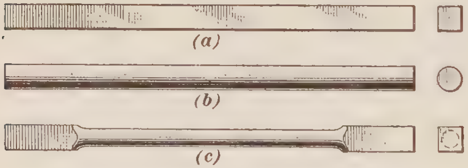


Fig. 3

8. Test Pieces for Torsion and Shearing.—Test pieces of convenient sizes for torsion tests are shown in Fig. 3. That shown at (*a*) is a bar about 20 inches in length and 1 inch square; that at (*b*) is of the same length, but is cylindrical and 1 inch in diameter; that shown at (*c*) is a piece formed from a square bar turned down at the middle to $\frac{3}{4}$ inch diameter. It is of the same length as the others.

Shearing tests of bars are seldom made. However, tests are sometimes made to determine the shearing strength of rivets. To do this, a test piece like that shown in Fig. 4 is employed. It consists of two plates riveted together. The plates are made of such strength that the rivets will fail first, by shearing off, when the entire piece is subjected to tension.

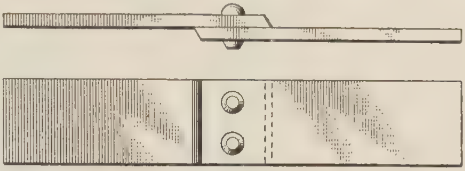


FIG. 4

APPARATUS FOR TESTING MATERIALS

9. The Testing Machine.—Physical tests of materials are usually made by means of a machine called a **testing machine,** although there are a few simple tests that can be made with ordinary tools, such, for example, as bending a piece of iron or steel while cold to determine its toughness and ductility, or dropping a weight on a bar of cast iron to determine its ability to resist shocks. The important tests, however, are accomplished by special machines designed for the purpose, one of which is shown in Fig. 5. This machine may be used to test materials in tension, compression, or flexure. As shown in the illustration, the machine is fitted up to make a tension test.

10. The bar *a* to be tested is held firmly by its ends in jaws or clamps at *b* and *c*. The lower clamp *b* is attached to the movable head *d*, which is raised and lowered by the turning of two heavy screws, one at each side of the head *d*.

One of these screws is shown at *e*. Large nuts are rigidly held in the head and the screws pass through them. Hence, when the screws are turned by the gears in the base of the

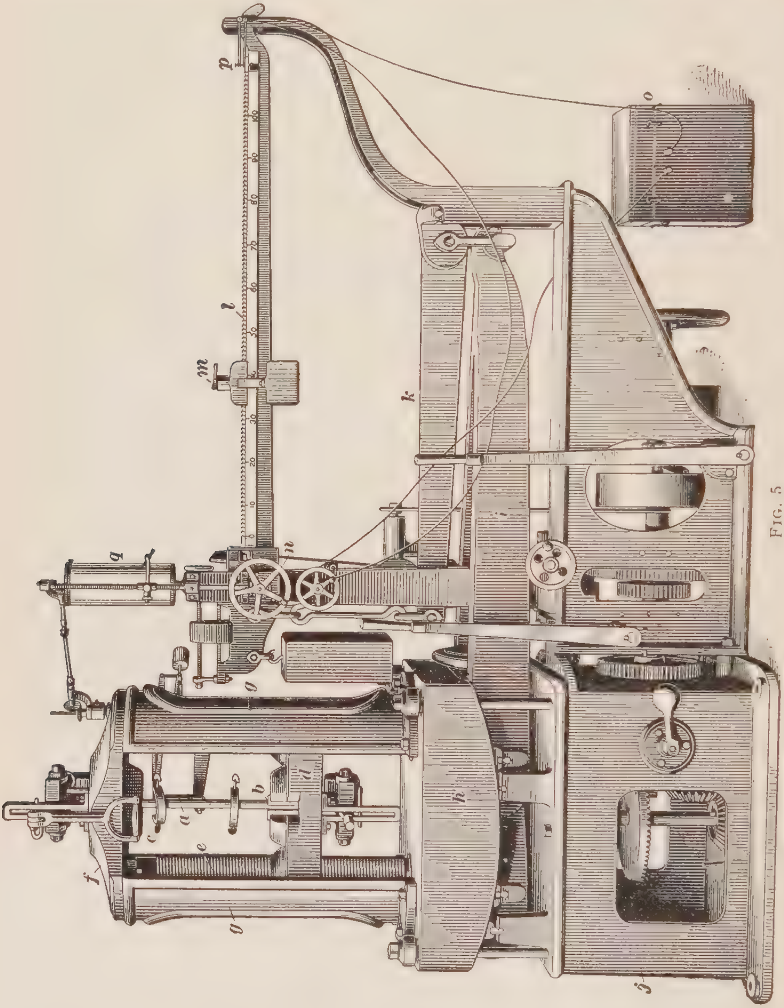


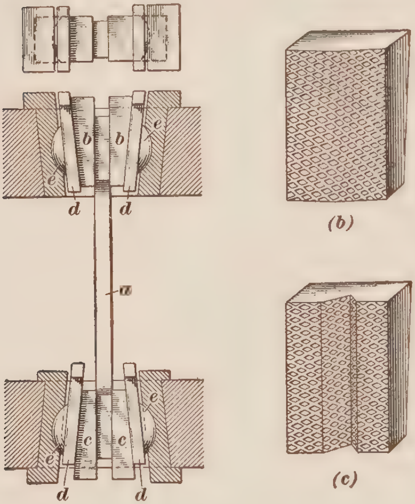
FIG. 5

machine, the head *d* is moved vertically. The upper clamp *c* is held immovable in the fixed head *f*, which is supported on the columns *g*. These columns are secured to the weighing platform *h*, which rests on knife-edge bearings on the lever *i*. This lever is fulcrumed on knife edges that are supported by the box frame *j*. Consequently, when the head *d* descends, so as to exert a tensile force on the piece *a*, this force is transmitted to the head *f* and through it to the columns *g*, which in turn press downwards on the weighing platform *h*. This pressure is then transmitted through the levers *i* and *k* to the scale beam *l*, which is graduated so that the load on the test piece can be read directly by moving the poise *m* to the point at which the scale beam floats or balances. This poise is moved by means of a long screw extending along the top of the scale beam. The screw may be rotated by means of the hand wheel *n*, or it may be operated automatically. A battery *o* is connected to an electromagnet, which operates a clutch through which the screw may be caused to rotate by power from the driving mechanism of the machine. The battery is so connected that when the scale beam rises the finger *p* closes the battery circuit by touching the beam, and the clutch is instantly operated by the magnet. The weight or poise is then moved out along the beam until the beam falls, breaking the contact at *p*, releasing the clutch, and stopping the outward movement of the poise. This action is repeated until the maximum load is reached. The poise must then be run back by turning the hand wheel *n*. To return the poise to the zero mark, a lever at the top of the poise is pressed down, releasing the nut which engages with the screw on the scale beam. The poise may then be moved by hand.

AUXILIARY APPARATUS

11. Jaws for Holding Test Specimens.—When a test piece is subjected to tension in a testing machine, it should be acted on only by a force applied in the direction of its axis. That is, the ends of the piece should be so held that there will be no tendency to bend the test piece. In Fig. 6 (*a*) is shown a test piece held in ball-and-socket jaws intended to counteract any tendency to bend. The upper end of the test piece *a* is held firmly between the jaws *b*, *b* in the fixed head of the machine, while the lower end is held by similar jaws *c*, *c* in the movable head. The jaws press against blocks *d*, *d* that have ball-and-socket bearings at *e*, *e*; such jaws permit the piece to be stretched without inducing any side stresses or bending forces. The jaws are wedge-shaped, and fit into tapered rectangular holes in the fixed and movable heads. On account of this wedging action, any increase in tension causes the piece to be held more firmly.

The faces of the jaws are roughened into file-like surfaces that hold the test piece tightly and prevent slipping. In Fig. 6 (*b*) is shown the roughened face of an ordinary jaw for holding a flat test piece, like that shown in Fig. 6 (*a*).



(b)

(c)

(a) FIG. 6

In Fig. 6 (*c*) is illustrated a jaw used for the purpose of holding test pieces with round ends. As in the previous types, the gripping surfaces are roughened so as to take firm hold of the test piece.

12. The Micrometer Caliper.—Before a test piece is placed in the testing machine, it is measured, so that its diameter, if it is a round specimen, or its breadth and thickness, if it is rectangular, is known. These measurements should be made very accurately.

In Fig. 7 is shown one kind of measuring instrument used for this purpose; it is known as a **micrometer caliper**, and is capable of giving very accurate results. It really consists of two micrometers, *a* and *b*, fastened to one base *c*. The piece to be measured is placed between the fixed point *d* and the movable point *e*, and the latter is then screwed up by means of the nurled wheel *f* until it rests lightly against the piece. The reading is then taken from the graduated scale on the bar *g* and the edge of the wheel *f*.

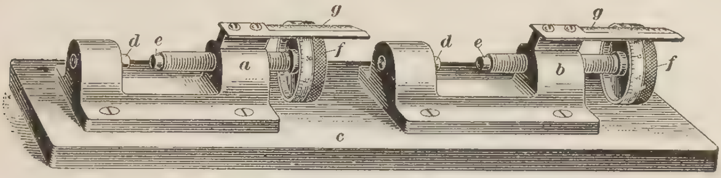
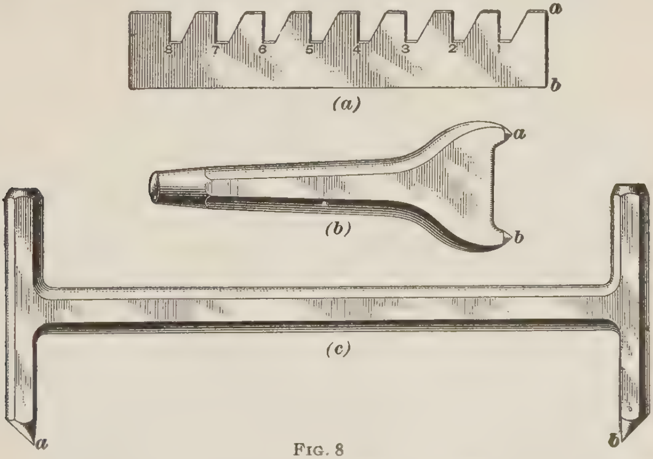


FIG. 7

In measuring rectangular specimens, it is advantageous to have a double micrometer, as shown, so that one may be set for the width and the other for the thickness. These micrometers are graduated so as to read to $\frac{1}{1000}$ inch, and may be used for measurements up to $2\frac{1}{2}$ inches. This instrument is used also in determining the size of the test piece after it is tested, as in tension or compression tests the dimensions of the test piece are slightly altered.

13. Spacing Instruments.—In a tension test on an 8-inch test piece, it is customary to divide the length into



(a)

(b)

(c)

FIG. 8

eight equal parts. After the test, the elongation of the piece is measured on this marked portion of the bar. An instrument adapted to the division of a test piece as described is shown in Fig. 8 (*a*). It consists of a steel rule having eight notches along its edge. The sides of these notches that lie parallel to the end *a b* are exactly 1 inch apart. Hence, by laying this rule on a flat test piece and making a scratch along each of the edges named the piece is easily marked off into eight equal divisions.

If the test piece is round, the division may be accomplished by the use of the instrument shown in Fig. 8 (*b*). It is simply a prick punch with two points instead of one, the points *a* and *b* being exactly 1 inch apart. Having scratched a straight line along the side of the piece, the eight 1-inch divisions are laid off by placing the prick punch in successive positions and tapping it lightly with a hammer. The marks thus .left are permanent, whereas simple scratches on the piece may easily become effaced. The purpose of laying off these divisions is to locate the point where the test piece fractures or breaks; in some tests, also, it is customary to measure the elongation in each inch of the piece.

In case two marks 8 inches apart are all that are desired, a double punch of the form shown in Fig. 8 (*c*) may be used. Its points *a* and *b* are 8 inches apart, and it is only necessary to place the points in position and tap each punch lightly with a hammer.

14. The Micrometer Extensometer.—A form of instrument used to measure the elongation of a piece is shown in Fig. 9. It is known as the **micrometer extensometer,** since the measurements are taken by means of micrometer screws. The instrument is tightly clamped to the test piece by clamps at *a* and *b*. The points that hold the clamps to the test piece are kept exactly 8 inches apart by the distance bars *c*, one on either side. The upper ends of these are hook-shaped; after the instrument is firmly fastened to the test piece the hooked ends are disengaged and moved to one side, as shown in the illustration. To the lower clamp is fastened a frame carrying two micrometer screws *d, d* attached to index wheels *e, e*. The same frame also carries the graduated scales *f*, *f*. The upper clamp carries a frame to which are attached posts *g*, *g* that are adjusted so as to come directly opposite the points *h*, *h* of the micrometer screws. These posts *g*, *g* are insulated from the other metallic parts of the instrument and are connected by wires to one pole of a battery *i*. Another wire connects the other pole of the battery to the lower clamp *b*, through an electric bell *j*. It is evident, therefore, that if the point *h* of one screw touches the corresponding post *g*, the bell will ring.

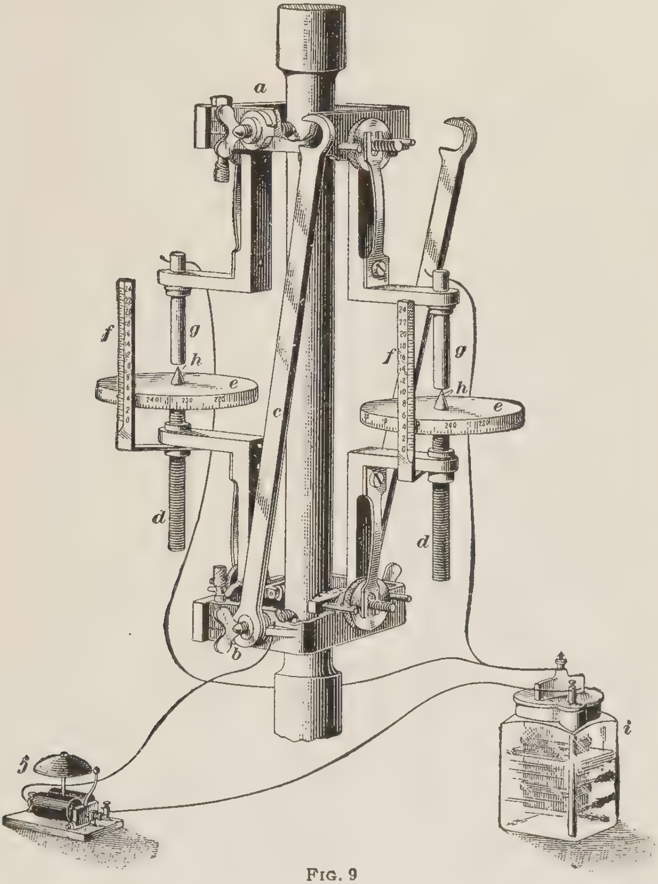


FIG. 9

In order to measure the extension of a piece, the reading of each micrometer is taken before any load is applied. Then, when a certain load is put on the test piece, it will elongate somewhat, thus separating the post *g* from the point *h*. The wheels *e*, *e* are then turned until the points *h*, *h* just fail to touch the posts *g*, *g*, and the reading of each is again taken. The difference between the average of the first two and the average of the last two readings is the elongation. This operation is repeated for each load, or as often as it is desired to take readings. In order to be certain that the point *h* just fails to touch *g*, the screw is turned upwards until the bell rings and then turned back slowly until the bell ceases to ring.

The index wheels are graduated into 250 equal divisions and the screws have forty threads per inch. Hence, for each movement of one division on the index wheel, the vertical movement of the screw is $\frac{1}{40} \times \frac{1}{250} = \frac{1}{10000}$ inch; that is, the instrument will read to $\frac{1}{10000}$ inch. The readings from both micrometers should be taken as nearly at the same instant as possible, which may be accomplished by having an observer to manipulate each screw. It is usually desirable to have a bell for each micrometer, so that there may be no possibility of error.

The extensometer should always be removed from the test piece before fracture occurs, as the jar due to the recoil after the rupture may damage the instrument. Frequently, the elongation beyond the elastic limit may be measured roughly with a pair of dividers after the extensometer has been removed. These measurements can be taken nearly up to the breaking point, and while probably not accurate to less than $\frac{1}{100}$ inch, they nevertheless may be used in determining the elongation near the point of rupture.

15. The Compressometer.—In a compression test or in a transverse test, it is desirable to know the amount by which the piece shortens or by which it deflects under various loads. An instrument known as a **compressometer** one form of which is shown in Fig. 10, is used for this purpose. It contains two micrometer screws *a* and *b*, having the same number of threads per inch and connected by the gears *c*, *d*, and *e*, so that when one screw is turned, the other turns an equal amount in the same direction. These screws are supported by the base *f* and carry the frame *g*, which, having once been set level, remains so, as the micrometer screws move equally.

In using this instrument, the points *h* of the rigid arms *i* are placed in contact with the platform of the machine, or with a block of convenient thickness lying on the weighing platform; these points do not have any vertical movement. The points *j* of the arms *k* are placed beneath the compression head of the machine and in contact with it. The arms *k* are fastened to a bar *s*, pivoted at *l*, which carries a thumbscrew at its end, the point *m* of the screw being opposite an insulated metal block *n* held in the frame *g*. The block *n* is connected to one terminal of a battery *o* through a bell *p*, the other terminal of the battery being connected to the frame of the instrument. Hence, if the point *m* touches *n*, the bell will ring.

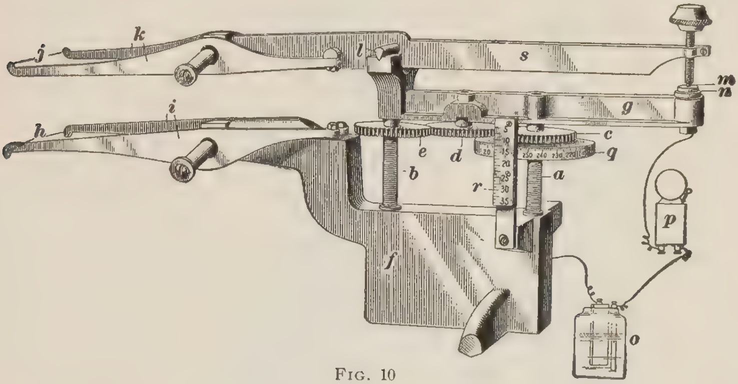


FIG. 10

Now, as the movable head of the testing machine descends, it depresses the points *j* and turns the bar *s* on the fulcrum *l*, separating the points *m* and *n*. The screw *a* is then turned down by means of the index wheel *q*, lowering the fulcrum *l* until the point *m* touches *n* and rings the bell.

The amount the fulcrum *l* is lowered is measured by the index wheel *q* and the scale *r*. This measurement represents the amount the test piece is compressed. There are 250 divisions on the index wheel, and each micrometer screw has forty threads per inch, so that this instrument may be used to measure compression to $\frac{1}{10000}$ inch. When used in connection with a transverse test, an instrument of this form is sometimes termed a **deflectometer.**

16. The Autographic Recording Apparatus.—At *q*, Fig. 5, is shown an **autographic attachment** by means of which the stress in a test piece and the corresponding deformation are automatically recorded throughout the test. An enlarged view of this apparatus is shown in Fig. 11. At *a* is a cylindrical drum, mounted on a vertical shaft supported by the frame of the machine. This drum has a sheet of paper *b* fastened tightly around it, the paper being

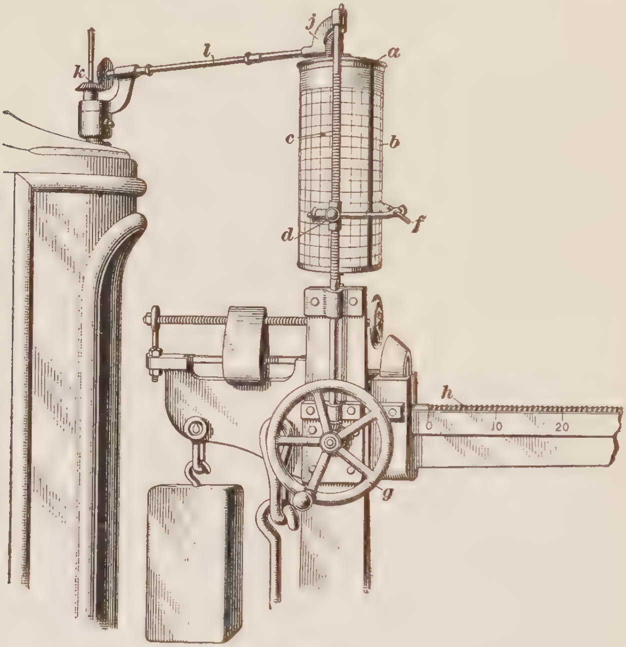


Fig. 11

divided into squares by horizontal and vertical lines equally spaced. Beside the drum and parallel to its axis is a vertical standard and a screw *c*, on which is a nut, at *d*, carrying an arm to which is attached a pencil *f*. This screw has a bevel gear on its lower end, which meshes with another gear on the shaft of the hand wheel *g*. Hence, any movement of the screw *h* on the scale beam will cause a corresponding vertical movement of pencil *f*. In other words, the vertical movement of the pencil is proportional to the distance traveled by the poise *m*, Fig. 5, and hence proportional to the load on the test piece.

The drum *a* is given a rotary motion by means of bevel gears at *j* and *k* and the shaft *l*. This rotation is due to the deformation of the test piece. By means of gearing, the amount of deformation of the test piece is transformed into motion of the shaft *l*, causing rotation of the drum. Thus the amount of rotation of the drum is at all times proportional to the deformation of the test specimen.

In making a test, the pencil is set at the point on the paper *b* corresponding to zero load and zero deformation. Then, as the test progresses and the load and deformation increase, the pencil rises vertically and the drum turns, and there is drawn on the sheet a curve that at all times represents the load and its corresponding deformation.

TENSION TEST

METHOD OF MAKING THE TEST

17. Preparations for the Tension Test.—The test piece having been first prepared and given a uniform cross-section, the area of this section is carefully determined by the use of the micrometer caliper. Next, if it is an 8-inch specimen, the piece is marked off into eight divisions of 1 inch each, care being taken to have this length of 8 inches extend over that portion of the piece that is uniform in cross-section. When the piece is thus marked with prick-punch marks, it will appear somewhat as shown in Fig. 12

The extensometer is then attached and the piece is placed in the jaws of the machine. At first, it will fit loosely, and in order to set the jaws firmly the machine is started and the movable head allowed to descend until the scale beam tips up, the poise having been previously placed at zero and the machine balanced. There is now a very slight tension in the test piece, but it is so small that it will not affect the piece appreciably. In case the autographic attachment is to be used, it is now attached, and the pencil is set at zero on the sheet. The latter should fit snugly to the drum, and there should be no lost motion in any of the attachments. This completes the preparation of the machine for the test.

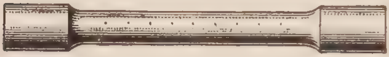


FIG. 12

18. Conducting the Test.—The readings of the scale beam and of the extensometer should be taken at the same instant. The interval between readings is not fixed, but may be assumed at pleasure. Some engineers prefer to take readings at intervals of 2,000 pounds; that is, to take a reading each time the tension becomes 2,000 pounds greater than at the preceding reading. Others take readings on a time basis, say every $\frac{1}{2}$ minute.

The machine should be run very slowly until the piece is stretched beyond the elastic limit, after which it may be speeded up. The application of the load to the test specimen should be gradual and increase uniformly. There should be no shocks or jerks. Neither should the test be made too quickly. At ordinary temperatures, a tension test covering several minutes will give practically the same results as those obtained by applying the load much more slowly. But a very rapid application of the load tends to give too high values for the elastic limit and the ultimate strength. The testing machine is so constructed that from four to seven speeds are obtainable, to suit different conditions.

Card of Tensile Tests and Reports of Same

TESTING DEPARTMENT

Tensile Test Specimen from ( ) Machinery Steel, (2) Wrought Iron, (3) Cast Iron

MARKED	SIZE	Area in Square Inches	Broke at in lb.	Strain per Square Inch in lb.	Limit of Elasticity in lb.	Limit of Elasticity per sq. in in lb.	Elongation in 6 inches	Elongation per cent. of Length	Area of Reduced Section in sq. in.	Reduction per cent. of Section
No 1	1.00 in diam	.7854	49510	63030	32500	41380	1.96	.33	.3019	.61
no 2	1.03 " "	.8332	41530	49840	27100	32525	1.58	.26	.5153	.38
no. 3	1"X1" square	1.00	15940	15940			.008 in 3"	.0026		

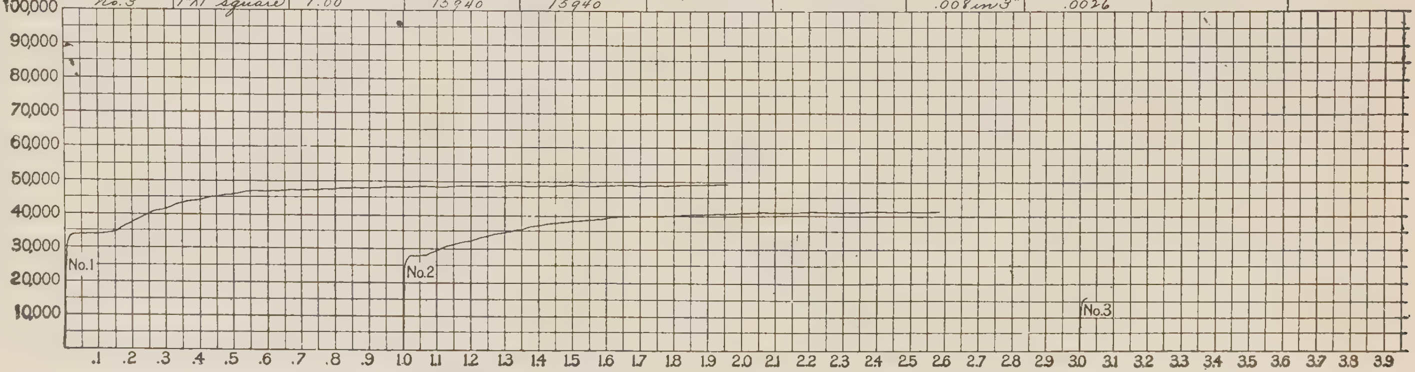


FIG. 13

RECORDS OF THE TEST

19. The Autographic Diagram.—When the autographic attachment is used in making a test, the diagram produced by it is known as an **autographic diagram** or a **stress curve.** Three of these diagrams, for three tension tests, are shown in Fig. 13, recorded on one sheet. The sheet on which the records are drawn is generally called a **card.** The three curves, together with the data at the top of the sheet, give full information concerning the tests. Curve No. 1 is for machinery steel, No. 2 is for wrought iron, and No. 3 is for cast iron. The specimens of steel and wrought iron are 6 inches long, and of cast iron 3 inches long. The sheet is divided into squares, and each division on the vertical scale represents a load of 5,000 pounds, while each division on the horizontal scale represents an elongation of .05 inch.

It will be observed that in the test of machinery steel, as represented by curve No. 1, the curve is approximately a straight line, until a stress of about 32,500 pounds is reached. This means that up to this point the load and the elongation are in the same ratio, or directly proportional to each other. Beyond this load, the elongation increases much more rapidly than the load, as shown by the approach of the curve to a horizontal line. The point where this sudden change of direction occurs is the elastic limit, since it is the point beyond which the elongation is no longer directly proportional to the load. The area of the test piece being .7854 square inch and the stress at the elastic limit being 32,500 pounds, the elastic limit of the material must be $32{,}500 \div .7854 = 41{,}380$ pounds per square inch. The diagram also shows a total elongation of 1.96 inches, but as this is the elongation in 6 inches of length, the elongation per inch is $1.96 \div 6 = .33$ inch, nearly, or practically 33 per cent. The area of the section at the point of rupture, measured after the piece is broken, is .3019 square inch. Consequently, the reduction of area, as compared with the original cross-section, is

$$\frac{.7854 - .3019}{.7854} = .61, \text{ or 61 per cent.}$$

TENSION TEST REPORT

Test of Wrought Iron *Specimen Mark* 3
Material from Greenway Iron and Steel Co.
Form of Section Circular *Machine used* #1
Initial: Length 8 *in. Dimensions of Cross Section* 1 *in. Area* .7854 *sq. in.*
Final: " 10 " " " " " $\frac{25}{32}$ " " .4794 " "
Position of Fracture Middle *Character of Fracture* Fibrous
Load per sq. in.: El. L. 30,558 *Max.* 51,337 *Breaking* 48,841
Elongation % 25 *Reduction of Area %* 39 *Modulus of Elasticity* 26,000,000

No.	LOAD		EXTENSOMETER READING			ELONGATION		
	Actual P	Per Sq. In. $S=\frac{P}{A}$	1	2	Mean	Actual	Diff.	Per In.
1	0	0	.0645	.1356	.10005	0	0	0
2	4000	5093	.0661	.1368	.10145	.00140	.00140	.000175
3	8000	10186	.0673	.1386	.10295	.00290	.00150	.000363
4	12000	15279	.0688	.1403	.10455	.00450	.00160	.000563
5	16000	20372	.0702	.1417	.10595	.00585	.00135	.000800
6	20000	25465	.0720	.1433	.10765	.00760	.00170	.000950
7	24000	30558	.0759	.1434	.10965	.00960	.00200	.001200
8	24810	31589	.0920	.1590	.12550	.02545	.01585	.003181
9	25540	32518	.1430	.1944	.16870	.06865	.04320	.008581
10	26660	33944	.1680	.2255	.19675	.09670	.02805	.012088
11	28910	36809	.2188	.2800	.24940	.14935	.05265	.018669
12	31540	40043	.3070	.3850	.34350	.24345	.09410	.030431
13	33370	42487	.3945	.5000	.44725	.34720	.10375	.043400
14	35310	44958	.5370	.6054	.57120	.47115	.12395	.058894
15	36520	46498	.6288	.7054	.66710	.56705	.09590	.070881
16	37470	47708	.7235	.8100	.76675	.66670	.09965	.083338
17	38500	49020	.8870	.9650	.92600	.82595	.15925	.103244
18	39000	49656	.9836	1.0757	1.02965	.92960	.10365	.116200
19	39350	50102	1.1400	1.1902	1.16510	1.06505	.13545	.133131
20	39660	50497	1.2250	1.3000	1.26250	1.16245	.09740	.145306
21	40260	51261	Extensometer			1.30000	.13755	.162500
22	40320	51337	Removed			1.60000	.30000	.200000
23	40070	51019				1.90000	.30000	.237500
24	38360	48841				2.00000	.10000	.250000

Date ____________ 19

Observers ____________

FIG. 14

20. The records on the card show similar data for both the wrought-iron and the cast-iron specimens. The point of zero stress for all of the curves has been taken on the base line of the diagram, but the point of zero elongation has been shifted to the right in the case of curves No. 2 and No. 3 in order that the curves may not intersect and thus become confused. In any case, the total elongation is found by subtracting the reading at the beginning of the test from the reading at the point of rupture. In the case of the wrought-iron specimen, the total elongation is 2.58 − 1.00 = 1.58 inches, while in the case of the cast-iron specimen it is but .008 inch.

21. Log of a Tension Test.—The record of a test, frequently termed the **log** of a test, may be kept in a number of convenient forms, one of which is illustrated in Fig. 14. It consists of a sheet divided into several columns, with blank spaces in which are written the data observed during the entire test.

The first column at the left gives the numbers of the readings, and the second gives the corresponding loads as read from the scale beam. In the third column are given the loads per square inch of section, these results being found by dividing the load P in the second column by the area A of the cross-section of the test piece, in square inches.

In the test from which this record was taken, the elongation was determined by the use of an extensometer like that shown in Fig. 9, and the poise on the scale beam was operated by hand. The fourth and fifth columns contain the readings of the two micrometers on the extensometer, the mean of each pair of readings being given in the sixth column, found by taking one-half of the sum of the two readings at any instant. In the seventh column are given the actual elongations corresponding to the separate readings. The actual elongation at any instant is equal to the mean extensometer reading for that instant minus the first mean reading of the extensometer. Thus, for the eighth reading, the elongation is .12550 − .10005 = .02545 inch; for the seventeenth reading, it is .92600 − .10005 = .82595 inch.

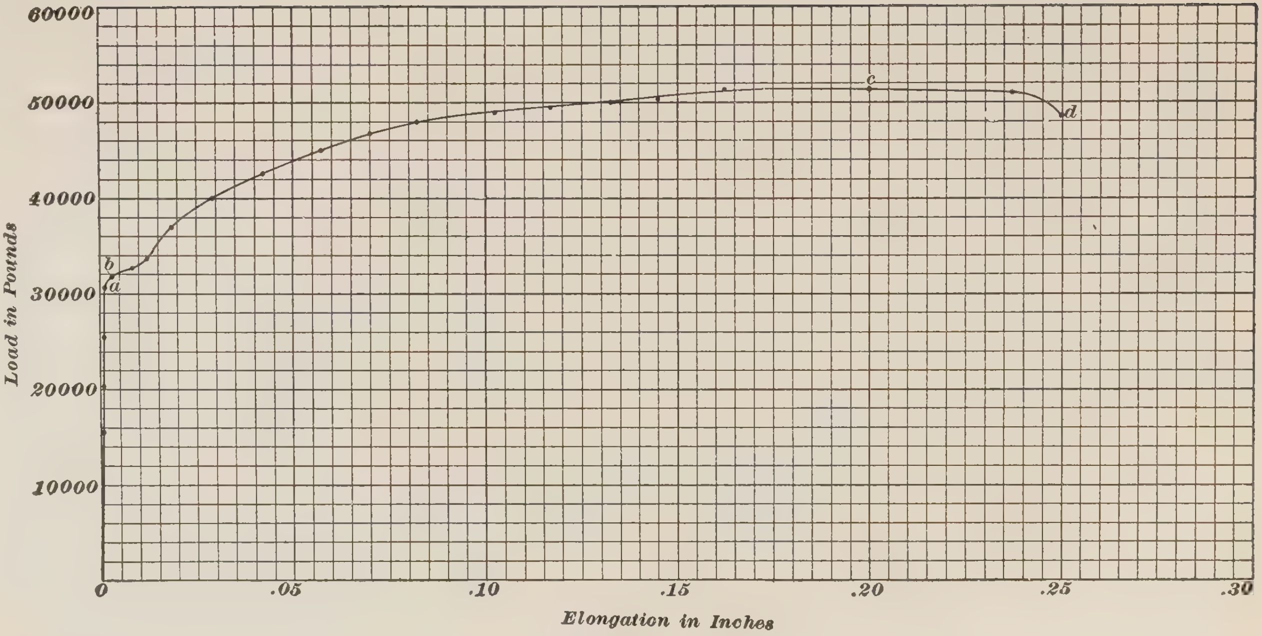

60000
50000
40000
30000
20000
10000
Load in Pounds
0
.05
.10
.15
.20
.25
.30
Elongation in Inches
a
b
c
d

FIG. 15

The last four readings in this column were secured by measuring the test piece with the aid of a pair of dividers after the extensometer was removed.

The eighth column gives the differences of elongation, each of which is found by subtracting from its corresponding actual elongation the preceding actual elongation. Thus, at the tenth reading, the difference of elongation is equal to .09670 − .06865 = .02805 inch. The last column gives the actual elongations per inch of length of the piece, these being found by dividing the values in the seventh column by 8, the length of the test piece, in inches.

The notes on the upper part of the report explain themselves, to a large extent. The final length is equal to the original length plus the total elongation up to the point of rupture of the piece. The final area of cross-section is determined by measuring the dimensions of the piece at the point where it broke. The percentage of elongation is found by dividing the total elongation by the original length of the piece; thus, $2 \div 8 = .25 = 25$ per cent. The percentage of reduction of area is found by dividing the difference between the initial and final areas by the initial area; thus,

$$(.7854 - .4794) \div .7854 = .3896, \text{ or say 39 per cent.}$$

The terms modulus of elasticity, maximum and breaking loads, and load at the elastic limit, will be explained later, in connection with the curve of stresses and elongations.

22. Plotting the Stress Curve.—After the log of the test has been prepared, as in Fig. 14, the stress curve may be plotted as shown in Fig. 15. This should be done on sheets ruled into small squares, known as **cross-section paper** or **squared paper,** since it is very convenient to use. The first step is to decide on the scales of stress and elongation. Suppose that each vertical division represents a load of 2,000 pounds, and each horizontal division an elongation of .005 inch per inch of length of the test piece. Taking the data given in the third and ninth columns in Fig. 14, the stresses are laid off in Fig. 15 as ordinates and the corresponding elongations as abscissas, and through the points thus located a curve is drawn; this curve is the stress curve. It may not be possible to draw a smooth curve to pass through all the points located. This is usually due to errors in observing the readings, and in such a case, the curve should be drawn so as to represent, approximately, the mean of the locations of the points.

23. Reading the Stress Curve.—In connection with curve No. 1, Fig. 13, it was shown that the elastic limit is represented by the point at which the curve first takes a decided slope from the vertical. In Fig. 15, the point *a* represents the elastic limit of the material, since that is the point where the curve rapidly departs from the straight line *O a*. At the point *b*, there is a still more decided departure from the vertical trend; this point is known as the **yield point.** It is at this point that the test piece begins to fail rapidly, showing that its structure is being broken down. The point *c* at which the curve is farthest above the base line indicates the point of ultimate strength of the material; that is, the point of maximum load per square inch. At *d*, the end of the curve, the piece fractures, and the height of this point above the base line represents the breaking load, which may or may not be equal to the ultimate strength.

The falling of the curve from *c* to *d* indicates that there is a rapid decrease of load on the test piece during this period of the test. After passing the point of ultimate strength *c*, the elongation becomes very rapid, and in order to keep the scale beam floating or balanced, it is necessary to move the poise backwards along the beam, which results in a gradual falling of the curve on the autographic record. As the piece yields, its area of cross-section rapidly diminishes. If the testing machine were so made that the load at *c* could be maintained steadily, the piece would break at that point. Hence, the fiber stress at the point *c* is called the maximum fiber stress, or ultimate strength, of the material.

Inspection of the stress curve and the test report shows that the load per square inch at the elastic limit *a* is 30,558 pounds. The maximum load, at *c*, is 51,337 pounds per square inch, while the breaking load, at *d*, is only 48,841 pounds per square inch. The modulus of elasticity is equal to the load per square inch, at any point below the elastic limit, divided by the elongation per inch at that load. That is, divide any value in the third column of Fig. 14, up to the elastic limit, by the corresponding value in the ninth column to obtain the modulus of elasticity.

The moduli of elasticity calculated from several readings will not agree exactly, but by taking the average of several results a close approximation to the true modulus may be secured. Thus, taking the corresponding readings in the fourth line of Fig. 14,

$$15{,}279 \div .000563 = 27{,}140{,}000, \text{ nearly;}$$

taking the fifth line,

$$20{,}372 \div .0008 = 25{,}460{,}000;$$

taking the sixth line,

$$25{,}465 \div .00095 = 26{,}800{,}000;$$

taking the seventh line,

$$30{,}558 \div .0012 = 25{,}460{,}000$$

The average of these four results is 26,215,000, so that the modulus of elasticity may be taken, approximately, as 26,000,000.

24. Important Points in a Tension Test.—There are six important points or factors to be determined by a tension test, of which a record should be kept by means of a suitable report or record, or by a diagram, or still better, by both. These factors are the modulus of elasticity, the elastic limit, the yield point, the ultimate strength, the percentage of elongation, and the percentage of reduction of area of cross-section.

COMPRESSION TEST

METHOD OF MAKING THE TEST

25. Preparation for a Compression Test.—Before testing a piece in compression, the clamping jaws are removed from the movable head of the testing machine, as they are not needed. A cylindrical steel block is then fitted to the under side of the movable head of the machine, as shown in Fig. 16, in which *a* represents the movable head and *b* the cylindrical block. This block has a lug or projection *c*, rectangular in shape, which fits into the rectangular opening in the head that had previously held the jaws. A stud bolt *d*, fastened to the block *b*, passes up through the hole in the head and through a cover-plate *e*. The nut *f* is then tightened, holding the block firmly against the head. A similar block *g* is then placed on the weighing platform *h* of the machine, where it is held by short dowel-pins *i*, *i*. Care should be taken to have the head, the blocks, and the platform quite clean, so that the blocks will have their faces parallel. The machine should then be balanced, with the poise on the scale beam set at zero.

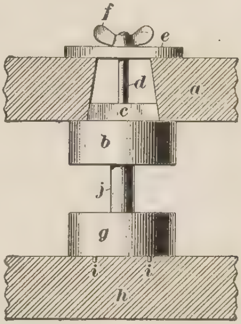


FIG. 16

26. The test piece should next be measured quite carefully, so as to obtain its length and its area of cross-section, these measurements being made with a micrometer caliper. The specimen should be placed between the blocks *b* and *g*, as shown at *j*, Fig. 16, and the movable head lowered until the piece is squeezed just tightly enough to be held in position. The compressometer should next be placed in position and adjusted, and connected with the bell and the battery.

27. Conducting the Compression Test.—The manner of conducting a compression test does not differ

COMPRESSION TEST REPORT

Test of Cast Iron Specimen Mark 17
Material from Cambridge Foundry
Form of section Circular Machine used #2
Initial: Length 1 in. Dimensions of Cross Section .798 in. Area .5 sq. in.
Final: " .8968 " " " " " .867 " " .590 " "
Position of Fracture Near middle Character of Fracture Crystalline 45° angle
Load per sq. in.—El-L 80,000 Max. 109,300 Breaking 107,200
Compression .1032 in. Compression % 10.32 Modulus of Elasticity 2,500,000

No.	LOAD		COMPRESSOMETER	COMPRESSION		
	Actual P	Per Sq. In. $S=\frac{P}{A}$	READINGS	Actual	Diff.	Per. In.
1	0	0	.5660	0	0	
2	1200	2400	.5688	.0028	.0028	.0028
3	4900	9800	.5710	.0050	.0022	.0050
4	8650	17300	.5720	.0060	.0010	.0060
5	14000	28000	.5780	.0120	.0060	.0120
6	22950	45900	.5840	.0180	.0060	.0180
7	32000	64000	.5940	.0250	.0070	.0250
8	38750	77500	.5980	.0320	.0070	.0320
9	44380	88760	.6140	.0480	.0160	.0480
10	48650	97300	.6210	.0550	.0070	.0550
11	51600	103200	.6400	.0740	.0190	.0740
12	53875	107750	.6580	.0920	.0180	.0920
13	54650	109300	.6692	.1032	.0112	.1032
14	53600	107200	Piece Broke			

Date ____________ Observers

FIG. 17

greatly from that used in a tension test. The machine should be run at a fairly slow speed during the test, and the readings of the scale beam and instruments should be taken at the same instant at each period of observation.

RECORDS OF THE TEST

28. Log of a Compression Test.—A complete record of the various readings taken in the test should be kept on properly prepared blanks, similar to those used in tension tests. Such a blank form, filled out with the results of a compression test of a piece of cast iron 1 inch long and .798 inch in diameter, is shown in Fig. 17. Since the length of the specimen is 1 inch, the total compression is the same as the compression per inch, and the fifth and seventh columns contain the same values.

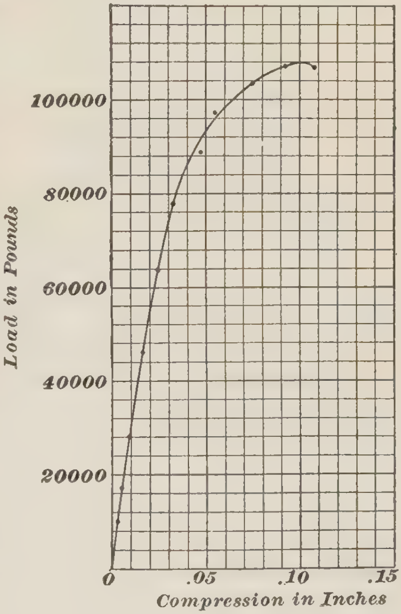


FIG. 18

29. The Stress Curve.—If the autographic attachment had been used, the curve of stresses and deformations would have been drawn on a sheet similar to that containing the curves in Fig. 13. It is an easy matter, however, to plot the curve from the data sheet in Fig. 17. Fig. 18 shows the stress curve thus plotted, each vertical division representing 4,000 pounds load, and each horizontal division a compression of .01 inch.

TRANSVERSE TEST

METHOD OF MAKING A TRANSVERSE TEST

30. Preparation for a Transverse Test.—To make a transverse test, the testing machine is fitted as shown in Fig. 19. A **V** block *a* is fitted to the under side of the movable head *b* of the machine, and two similar blocks are placed on the weighing platform, at equal distances on each side of *a*, as at *c* and *d*. These blocks have steel edges *e*, *e*, on which the test piece rests and which are from 4 to 6 inches wide, measured along the steel edges. The edges of the three blocks should be parallel, as they will be if properly adjusted. The machine should then be balanced.

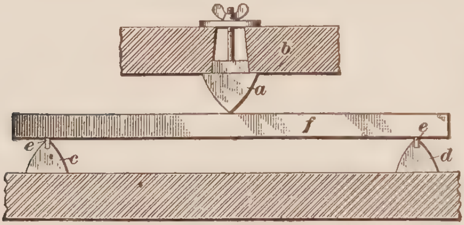


FIG. 19

31. The test bar used in a transverse test is usually rectangular in shape. This should be carefully measured by a micrometer caliper, and the breadth and thickness, as well as the distance between supports *c* and *d*, Fig. 19, recorded. The test bar should then be put in place, as at *f*, and the head lowered until the block *a* just touches the bar. Next, the compressometer should be placed on the platform and adjusted properly. It should be set exactly beneath the edge of the block *a*, since the deflection of the test piece will be greatest at its middle point; but it should be removed, to prevent damage, when the piece shows signs of rupture.

32. Conducting a Transverse Test.—The method of conducting a transverse test does not differ from that employed in either of the two tests already described. It will be found that the deflections will be considerable, while the loads on the scale beam will be small. The machine should be run at a very slow speed.

TRANSVERSE TEST REPORT

Test of Wrought Iron *Specimen Mark* 6044

Material from Hartwell Rolling Mills *Machine Used* # 3

Form of Section Square *Moment of Inertia* .0833

Dimensions of Section: Width 1 *in. Depth* 1 *in. Area* 1 *sq. in.*

Length between Supports 24 *in. Position of Load* Middle

Actual Load: At Elastic Limit 1150 *lb. Maximum* 1465 *lb.*

Deflection: " " " " .151 *in.* " .790 *in.*

Stress in Outer Fiber: Lb. per sq. in. El. L. 41,400 *Max.* 52,740

Shearing Stress: " " " " " 575 " 733

Rupture None *Modulus of Elasticity* 28,000,000

No.	LOAD	COMPRESSOMETER		TOTAL DEFLECTION
		READING	DIFFERENCE	
1	0	0		0
2	100	.010	.010	.010
3	200	.020	.010	.020
4	300	.035	.015	.035
5	400	.050	.015	.050
6	500	.064	.014	.064
7	600	.078	.014	.078
8	700	.091	.013	.091
9	800	.105	.014	.105
10	900	.119	.014	.119
11	1000	.132	.013	.132
12	1100	.147	.015	.147
13	1200	.167	.020	.167
14	1250	.205	.038	.205
15	1300	.360	.155	.360
16	1400	.620	.260	.620
17	1465	.790	.170	.790

Date __________

Observers __________

FIG. 20

RECORDS OF THE TEST

33. Log of Transverse Test.—The log or record of a transverse test is shown in Fig. 20, the blank form being similar to that used in tension and compression tests. The specimen was a wrought-iron bar 28 inches long and 1 inch square, the distance between supports was 24 inches, and the load was applied at the middle. The distance of the outermost fiber from the neutral axis is one-half the depth of the bar, or $\frac{1}{2}$ inch, and the moment of inertia, according to *Strength of Materials*, Part 2, is $I = \frac{b\,d^3}{12}$. But the breadth b and the thickness or height d are equal, so that

$$I = \frac{b\,d^3}{12} = \frac{b^4}{12} = \frac{1}{12}$$

The first column, as usual, contains the numbers of the readings. The second column gives the actual loads on the specimen. The load per square inch is not used in transverse tests. The third and fourth columns are for micrometer readings and differences of readings. In this test, the readings of the compressometer are taken from zero, and consequently the readings as given in the fifth column are the same as those in the third column. The value of the modulus of elasticity for center loading is found by the use of the formula from *Strength of Materials*, Part 2, $s = \frac{W\,l^3}{48\,E\,I}$, which, when solved for E, becomes

$$E = \frac{1}{48}\,\frac{W\,l^3}{I\,s}$$

The value of W, the load, is found in the second column; the length l is 24 inches; the moment of inertia I is $\frac{1}{12}$ and the deflection s is given in the fifth column. Hence, by calculating values of E, using values from the table that lie within the elastic limit, and taking the average of the results, the modulus of elasticity is found to be approximately 28,000,000.

Since the piece is loaded at the middle, the shearing stress is greatest at the points of support and is equal to one-half the load. Hence, at the elastic limit, the shearing stress is

$$1{,}150 \div 2 = 575 \text{ pounds},$$

and at the maximum load, it is

$$1{,}465 \div 2 = 733 \text{ pounds}.$$

The fiber stress is found by the formula in *Strength of Materials*, Part 2, $M = \frac{S\,I}{c}$, which, when transposed, gives $S = \frac{M c}{I}$, S being the stress in the outer fibers, c the distance from the neutral axis to the outer fibers, and M the bending moment in inch-pounds. In the case of a bar loaded at the center, $M = \frac{1}{4}\,W l$. Hence, at the elastic limit, $M = 6{,}900$ inch-pounds, and at the point of maximum load it is 8,790 inch-pounds. Substituting these values for M, making $c = \frac{1}{2}$ and $I = \frac{1}{12}$, the stresses in the outer fibers are found to be 41,400 pounds and 52,740 pounds, respectively. The piece did not rupture, but merely failed by bending, which was to have been expected of a bar of wrought iron.

34. Plotting the Curve for a Transverse Test. The method of plotting the stress diagram does not differ in principle from the method already explained in connection with tension and compression tests. The curve is shown in Fig. 21, in which one division represents 100 pounds load and .04 inch deflection, on the vertical and horizontal scales, respectively.

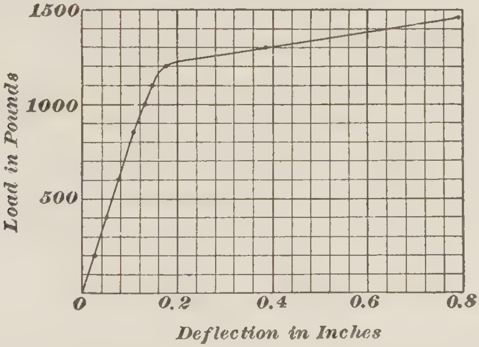


FIG. 21

SHEARING AND TORSION TESTS

35. As already stated, tests for shearing and torsion are less important and made less frequently. Shearing tests are usually made on riveted pieces like that shown in Fig. 4, which are placed in a testing machine and pulled apart in the same manner as a tension test piece.

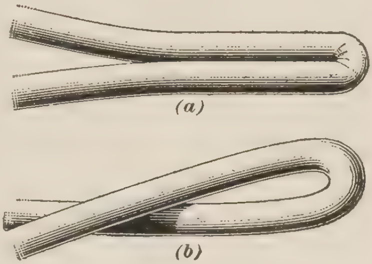
Torsion tests are made by use of a machine having one fixed head and one movable head, the latter having a rotary motion. The test bar is held by jaws in both heads of the machine and the movable head is turned, twisting the specimen until it breaks. The twisting moment is registered by a scale beam, and the angle through which the piece is twisted is read from a scale near the movable head.

36. Relation Between Shearing and Torsion.—It is evident that a bar subject to torsion is actually subjected to shearing. Suppose the bar to be composed of an infinite number of thin sections lying at right angles to the axis, much like a long roll of coins lying on their edges. Then, as the bar is twisted, those sections nearest the fixed head twist but little, while those nearest the movable head turn considerably; the result is that each section tends to slide on its neighbor, thus setting up shearing stresses in the fibers in planes at right angles to the axis of the bar. When a piece of wrought iron or steel is broken by a torsion test, the break occurs in a plane almost at right angles to the axis of the bar. In the case of cast iron, the break is more irregular, often approaching a helical or spiral shape.

MISCELLANEOUS TESTS

37. The Cold-Bending Test.—An excellent method of testing the ability of wrought iron and soft steel to withstand the effects of severe distortion when cold, either during processes of manufacture or during subsequent use, is by cold bending. A test of this kind tends to expose any objectionable qualities in the metal, such as brittleness and lamination, which may be caused either during the rolling or the annealing processes. A steel that suffers a large percentage of reduction in area under a tension test will withstand severe cold bending tests, so that the result of such a test really forms an index to the character of the structure of a metal.

This bending test has the advantage of requiring only a heavy hammer and an anvil for its application; a test of this kind is practicable when means for the more complete and elaborate tests are unobtainable. But the methods of making the test and the interpretation of the results have never been standardized, like those relating to the foregoing tests. Fig. 22 (*a*) shows a specimen bar of mild steel that has been subjected to a bending test of 180°, that is, bent back on itself and flattened under the blows of a hammer. Fig. 22 (*b*) shows a similar specimen that has been bent through more than 180°, but without flattening. It will be seen that there are no indications of cracks or laminations in either piece. The latter is a round bar $\frac{3}{4}$ inch in diameter, and it has been bent into a curve whose inner radius is less than $\frac{3}{8}$ inch. This is a very severe test and is seldom demanded of any material except soft steel, or that having a tensile strength of less than 65,000 pounds per square inch. These bending tests may be made under steady pressure, if desired, instead of under blows from a hammer.


(*a*)

(*b*)

FIG. 22

SOME TESTS OF CAST IRON

38. Special Terms Relating to Cast Iron.—In addition to the numerous terms used in connection with the testing of wrought iron and steel, there are several that relate especially to cast iron and are explained as follows:

1. **Density.**—By a dense iron is meant iron having a fine, close grain, as distinguished from iron having a coarse,

open grain. The greater density of the former may amount to as much as 50 or 60 pounds difference in weight per cubic foot. A cubic foot of white iron weighs about 475 pounds, while a cubic foot of gray iron may weigh only 425 pounds.

2. **Brittleness.**—Cast iron is said to be brittle when it breaks easily. White iron and irons containing certain impurities are brittle and should not be used in parts that are subject to jars, shocks, or sudden changes of load.

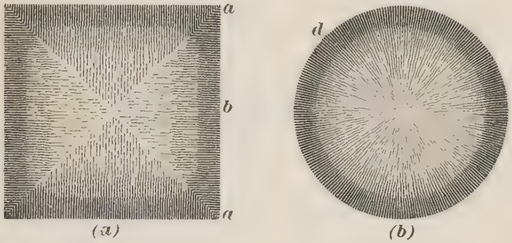
3. **Strength.**—By the strength of cast iron is meant its ability to withstand stresses of various kinds without yielding or breaking. Tests are usually applied to cast iron transversely and by impact, since it is generally used to resist transverse and crushing stresses and those resulting from impacts or blows. Tensile tests are very seldom made on cast iron.

39. Transverse Test Bars of Cast Iron.—The size and form of cast-iron bars for transverse tests vary greatly. and therefore it is seldom possible to compare records obtained from different sets of tests. Both round and square bars are used, their lengths varying from 12 to 50 inches and their cross-sections from $\frac{1}{2}$ square inch to 5 or 6 square inches.

The test bar should be of such form and size that it will be as little affected by variations in the dampness of the molding sand as possible. The moisture in the sand causes the outer surfaces of the bar to chill and produces a closer grain. The bar should be of such form and cast in such a position that the rate of cooling and its structure will be uniform throughout.

It is not good practice to use a test bar having a sectional area of less than 1 square inch, as the dampness of the sand in the mold will greatly affect a small bar. A test bar 1 inch square is best adapted to soft or medium grades of iron, and the larger sizes give the best results for the harder grades. Any iron that takes a chill easily requires a large test bar, so that the surface of the bar may bear a smaller ratio to the area of the cross-section. An important consideration is to keep the conditions the same in doing the work, so that useful comparisons may be made.

40. Structure of Transverse Test Bars of Cast Iron. Having decided on the dimensions of a test bar, the next point is to obtain a uniform structure. It has been found that a square bar is surrounded or encased in a shell of denser metal, as indicated in Fig. 23 (*a*); the heavier shading represents the denser metal. It is seen that this shell is thicker at the corners *a* than at the sides *b* of the bar. From this it is apparent that a uniform structure cannot be expected of a square bar, no matter what its position when cast.



(a) (b)

Fig. 23

A round bar, shown in section in Fig. 23 (*b*), will also have a shell *d* of denser metal, but this shell will be uniform all around the bar, in bars that are cast on end, and for this reason the round bar, cast on end, is often preferred for both tensile and transverse tests.

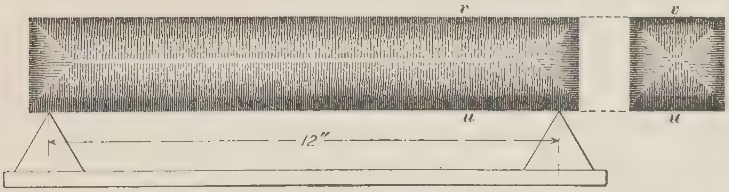


Fig. 24

In the case of a bar cast on its side, if a transverse load is applied to the surface that was uppermost in casting, the bar will sustain a greater stress than it would if the load were applied to the surface that was at the bottom when the bar was cast. The structure of a 12-inch bar cast on its side is shown in Fig. 24. The lower shell *u* is much thicker than the upper one *v*, and there will be a difference in the results of tests made with the bar in the two positions. Hence, if bars are cast on their sides, they should be tested in the same positions. This may be easily assured by casting some mark on the upper surface of the bar.

41. Testing Cast Iron for Contraction.—A knowledge of the amount of contraction of the several grades of iron will greatly assist the molder in proportioning the parts of a mold and deciding on the grade to use for a given casting. In making test bars for contraction tests, it must be remembered that here, also, the dampness of the mold affects results, and hence thin or small bars should be avoided. A difference in the moisture of two molds may cause a difference of as much as $\frac{1}{32}$ inch per foot in the contraction of the same grade of iron when cast in small bars.

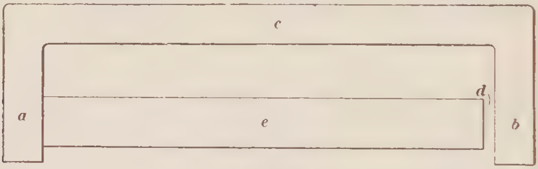


FIG. 25

One method of measuring the contraction of a test bar of cast iron is to cast the bar between the points *a* and *b* of a yoke *c*, as shown in Fig. 25. The yoke *c* is placed in the mold and a pattern for the bar is placed between the points *a* and *b*. The yoke is left in the mold during the casting operation, and the iron shrinks away from it in cooling. In the illustration, the casting *e* is shown in place in the yoke in which it was cast, and the distance *d* between the end of the bar *e* and the end *b* of the yoke indicates the amount of contraction.

CHARACTERISTICS OF MATERIALS

WROUGHT IRON AND STEEL IN TENSION

42. Elastic Limit and Yield Point.—In tensile tests of wrought iron and steel, it is found that from the beginning of the test the scale beam tends to rise continually to the yield point. At this point, the beam will suddenly fall and remain down for a short time, after which it will rise again. The load registered by the scale beam at the yield point represents the extreme limit to which the material may be loaded in actual work. The yield point usually occurs at from 60 to 70 per cent. of the ultimate strength, while the elastic limit is reached at from 50 to 70 per cent. of the ultimate strength.

If wrought iron and some grades of steel are stretched beyond the elastic limit, and the load is then removed for a short time, it will be found, on applying the load again, that the elastic limit of the material has been raised to the highest point to which the material was originally loaded; in some cases, this has been repeated until the elastic limit nearly corresponded to the ultimate strength.

43. Elongation and Reduction of Area.—As a tension specimen is pulled, it lengthens and its cross-section diminishes, so that the force applied is acting on a constantly decreasing cross-sectional area instead of on a constant area. However, since this is the condition to which materials are subjected in actual machines, the stresses are always expressed in pounds per square inch of the original area of cross-section, unless otherwise specified.

There are two distinct stages in the elongation of a tension test piece. The first stage ends with the time during which the stretching and the reduction of area are uniform throughout the whole length of the bar. The second stage ends when the ultimate strength is reached, and when the test piece begins to **neck** or contract in area at one point, this being the point at which rupture will finally occur. The elongation then occurs wholly at this point instead of being uniformly distributed, as before.

Fig. 26 will serve to make clear the nature of this action. At (*a*) is shown the original specimen, before testing. At (*b*) is shown the same piece after it has been stretched to the point of ultimate strength. At (*c*) is shown the specimen when it is about to rupture at the necked part.

The elongations of specimens having the same cross-sectional area, but of different lengths, are proportional to the lengths within the limit of ultimate strength. Beyond that, the necking is about the same for all specimens of the same area, regardless of length. Hence, the shorter pieces show a larger percentage of total elongation than the long ones, which necessitates a standard length of test specimen. The percentage of elongation indicates the ductility of the metal.

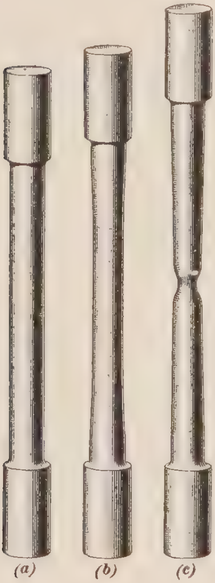

(*a*) (*b*) (*c*)

FIG. 26

44. Although the reduction of cross-sectional area is not extremely important, it is usually recorded with the other data of the test. In some of the more ductile irons, it may reach 50 or 60 per cent. of the original cross-section. The shape of the fracture and the appearance of the metal at the break are usually noted also.

CAST IRON IN TENSION

45. Elongation, Tensile Strength, and Ductility. Cast iron subjected to tensile stresses elongates very little. Good gray cast iron will stretch about $\frac{1}{8}$ inch in every 58 feet, per ton of pull per square inch, up to about one-half the breaking load. The tensile strength of cast iron may vary from 7,000 to 40,000 pounds per square inch, depending on its composition and the precautions observed in casting the bar. It possesses little or no ductility, and breaks very suddenly under the maximum load. There is no necking to indicate where the break will occur, as in the case of wrought iron and steel. The reduction of area is so small that it is scarcely measurable.

IRON AND STEEL IN COMPRESSION

46. Form of Break or Fracture.—There are two distinct ways in which test specimens of iron and steel fail when subjected to compression. In one, the metal spreads out as the load increases, so that the area of cross-section of the specimen grows greater as the piece is compressed.

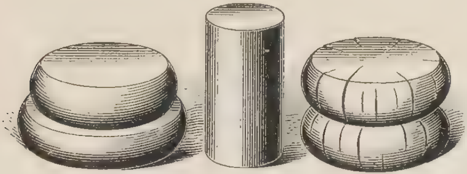

(c) (a) (b)

Fig. 27

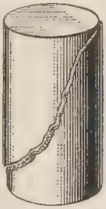


Fig. 28

This is shown in Fig. 27, in which (*a*) represents the form of the original test piece, (*b*) the final appearance of two steel specimens, and (*c*) the final appearance of two wrought-iron pieces.

In compression tests of more brittle materials, as cast iron and hardened steel, the test specimens usually fail by shearing off diagonally, as shown in Fig. 28. The fracture will occur at the maximum strength of the piece, and there will be little compression recorded by the compressometer.